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STRESS AND DEFLECTION ANALYSIS OF MECHANICALLY FASTENED JOINTS

HARRY G. HARRIS

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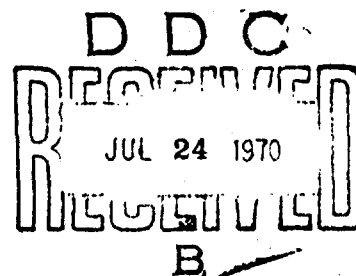
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*Grumman Aerospace Corporation
Bethpage, New York*

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FOREWORD

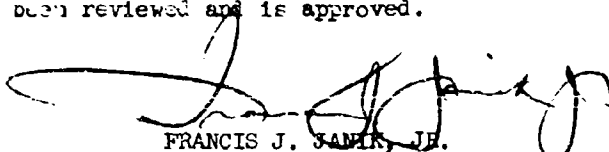
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The experimental effort was supervised by Robert Zoellner of the Elements and Materials Test Group.

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This technical report has been reviewed and is approved.



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ABSTRACT

This report presents analytical techniques for predicting both the linear and nonlinear stresses and deformations of mechanically fastened joints. The idealization used is a set of stacked parallel plates which transfer planar loads among themselves by means of transverse fasteners. The plates are treated by finite element methods of matrix structural analysis in which each element is assumed to be in plane stress for both elastic and plastic stress states. The fasteners, which are treated by short-beam theory, interact with the plates under the assumption that the plates may be represented by an equivalent elastic foundation.

Application of the present analytical techniques was made to a variety of problems including: the combined elastic-plastic behavior of plates with unloaded holes, the load-deflection behavior of single-fastener joints, the residual stress distributions in plates with squeeze rivets, the effect of fastener bending and shear deformation on the bearing stress distribution between the fastener and the plate, and the prediction of the fatigue life of typical mechanically fastened joints. In all these cases, comparisons with test results generally gave very good correlation.

Parametric studies were performed to determine the effects on stress and deflection distributions of variables such as: initial clearance or interference between fastener and hole, load level, geometry and material properties. The effect of these variables upon fatigue life of single and multi-fastener joints under realistic loadings was also assessed.

For the range of parameters studied, the effects of hole clearance and fastener interference and geometric configuration appear to play the dominant roles in determining the stress distribution and hence, the fatigue life of mechanically fastened joints.

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LIST OF SYMBOLS

a	material constant in member equation
A	fastener cross-sectional area (in ²); constant
B	constant
C, C ₁ , C ₂ C ₃ , C ₄ , C ₅ , C ₆ }	constants
d	fastener diameter (in.)
D, D ₁	diameter of hole (in.); constant
ΔD	change in diameter
e, e ₁ , e ₂	plate edge distances measured from the hole center, (in.)
E	Young's modulus (psi)
f	nominal stress, (ksi)
f _m	mean nominal stress, (ksi)
f _{max}	maximum nominal stress, (ksi)
f _{min}	minimum nominal stress, (ksi)
G	shear modulus of fastener material, (psi)
I	fastener cross-sectional moment of inertia, (in. ⁴)
k	"spring constant" or slope of the load-deflection curve (lb./in.); foundation modulus (psi)
k ₁	foundation modulus of plate layer 1 (psi)
k ₂	foundation modulus of plate layer 2 (psi)
k _{eff}	effective joint stiffness (lb/in)
K ≡ S.C.F.	stress concentration factor
k _{fp}	Stowell plastic stress concentration factor
K _N	Neuber stress concentration factor
K _T	theoretical elastic stress concentration factor
l	beam length (in); gage length (G.L.)

M	cross sectional moment (in-lb)
M_1	cross sectional moment at beam station 1 (in-lb)
M_2	cross sectional moment at beam station 2 (in-lb)
M_H	cross sectional moment at head attachment (in-lb)
N	cycles to failure
p	radius of hole or notch in Neuber equation, (in.)
P	applied load; squeeze rivet applied load, (lb.)
P_1	redundant radial load; load at a particular offset of δ_1 in the load-deflection curve, (lb.)
P_j	load at an offset of δ_j (lb.)
P_0	statically determinate radial load in contact problem, force transmitted by fastener-held joint (lb.)
P^*	by-pass load in multi-fastener joint, (lb.)
P_u	ultimate load, (lb.)
P_2	load at an offset of 0.002 inches, (lb.)
P_5	load at an offset of 0.005 inches, (lb.)
P_{12}	load at an offset of 0.012 inches, (lb.)
q	distributed plate-interaction load per unit of fastener length (lb/in)
Q	cross sectional shear; fastener transfer load in multi-fastener joints, (lb.)
S, S_w	hole spacing, (in.); hole spacing across plate width, (in.)
S_L	hole spacing along plate length, (in.)
W, W_1, W_2	plate width, (in.)
x	fastener axial coordinate, (in.)
\bar{x}	fastener axial coordinate with shift in origin, (in.)
X	cartesian coordinate
y, y_1, y_2	relative beam deflections, (in.)
Y	cartesian coordinate

β	fastener cross sectional shearing-angle rotation (radians)
$\bar{\beta}$	fastener-plate bending stiffness parameter (in^{-4})
γ	fastener-plate shearing stiffness parameter (in^{-2})
γ_1, γ_2	fastener-plate shearing stiffness parameters (in^{-2})
δ	displacement, (in)
δ_e	elastic component of displacement, (in)
δ_p	plastic component of displacement, (in)
δ_o	correction to p vs. δ curve, (in)
δ_T	overall joint deflection, (in)
δ_{T1}	total displacement corresponding to P_1 on load-deflection curve, (in)
δ_{Tj}	total displacement corresponding to P_j on load-deflection curve, (in)
Δ	denotes incremental quantity (dimensionless)
ϵ	strain, in/in
ϵ_a	strain amplitude, in/in
ϵ_{elr}	elastic strain range, in/in
ϵ_{max}	maximum strain, in/in
ϵ_t	equivalent fully reversed strain amplitude in/in
ϵ_{tr}	equivalent fully reversed strain range in/in
ϵ_1	major principal strain; strain range for initial stage of unload, in/in (Fig. 55)
ϵ_2	intermediate principal strain; strain range for final stage of unload, in/in (Fig. 55)
ϵ_3	minor principal strain, in/in
$\bar{\epsilon}$	"effective strain", (in/in)
$\epsilon^{(k)}$	uniaxial strain at k^{th} cycle, (in/in)
$\bar{\epsilon}^p$	plastic component of "effective strain", (in/in)
η	applied loading cycles

θ	angular measure around hole circumference
λ	cross sectional shear parameter (dimensionless)
ν	Poisson's ratio (dimensionless)
σ	stress, (ksi)
σ_f	stress at fracture in static test, (ksi)
σ_{\max}	maximum stress, (ksi)
$\sigma_{\max R}$	value of σ_{\max} when $E/k \rightarrow \infty$
σ_{\min}	minimum stress, (ksi)
σ_m	mean stress at edge of hole or notch, (ksi)
σ_{net}	net section stress, (ksi)
σ_0	mean net section stress due to interference, (ksi)
$\sigma_{\text{appl.}}$	net section stress due to applied load, (ksi)
σ_r	radial stress
σ_R	ratio of σ_{\max} to $\sigma_{\max R}$
σ_θ	circumferential stress, (ksi)
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
$\bar{\sigma}_N^{(k)}$	effective stress of the N^{th} element at k^{th} cycle
$\sigma_{3N-2}^{(k)} = \sigma_x^{(k)}$	for the N^{th} element at k^{th} cycle
$\sigma_{3N-1}^{(k)} = \sigma_y^{(k)}$	for the N^{th} element at k^{th} cycle
$\sigma_{3N}^{(k)} = \tau_{xy}^{(k)}$	for the N^{th} element at k^{th} cycle
$\sigma^*(k)$	relaxed uniaxial stress at k^{th} cycle
$\Delta\sigma_1$	stress range for initial stage of unload, ksi (Fig. 55)
$\Delta\sigma_2$	stress range for final stage of unload, ksi (Fig. 55)
ψ	cross sectional rotation, (radians)

Matrices:

$\{D_A\}$	allowable relative radial displacements
$\{D_B\}$	relative radial displacement caused by by-pass load
$\{P\}$	load vector
$\{P^{(k)}\}$	load vector at the k^{th} cycle
$\{\delta\}$	nodal displacement vector
$\{\delta^{(k)}\}$	nodal displacement vector at the k^{th} cycle
$\{\epsilon_1\}$	initial strain vector
$\{\epsilon_1^{(k)}\}$	initial strain vector at the k^{th} cycle
$\{\Delta_r\}$	relative radial displacement vector
$\{\sigma\}$	stresses of all elements
$\{\sigma^{(k)}\}$	stresses at the k^{th} cycle
$\{L\}$	loading conditions
$[A]$	flexibility coefficients of hole for unit radial loads
$[D]$	influence coefficients relating induced strains to nodal displacements
$[D_L]$	relative radial displacements for various loadings in statically determinate structure
$[G]$	influence coefficients relating induced strains to element stresses
$[\Delta]$	influence coefficients relating applied loading to nodal displacements
$[\Gamma]$	influence coefficient relating applied loading to element stresses.

CHAPTER 1. SUMMARY OF PRESENT EFFORT

1.1 Introduction

The accurate structural analysis of mechanically fastened joints is an important aspect in the efficient design of aircraft structures where minimum weight is a prime consideration. As a first step in such an analysis, it is necessary to know the distribution of the applied loads among the different fasteners in a multi fastener joint. Previous investigations (Refs. 1 and 2) have developed convenient, practical methods for determining such gross load distributions in multi fastener joints. However, one of the necessary input quantities for the use of such methods is the load-displacement relationship for an appropriate single fastener joint. In the studies of Refs. 1 and 2 resort to test data was made in obtaining the input information necessary for such gross analysis methods.

In addition, another input to these procedures is the proper spring constants representing the sheet material between fasteners. The spring constants involve uniformly loaded "effective widths" because of the non uniform stress fields that exist in the sheets in the vicinity of the fasteners. Here again the techniques of Refs. 1 and 2 employ semi-empirical inputs to account for the plate spring constants needed in the analysis.

The methods developed in the present study provide an alternative to the empirical approach by supplying analytical predictions of the plate-fastener spring constants for the complete range of loading up to and beyond yielding, using nonlinear planar analyses. Methods are also developed for predicting the non-planar elastic portion of the load-deflection curve of a single fastener. A more general inelastic procedure for predicting the inelastic portion of this load-deflection curve is also indicated but not implemented. The total joint deformation resulting from both plate and fastener deformations are then determined and some analytical results are compared with those obtained from a concurrent, small scale experimental program.

After the transfer loads and by pass loads for the individual fasteners are obtained by procedures such as those of Refs. 1 and 2, the peak stresses

that occur in the vicinity of the fastener holes must still be determined. These total peak stresses are particularly significant in fatigue studies. The procedures presented in this report, which employ matrix methods of structural analysis in conjunction with finite element techniques, enable the determination of these stresses in plates having either loaded or unloaded holes. Furthermore, the effects of fastener bending and shear deformations on the distribution of bearing stress between fastener and plate, through the plate thickness, are analyzed and the resulting increases in the peak bearing stress determined.

To determine the applicability of the analytical results obtained, the fatigue analysis prediction of an aluminum joint with both hole clearance and interference fit is made. Finally, a comparison is made of the analytical life prediction of an actual titanium joint with test results showing the effect of clearances for a spectrum type loading.

1.2 Methods of Analysis

1.2.1 General

A mechanically fastened joint is, in general, a highly redundant structure in which the mechanism of load transfer creates a nonlinear, three-dimensional, combined elastic-plastic stress state. Since the exact analysis of such problems is clearly beyond the present state of the art, a more modest approach, based upon justifiable engineering simplifications, is employed.

Basically, the approach taken is to create a series of two-dimensional plate idealizations which include as variable parameters the important planar effects such as material properties, geometric configuration, hole clearance and interference fit. In addition, a one-dimensional model capable of accounting for fastener shear and bending, head stiffness effects and bearing stress variation through the plate thickness is used. This component analogue to simulate the actual mechanically fastened structure is illustrated schematically in Fig. 1.

1.2.2 Two-Dimensional Problems

The planar problems are treated by the finite element method of structural analysis. Two types of problems are considered in the present study: (1) elastic-plastic analysis of plates with unloaded holes, and (2) elastic nonlinear contact problem analysis of plates with holes loaded by

rigid circular pins having arbitrary initial fit conditions. The analysis of problems of the first type is given in Chapter 2 and problems of plates with loaded holes are treated in Chapter 3

1.2.3 Three-Dimension Effects

The three-dimensional fastener-plate interaction is approximated through idealization of this system as a short beam on an elastic foundation. The fastener deformations are treated by various beam theories which include either shear or bending effects, while the plates are replaced by a continuous set of mutually uncoupled springs whose moduli are determined from the concurrent two-dimensional finite-element study described in Section 1.2.2.

1.3 Application of Present Analytical Results

1.3.1 Effective Width and Inelastic Stiffness of Plates with Holes

An elastic-plastic planar analysis capable of generating "spring constants" for the plates in multi-fastener joint analyses as described in Refs. 1 and 2 is applied to several plates with unloaded holes. The stiffness characteristics of these structures are evaluated at load levels in the elastic and plastic regions. From the planar finite element analysis the "effective width" of the plate is determined. Present results reveal that this parameter is strongly influenced by the hole spacing-to-hole diameter ratio (S/D) and to the hole spacing-to-plate width ratio (S/W). For usual hole spacing, say of the order of $4D$, it is found for the values of S/W considered, that the effective width of the plate is approximately 75 to 90 percent of the original width.

1.3.2 Prediction of Load-Deflection Behavior for Single Fasteners in Single and Double Shear

The bending and shear deflections in a fastener are evaluated separately by solving the differential equations for a beam on an elastic foundation, where the modulus of the foundation is the local spring constant of the plate. The cases considered are a fastener in antisymmetrical single shear and a fastener in symmetrical double shear. The relative magnitudes of the shear and bending deformations are estimated for several cases through numerical applications. In addition, the effects of head fixity are estimated through consideration of the two limiting cases, i.e., a rigidly clamped head and one with negligible rotational resistance.

The effects of fastener flexibility on the bearing stresses between fastener and plate through the plate thickness are examined. The increase in the bearing stress, at some plate thickness locations, above the nominal bearing stress obtained with a rigid fastener are evaluated for a range of fastener diameter to plate thickness ratios and the results presented in graphical parametric form. If shear rigidity of the plate in the thickness direction may be neglected, these local increases in bearing stress result in corresponding increases in all the plate stresses at the same plate thickness locations.

1.3.3 Fatigue Analysis Under Constant Amplitude and Spectrum Loading

The effect of fastener fit (ie. clearance and interference) on fatigue life of mechanically fastened joints is studied for conditions of constant amplitude and spectrum type loading. In general, results indicate that fatigue life is reduced for cases where there is a clearance between fastener and hole and increased where there is an interference fit. The degree of improvement or reduction in fatigue life is shown to be a function of the amount of clearance or interference in the joint.

There appears to be a fairly uniform decrease in fatigue life in going from a neat fit condition up to clearances of approximately 2% dia., after which the effect is progressively less significant. For interference fit cases, results indicate the possibility of an optimum interference for a given stress level and joint geometry. It should be noted however, that excessive amounts of interference may be detrimental to fatigue life.

1.3.4 Correlation With Experimental Results

(1) Static Tests

Overall joint flexibilities for single fasteners in double shear are determined both experimentally and analytically. A comparison of the results reveals a reasonable correlation for the modest number (15) of tests performed. Both tests and analytical predictions reveal a greater sensitivity of results with the type of fit between fastener and hole (i.e., loose or interference) rather than with the fastener material employed.

(2) Fatigue Tests

Fatigue test data for double-shear and multi fastener aluminum alloy and titanium alloy joints, which include both constant amplitude and spectrum type fatigue loading, are compared to analytical predictions based on the results of this study. Although the data exhibit a fairly large scatter, the trend to reduction in fatigue life as clearance between fastener and hole is increased is clearly demonstrated. Improvement in fatigue life is generally obtained in going from a neat fit or clearance to an interference fit, but there are optimum degrees of interference for given stress levels and joint geometries. It is notable that all the basic trends are predictable by analysis.

Limited experimental test data are also presented to show the effect of clamp-up due to installation torque on fatigue life. A multi fastener double-shear aluminum alloy joint was tested in constant amplitude fatigue for conditions of normal clamp-up and no clamp-up. Considerable increases in fatigue life are shown for the clamp-up condition.

1.4 Conclusions and Recommendations

Analytical methods for studying the complex structural three-dimensional stress and deflection behavior of mechanically fastened joints have been presented in this report. These techniques have proved to be very useful in obtaining basic data so as to gain a better understanding of the problem and to narrow down the most important parameters which affect structural joint behavior. The results of planar analyses of plates with loaded and unloaded holes have provided input information for both static load-deformation characteristic studies and fatigue life predictions of joints.

An adequate engineering method of determining the separate effects of fastener shear and bending flexibility on overall joint flexibilities and local stress distributions has been developed. For the cases considered, the effects of bending flexibility were generally less significant than the effects of shear flexibility.

Possible extensions of the techniques presented include a more refined beam theory to include the effects of fastener shear and bending simultaneously, as well as countersink head effects and clamp-up force upon overall joint compliance and stress concentration.

It is concluded from these studies that the fatigue lives of mechanically fastened joints are significantly affected by the degree of clearance or interference between the fasteners and holes. Basic trends shown in experimental joint fatigue test data have been reasonably well predicted by the fatigue analysis. It was also possible to account for certain aspects of joint fatigue behavior through a study of the elastic stress variation around plate holes, as related to nominal stress level and hole location in the joint.

It is concluded that clearances between fasteners and holes will reduce fatigue life beyond that obtained with a neat fit condition. In addition the effect on an interference fit will generally improve fatigue life, although behavior under this condition is not as predictable as in the case of clearance fits. This is due to the degree of plasticity encountered as a result of the initial interference fit. Trends shown by the fatigue analysis, backed up by limited test data, indicate the possibility of the existence of an optimum interference for a given joint geometry, and that additional interference beyond these levels may tend to reduce fatigue life.

A limited study of the effect of clamp-up in a joint has shown that fatigue life can increase significantly as a result of clamp-up. This effect can be accounted for by analysis if a reasonable estimate of the magnitude and distribution of frictional forces is made.

It is recommended that further attention be given to the prediction of stress distribution in the region of fastener holes in joints, for the conditions of clearance and interference fits. For the interference cases in particular, extension of the present work to include the effects of plasticity would result in a closer representation of the stress-strain behavior at a hole or notch. If the strain amplitude in a fatigue cycle can be estimated to a reasonable degree of accuracy, the use of strain-life data enables a good estimate of fatigue life to be made. The trends indicated by the qualitative analysis methods employed in the present study are sufficiently well defined to show the value of a continuing effort in this field.

CHAPTER 2 INELASTIC ANALYSIS OF PLATES WITH UNLOADED HOLES

2.1 Introduction

The elastic-plastic analysis of planar structures having stress concentrations caused by unloaded holes is considered in this section. To perform such analyses two computer programs (based on finite element methods) are used. The first is an elastic analysis used to generate influence coefficients relating the applied loading and plastic strains to the stresses and displacements throughout the structure. The second step in the analysis procedure is an incremental plasticity program to follow the load well into the inelastic range. For this purpose a previously available computer program designated here as the Grumman-Air Force plasticity program was modified and expanded to accommodate the present computational requirements. This program is fully documented in the present report.

2.2 Inelastic Stress and Deflection Analysis of Planar Structures

2.2.1 Basic Considerations

The Grumman-Air Force program which forms the basis of the present plasticity analyses is fully documented in Ref. 3. The theoretical background of the program and the necessary modifications to provide for an inelastic deflection analysis capability are summarized here.

The program uses the "initial strain" method of plastic analysis where plastic strains are interpreted as initial strains. The basis of the method resides in the formulation of the stiffness analysis problem as a series of linear analyses each of which involves representation of a state of initial strain in the structure. The general formulation for this type of problem can be written in matrix form:

$$\{\sigma\} = [\Gamma] \{P\} + [G] \{\epsilon_1\} \quad (1)$$

Here, the σ 's are the stress values which characterize the state of stress in the respective elements of the structure. These may be defined, in a given element, e.g., at its centroid, or at nodal points of the finite element idealization. The matrix $[\Gamma]$ relates the stresses $\{\sigma\}$ to the applied loading

{P}, on the basis of a conventional elastic matrix displacement analysis of the structure. The matrix [G] relates the stresses to the initial strains $\{\epsilon_1\}$.

A similar expression to the element stresses can be written for the generalized nodal displacements since these must be determined at each step of the loading in order that a complete analysis of a structural joint can be made. This equation takes the form:

$$\{\delta\} = [\Delta] \{P\} + [D] \{\epsilon_1\} \quad (2)$$

In this expression the values of the nodal displacements $\{\delta\}$ are computed in an analogous fashion to the stresses at each step in the load increment. The matrix $[\Delta]$ is the flexibility matrix relating displacements to applied loads {P}, and the matrix [D] relates nodal displacements to the initial strains. Both the $[\Delta]$ and [D] matrices are determined from an elastic analysis of the structure together with the matrices [F] and [G] needed for the stress analysis. These four matrices then act as inputs to the plasticity program.

In the present approach, the initial strains ϵ_1 are taken as the plastic strains ϵ^p at the specified load level. The load is applied incrementally and the above linear analyses are applied for each increment. An examination of the numerical techniques used to solve such nonlinear problems reveals two possible approaches: (1) a non iterative step-by-step calculation in which all quantities, including the initial (plastic) strains, ϵ_1 , are incremented and (2) an alternate iterative procedure for any particular load level. The first approach, which is the most straightforward, is the one used in this analysis.

Two methods for the non iterative step-by-step procedure have been suggested in Ref. 4 but subsequent evaluation of these methods (Ref. 3) indicates that only one of them is free from inherent computational instabilities. In this method the stresses in the plate are expressed by the following modified form of

$$\{\sigma^{(k)}\} = [F]\{P^{(k)}\} + [G]\{\epsilon_1^{(k-1)}\} \quad (3)$$

where k is the load increment designation. In a similar fashion the computations for the displacements can be made by rewriting Eq. 2 in the form

$$\{\delta^{(k)}\} = [\Delta] \{P^{(k)}\} + [D] \{\epsilon_1^{(k-1)}\} \quad (4)$$

where k is the same load increment designation as before. In Eqs. 3 and 4 the stresses and displacements in the k^{th} increment are expressed in terms of the initial (inelastic) strains from the $(k-1)^{\text{th}}$ increment.

The actual method used for determining the stresses and strains in the k^{th} increment is the "constant strain" method, which is illustrated in Fig. 2 for a uniaxial stress-strain case. The method proceeds as follows. One enters the k^{th} increment with applied loads $\{P^{(k)}\}$ and initial strains $\{\epsilon^{(k-1)}\}$, the latter evaluated during the preceding increment. The first operation of the k^{th} increment is to determine an approximation to $\{\sigma^{(k)}\}$ from Eq. 3 by direct substitution. Referring to Fig. 2 point A is thus determined with stress-strain coordinates $\sigma^{(k)}$ and $\sigma^{(k)}/E + \epsilon^{(k-1)}$. Since point A will probably not lie on the stress-strain curve of the material, the stress at point A is relaxed to $\sigma^{*(k)}$ (while the total strain is held constant, i.e., "constant strain"), corresponding to point B, which does lie on the stress-strain curve. Point B, then, defines the final, corrected values of stress, $\sigma^{*(k)}$, and initial strain, $\epsilon_1^{(k)}$, in the k^{th} cycle, as indicated in Fig. 2.

2.2.2 Isotropic Elastic-Plastic Analysis

In the general biaxial stress state which is of interest in the analysis of planar structures, the notion of an "effective" stress-strain relationship as shown in Fig. 3 is used in conjunction with the Von-Mises yield criterion and the Prandtl-Reuss flow relations of incremental plasticity theory.

The biaxial analysis employs the same step-by-step procedure described above, with modifications for biaxial stress states. It can be summarized in the following algorithm for the k^{th} load level:

1. Obtain the stress components at each node using the basic Eq. (3) by assuming the initial strains from the previous load level.

2. Using these stresses, calculate an "effective" stress at each node.
3. Assuming that the effective stress-strain relation for the material, modified by including the elastic strain, corresponds to data measured in a simple uniaxial tension test, determine the "effective" strain corresponding to the effective stress. Calculate the corresponding effective plastic strain and the increment in effective plastic strain over the previous load level.
4. Using the incremental flow relations, calculate the inelastic strain increments. The proportionality constant in these equations is the ratio of the effective plastic strain increment to the effective stress.

At this point in the calculation, the applied load can be incremented again and the cycle repeated. The details in implementing each of these four steps in the algorithm follows:

Step (1) When calculating the stresses $\sigma_N^{(k)}$ at the node point, N , for the k^{th} load level using the initial strains from the $(k-1)^{\text{th}}$ load level in Eq. (3), it is convenient to identify the three stress components (one shear and two normal) at the node point by means of subscripts $3N$, $3N-1$, and $3N-2$, as follows:

$$\begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix} \sigma_N^{(k)} = \begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \sigma_{3N-2}^{(k)} \\ \sigma_{3N-1}^{(k)} \\ \sigma_{3N}^{(k)} \\ \cdot \\ \cdot \end{Bmatrix} \quad (5)$$

The stresses are thus arranged in groups of three components at each node point N.

Step (2) We now calculate the corresponding effective stresses $\bar{\sigma}_N^{(k)}$ for each of the nodes from the Von-Mises expression:

$$\bar{\sigma}_N^{(k)} = \left[\left(\sigma_{3N-2}^{(k)} \right)^2 - \left(\sigma_{3N-2}^{(k)} \right) \left(\sigma_{3N-1}^{(k)} \right) + \left(\sigma_{3N-1}^{(k)} \right)^2 + 3 \left(\sigma_{3N}^{(k)} \right)^2 \right]^{\frac{1}{2}} \quad (6)$$

Note that by definition $\bar{\sigma}_N^{(k)}$ must be positive and is proportional to the octahedral shear stress. This expression together with the stress-strain data constitutes the strain hardening criterion.

Step (3) Assuming that the effective stress-strain curve ($\bar{\sigma}$ vs. $\bar{\epsilon} / E + \bar{\epsilon}^P$) is the same as the tensile stress-strain for the material of interest, we use the curve, together with the constant strain method, to determine the corrected (relaxed) value of $\bar{\sigma}_N^{(k)}$ and the corresponding effective plastic strain $\bar{\epsilon}_N^{P(k)}$. The increment in the effective plastic strain $\Delta \bar{\epsilon}_N^{P(k)}$ over that of the preceding interval will be either positive or zero, depending upon whether plastic loading or elastic unloading is taking place. Thus

$$\Delta \bar{\epsilon}_N^{P(k)} = \bar{\epsilon}_N^{P(k)} - \bar{\epsilon}_N^{P(k-1)} \quad (7a)$$

when $\bar{\sigma}_N^{(k)}$ is greater than any previous $\bar{\sigma}_N$ (inelastic strain increasing), and

$$\Delta \bar{\epsilon}_N^{P(k)} = 0 \quad (7b)$$

when $\bar{\sigma}_N^{(k)}$ is smaller than a previous $\bar{\sigma}_N$.

Step (4) The increments in the ordinary plastic strain components may now be obtained using a Prandtl-Reuss incremental relationship.

$$\Delta \epsilon_{3N-2}^{P(k)} = \frac{\bar{\epsilon}_N^{P(k)}}{\bar{\sigma}_N^{(k)}} \left[\sigma_{3N-2}^{(k)} - \frac{1}{2} \sigma_{3N-1}^{(k)} \right] \quad (8a)$$

$$\Delta \epsilon_{3N-1}^{p(k)} = \frac{\Delta \bar{\epsilon}_N^{p(k)}}{\bar{\sigma}_N^{(k)}} \left[\sigma_{3N-1}^{(k)} - 1/2 \sigma_{3N-2}^{(k)} \right] \quad (8b)$$

$$\Delta \epsilon_{3N}^{p(k)} = \frac{\Delta \bar{\epsilon}_N^{p(k)}}{\bar{\sigma}_N^{(k)}} \left[\sqrt{3} \sigma_{3N}^{(k)} \right] \quad (8c)$$

The total, ordinary plastic strain components are obtained by addition,

$$\begin{Bmatrix} \vdots \\ \epsilon_{3N-2}^{p(k)} \\ \epsilon_{3N-1}^{p(k)} \\ \epsilon_{3N}^{p(k)} \\ \vdots \end{Bmatrix} = \begin{Bmatrix} \vdots \\ \epsilon_{3N-2}^{p(k-1)} \\ \epsilon_{3N-1}^{p(k-1)} \\ \epsilon_{3N}^{p(k-1)} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \Delta \epsilon_{3N-2}^{p(k)} \\ \Delta \epsilon_{3N-1}^{p(k)} \\ \Delta \epsilon_{3N}^{p(k)} \\ \vdots \end{Bmatrix} \quad (9)$$

These components together with the new applied loads $P^{(k+1)}$ may be substituted in Eq. (3) to obtain $\sigma_N^{(k+1)}$ in the next load cycle.

2.2.3 Computer Program

The computer program which incorporates the theory described above is explained in Appendices A through D. Appendix A is essentially a user's manual for the program and describes the various options available. The logic of the program is given by means of extensive flow charts in Appendix B. The complete FORTRAN listing of the program is given in Appendix C. By necessity, the IBM 7094 version of the program provided for the Flight Dynamics Lab is small because of core limitations. Nevertheless, with a maximum number of 34 plastic nodes available, a moderate size stress concentration problem can still be run. An example of the output of the program is given in Appendix D which presents the results of a two hole aluminum plate at only three of the total of 150 load increments used in the solution up to a maximum load of 18,000 lbs.

2.3 Results and Discussion

2.3.1 Finite Element Idealizations

For the planar structural analyses of the present study, the finite element approach was used. The analysis technique, described in Section 2.2, requires an elastic analysis of the structure to be made in order to generate the four influence input matrices for the plasticity analysis. The Grumman COMAP-ASTRAL structural analysis program was used for this purpose. The details of this program will not be presented in this report but can be found in Refs.

5 and 6. Three different finite elements, available in the COMAP-ASTRAL library of elements, were examined and compared in the early stages of the study. These included the constant strain triangular element, the constant strain quadrilateral element (composed of four constant strain triangles), and a linearly varying strain triangle. The comparison of the three elements showed excellent convergence properties for the linearly varying strain triangle and somewhat slower convergence for the constant strain elements. However, the quadrilateral has a decided advantage over the triangle because more refinement in the grid size can be accomplished, for topological reasons, by using the same number of quadrilaterals as triangles for any particular structure. At the time when the work of the present effort was performed the COMAP-ASTRAL program did not have a matrix stacking procedure for the induced strain matrices and therefore considerably more data handling would have been necessary in using the linearly varying strain triangular element. For this reason, the constant strain quadrilateral element was used in all the parametric studies of this chapter. In the case of the constant strain elements the induced strain matrix reduces to a straight-forward matrix multiplication and is easily accomplished in COMAP, which is basically a matrix manipulation package.

2.3.2 Rectangular Plates

The mathematical model used to study the effective width of a plate which is part of a structural joint, as well as the effect of fastener spacing, is shown in Fig. 4. Due to symmetry, only a quadrant of the plate shown need be analyzed. Note that the grid chosen has a greater refinement near the region of high strain gradient, which for this type problem is perpendicular to the direction of the load. The number of elements used in this idealization

was 54. This is close to the maximum of 60 allowed for the IBM-360 version of the plasticity program used, in which all computations are done in core. Table 1 and Fig. 4 give the geometric properties of the six plate problems analyzed in this series. A graphical option in the input data, when using the Grumman structural analysis program COMAP-ASTRAL, permits the use of a graphical check to the input. Using this option to check the geometry and member data, plots of the six plates were made by means of an Orthomat drafting machine as shown in Figs. 5 and 6. All the plates were assumed to be of 2024-T4 aluminum alloy having the stress-strain curve shown in Fig. 7.

The output of the plasticity program includes all the nodal displacements and all the element centroidal stresses (when using constant strain elements) or nodal stresses (when using the linearly varying strain triangular element) at all increments of loading. From these results both the load-deflection characteristics and stress concentration factors of the structure can be computed. For purposes of illustration of the analytical results, the net section tangential stress distributions at various load levels are shown in Fig. 8 for plate problem P 1.1. The mean net section stress is also indicated in Fig. 8 at each level of load. The effects of plasticity are such that a leveling in the peak stresses at the hole boundary occurs as the load increases. The stress concentration factor, which is the maximum stress divided by the net section stress, decreases with load as shown in Fig. 9. Note also from this figure that the theoretical stress concentration factor is underestimated by about 20% using this particular idealization.

The load-deflection characteristics of plate No. P 1.3 shown in Fig. 10a. At any level in the loading the tangent to the load-deflection curve indicates the stiffness, k , of the structure. This has been plotted vs. applied load in Fig. 10b and shows a considerable drop in stiffness at a load level above 7 kips. Data such as the above can be readily used in the multi fastener joint analyses described in Refs. 1 and 2 which require the spring rates of the plate between fasteners as an input.

One of the objectives of the present study was the analytic determination of the "effective width" of a strap in tension pierced by several holes. The load-deflection curve obtained by finite element analysis is used to determine

the elastic spring constant, k_e , shown in Fig. 11 for the three plates with an S/W ratio of 2. Fig. 12 shows similar load-deflection results for three plates with an S/W ratio of 1. An "effective width" can be computed using the relations:

$$k_e = \frac{A_e E}{S}$$

and $A_e = (W - D_e)t = W_e t$

where,

- A_e is the "effective" cross sectional area
- E is the Young's modulus
- S is the spacing of the holes or pitch
- W is the distance between fastener lines
- D_e is the "effective diameter"
- W_e is the "effective width"
- and t is the plate thickness

Using the values of k_e obtained from Figs. 11 and 12, values of the "effective width" were computed. The results are shown in Fig. 13 for the six plates analyzed. It should be noted from Fig. 13 that as the fastener spacing to diameter ratio (S/D) decreases, the "effective width" decreases rapidly. At the "standard" spacing of 4D for example, the value of the "effective width" is between 75 and 90 percent of the original width for the two S/W ratios shown. The corresponding value of D_e for an S/D of 4 is 0.38D for S/W = 1 and 0.60D for S/W = 2. This appears much lower than the 0.80D recommended by McCombs et. al. (Ref. 1) as a semi-empirical value.

2.3.3 Plates Tapered in Width

A series of four tapered plate analyses were performed in order to study the effect of taper in width on the stiffness characteristics of the plate. A summary of the geometric properties of the plates is given in Table 2. The idealization used on this symmetric structure is shown in Fig. 14 using constant stress quadrilateral elements. Orthomat drawings of the four problems were made to check the geometry and member data input. These are shown in Figs. 15 to 18. The four tapered plates are assumed to be made of 2024-T4 aluminum alloy and have the stress strain behavior shown in Fig. 7. No

yielding is assumed below a stress level of 40,000 psi. The stress distribution across the two net sections of plate P 3.1A, as a function of applied load, is shown in Fig. 19 where the mean net section stress is also indicated. In Fig. 20 the variation of the stress concentration factor with increasing applied load is shown. The same decreasing effect with increasing load mentioned in connection with plate P 1.1 (Fig. 9) is obtained in this case also. The load-deflection characteristics of the four plates are summarized in Fig. 21. The effect of decreasing taper and the resulting decrease in net-section area shows a substantial reduction in the elastic modulus, k_e , as the taper is decreased from 1:4 (P 3.1B) to 1:16 (P 3.3B) the k_e drops from 432,000 lb/in to 343,000 lb/in. Plate P 3.3B has a relatively low carrying capacity compared to the other three. An examination of the development of yield zones with increasing load as shown in Fig. 21 will explain this behavior. At a load level of 5,200 lb. the yielded elements have transversed the whole net section of the edge holes with the resulting reduction in stiffness. The strain hardening modulus of 2024-T4 aluminum alloy is taken to be 360,000 psi (Fig. 7), which is approximately 30 times smaller than the elastic modulus. Hence, when the net section elements have yielded, the overall stiffness values will be very low compared to the elastic range stiffnesses.

CHAPTER 3 PLATES WITH LOADED HOLES

3.1 Nonlinear Contact Problem - Elastic Analysis

3.1.1 Introduction

In a typical multi-fastener joint under load the stress distribution around a particular hole is very complex and depends on a variety of geometric, material and manufacturing parameters. This chapter treats the problem of plates loaded by fasteners having either initial clearance or interference. A basic assumption in the analyses to be presented is that the stress distribution across the plate thickness is constant. This restraint will be relaxed in Chapter 4 which analyzes the three dimensional effects causing a non-uniform distribution across the plate thickness.

The load transfer from one plate of a joint to another through a fastener creates a nonlinear contact problem. The reason is that as the load in the joint changes the area of contact between the plate and the fastener changes and hence there is gradual load redistribution. The amount of redistribution is a function of the load level and the initial amount of clearance or interference between the fastener and the hole. Frictional effects also play an important role on the area of contact but for the present analysis these are assumed negligible so that the load direction at any point on the contact surface is always assumed to be radial in direction. Only radial compressive loads are permitted in the analysis.

3.1.2 Analytical Model

The mathematical model chosen in order to study the local effects of the load transfer and by-pass load around a single fastener is shown in Fig. 23. This analysis assumes that a region around the fastener can be isolated from the rest of the joint (Fig. 23a) in which the stresses influenced primarily by the fastener are localized. Another assumption is that the plate structure is symmetric and is loaded symmetrically as shown in Fig. 23b. The problem is one where on a specified number of contact points around the periphery of the hole the redundant radial reactions must be determined for a given load level and a particular initial fit condition.

Using the notation shown in Fig. 23b an expression can be written for the relative radial displacements between the plate and the rigid

fastener. This expression takes the form:

$$\{\Delta_r\} = [A] \{P\} + [D_L] \{\mathcal{L}\} + \{D_A\} + \{D_B\} \quad (10)$$

where,

- $\{\Delta_r\}$ are the relative displacements or "gaps" that exist between the plate and the fastener,
- $[A]$ is the flexibility matrix for unit radial load,
- $\{P\}$ are the redundant radial loads which exist on the contact surface,
- $[D_L]$ are the relative radial displacements that exist in the statically determinate structure ($P_i = 0$) for various loading conditions, $\{\mathcal{L}\}$,
- $\{\mathcal{L}\}$ load levels or loading conditions,
- $\{D_A\}$ set of relative radial displacements caused by initial clearance or interference between the fastener and the plate hole,
- $\{D_B\}$ relative radial displacements in the statically determinate structure caused by the applied by-pass load, P^* .

The condition for obtaining the redundants P is to set the relative radial displacements to zero.

$$\{0\} = [A] \{P\} + [D_L] \{\mathcal{L}\} + \{D_A\} + \{D_B\} \quad (11)$$

$$\text{Thus, } \{P\} = -[A]^{-1} \left\{ [D_L] \{\mathcal{L}\} + \{D_A\} + \{D_B\} \right\} \quad (12)$$

If any of the P 's are positive (tensile) these are put to zero. By a trial and error procedure the redundants are computed such that all are compressive and the computed relative radial displacements create a compatible deflected condition between the plate and the fastener. The details of the various steps used to generate the matrices needed for solving the contact problem are given in Appendix I.

3.1.3 Computer Program

The iterative solution of Equation 12 is accomplished by means of the nonlinear contact program documented in Appendices E through H. Appendix E

explains the use of the program. Extensive flow charts of the program are provided in Appendix F and the FORTRAN listing is given in Appendix G. A sample problem output is shown in Appendix H for illustrating the form of the output. The case run is the aluminum plate Problem P 2.3 with fit condition C2 which is equivalent to a $\frac{\Delta D}{D}$ of 0.4 percent. The initial lack of fit indicated in the program refers to the percentage based on the radius of the hole, not the diameter. Once the components of the radial loads are obtained as illustrated in Appendix H for each load level, these are then applied to the plate structure to determine the elastic stress and displacement distribution throughout.

3.1.4 Effect of Load Level and Fit on the Elastic Stress Distribution

(1) Aluminum Plates

Using the analysis techniques described in Section 3.3.1 and Appendix I, four Aluminum joints loaded by means of a single rigid fastener were studied. The geometric properties of these are given in Table 3. Orthomat drawings, used as a means of checking the input, were made and are shown in Figs. 24 and 25. Problem P 2.1 has the same geometry as the double shear single fastener joints used in the test program and described fully in Chapter 7. Results of the analysis to be presented in this section coupled with the three dimensional effects caused by fastener bending and shear given in Chapter 4 will be used as a comparison of the present analysis with the experimentally determined load deflection curve.

The effects of load level and initial fit on the load distribution at the contact surface between the fastener and the plate are illustrated in Figs. 25 to 28 for plate P 2.1. In these figures the loads on the rigid fastener are drawn vectorially to scale for three different load levels: 500 lb., 2000 lb. and 8000 lb. applied load. It should be noted that the loads acting on the plate structure are equal and opposite to those shown in these figures. For the initial lack-of-fit cases it can be seen that as the load increases the fastener deforms into the plate with the consequence that more and more points around the periphery come into contact. The effect of interference fit on the load distribution is to create a pre-compression on the fastener or a pre-tension or hoop stresses around the periphery of the hole. As the load of the fastener is increased the initial load distribution changes

and for the smaller values of interference ($\frac{\Delta D}{D} = 0.004$ and 0.02) the applied load is enough to pull the fastener away from the top of the hole (Fig. 28).

The variation of the maximum tangential stresses at the net section at different load levels is illustrated in Figs. 29 to 31. Fig. 29 gives the maximum tangential stress distribution for a clearance of $\frac{\Delta D}{D} = 0.004$ of plate P 2.1 and Fig. 30 gives the distribution for a clearance of $\frac{\Delta D}{D} = 0.02$.

The variation of tangential stress at a section $\theta = 81^\circ$ for four different interference fits is illustrated in Fig. 31. These results although obtained from an elastic analysis can still be very useful for generating stress concentration factors needed in a fatigue analysis. The application of these results to fatigue analysis of typical joints is illustrated in Chapter 5. The stress concentration factors vary with amount of clearance and level of load as illustrated in Figs. 32 to 34 for problem P 1.1 to P 1.3 respectively. Note from these figures that the clearance increases the stress concentration factor substantially and hence it may be anticipated that the fatigue life of the joint will consequently be decreased.

In the case of interference fit fasteners the stress concentration factor, as defined in this study (maximum stress divided by net section stress), must be modified. At zero applied load a substantial tensile tangential stress can exist at the edge of the hole depending on the magnitude of the interference and the position around the hole. The method of defining stress concentration factors for interference fit fastener was to use the net section stress, σ_{net} , as follows:

$$\sigma_{net} = \sigma_0 + \sigma_{appl.} \quad (13)$$

where,

σ_0 = mean net section stress at zero applied load

$\sigma_{appl.}$ = mean net section stress corresponding to the applied load

Using this approach the modified stress concentration factor for plate P 2.3 with an interference fit $\frac{\Delta D}{D} = 0.004$ was computed at six different cross sections and plotted vs. increasing applied net section stress in

Fig. 35. The results of Fig. 35 indicate that for a net section stress below 25 ksi the critical section is at 9° to the direction of load. As the load increases, however, the critical section rotates by 90° . This phenomenon can be explained by the fact that at a net section stress above 25 ksi the fastener has completely overcome the radial clamping effect of interference and is beginning to pull from the top of the hole. This has also been observed experimentally in Ref. 7 .

For the same plate a ten times larger interference ($\frac{\Delta D}{D} = 0.04$) indicates similar trends but at much higher net section stresses as shown in Fig. 36. A plot of maximum modified stress concentration factor vs. applied net section stress is shown in Fig. 37 for the same plate with three different interference fits.

Another important aspect of the plate with a single loaded fastener analysis is the use of the plate "spring constant" as input to the study of three dimensional effects (Chapter 4). The load-deflection curves of two of the aluminum joints are shown in Figs. 38 and 39 for all the different initial fit conditions indicated.

(2) Titanium Plates

The problem of a two fastener titanium joint in double shear was also analyzed using the contact problem program. The dimensions of the outer plates of the joint are shown in Fig. 40 together with the finite element model used in the analysis. Results of the effect of different clearances on the stress concentration factor are presented in Fig. 41 with increasing net section stress. The effect of load on the stress concentration factor with increasing load is not as large as in the case of the aluminum joints (Figs. 32 to 34) which had a different geometry. The results of Fig. 41 were used in Chapter 5 to compare the analytical fatigue life predictions with actual spectrum test results. The comparison of these is shown in Fig. 61 and the experimental work pertaining to the titanium joint is described in Section 7.2.2.

3.2 Elastic and Plastic Interference Stresses in Plates With Squeeze Rivets

3.2.1 Introduction

In order to increase the fatigue life of joints, a variety of techniques have been proposed including the use of interference fit fasteners. Several commercial fasteners using this technique have now become available. The difficulty in using some of these fasteners, however, is the need for special drills and tools for their installation.

A cheaper alternative is to squeeze a relatively soft rivet into the joint hole under a large enough force to obtain appreciable plastic flow in the rivet with enough radially outward bulging of the rivet to produce yielding in a substantial region of the plate surrounding the hole.

When the squeeze force is released, the rivet and plate spring back, with the radial spring-back of the plate tending to exceed that of the rivet at the hole boundary. The result is to create an interference fit between the rivet and the plate and a state of residual stresses in the plate which some test results indicate gives a beneficial effect on the fatigue life of the joint.

This study was undertaken in order to study the effect of material selection and squeeze force on the resulting residual stresses around the rivet hole.

3.2.2 Method of Analysis

As a first step in the solution procedure, a plastic analysis of the plate, into which the rivet is squeezed, is performed. The specific analyses in this section were made using a finite element plasticity program developed for the government by Grumman and presented in Refs. 8 and 9. Basically the initial strain approach used in the inelastic stress and deflection analysis program described in Chapter 2 is used in this program also. However, the program has additional capabilities in being able to handle complete stress reversal into the plastic range (not merely elastic unloading). In addition it can account for the Bauschinger effect, which in the case of uniaxial stress is characterized by a reduction in compressive yield stress due to prior yielding in tension and vice versa, and is thus significant for unloading and reversed loading situations.

The idealization in the analysis is shown in Fig. 42 . The axisymmetric model, representing one quarter of the structure is composed of 144 planar triangular elements in which stress and strain vary linearly.

The rivet is assumed to be a cylindrical structure which fits neatly into the plate hole prior to squeezing under high pressure. During squeezing it is assumed to undergo plastic deformations based on the deformation theory of plasticity. In Ref. 10 both incremental and deformation plasticity theories were used to study a shear lag problem in which the answers obtained by the two theories agreed very well. The rivet and plate structures are coupled by means of a semi-graphical technique described fully in Ref. 11. Basically the method combines the stress strain behavior of the two structures at the point of contact to obtain a compatible solution during maximum squeezing or subsequent unloading.

3.2.3 Results of Analysis and Discussion

(1) Aluminum Plate with Aluminum Rivet

An elasto-plastic analysis was made of a 3/8 in. thick aluminum plate made from 2024-T351 alloy. A 3/8 in. diameter rivet of the same material was squeezed with a 20 kip force. The objective of this study is to determine the residual stress distribution in the plate when the squeeze force is removed.

The finite element solution for the idealization shown in Fig. 42 is presented in Fig. 43 together with the solutions of the coupled structure (plate and rivet). Fig. 43 shows the variation in the circumferential stress, σ_θ , at the hole boundary with increasing applied radial stress, σ_r , at the hole boundary during loading and subsequent unloading from several values of σ_r . The corresponding positions of the yield boundary for each of these radial stresses is shown in Fig. 44.

It is seen from Fig. 43 that yielding occurs when the radial stress exceeds 23,000 psi. As the radial stress increases beyond 23,000 psi the circumferential stress first shows a relatively small increase and then begins to decrease. If the applied radial stress

is then reduced, the plate first unloads elastically, along lines parallel to the original elastic loading. Fig. 43 shows the straight line elastic reduction in circumferential stress. As the unloading proceeds, the initially tensile circumferential stresses change sign and become compressive. Further unloading can cause these circumferential compressive stresses to become large enough to result in yielding opposite to that which occurred during loading. The effect of yielding during loading on yielding in the opposite direction during unloading is accounted for by assuming an ideal Bauschinger effect. Still further unloading causes additional yielding, with a consequent reduction in the magnitude of the circumferential stress at the hole boundary as shown in Fig. 43.

A plot of the variation of circumferential stress with distance from hole edge is shown in Fig. 45 for the peak squeeze force condition ($\sigma_r = 53.2$ ksi) and two levels of unloading. The curve for $\sigma_r = -38.5$ ksi is the solution taking into account the plate-rivet interference. The curve for complete unloading to $\sigma_r = 0$ and the rivet removed, including the Bauschinger effect, is also shown.

(2) Titanium Plate with Steel Rivet

Residual stresses were evaluated for the case of an annealed titanium plate (Ti-6 Al-6V-2Sn) into which a 3/8 in. A286 steel rivet was squeezed with a 66 kip force. The plate has the same configuration as the aluminum plate in the previous problem. The variation of the circumferential stress at the edge of the hole with increasing radial stress and subsequent unloading from 3 levels of radial stress is shown in Fig. 46. The yield surface boundary for these same levels of radial stress is shown in Fig. 47.

The relationship between applied squeeze force and the residual circumferential and radial stresses at the hole boundary is shown in Fig. 48 (a) and (b) respectively. Two curves are shown in each case to account for the fact that the applied squeeze force is not known exactly since all the force is not transmitted to the shank of

the rivet. In the present analysis it is assumed that the axial stress in the rivet shank is somewhere between one-half to two-thirds of the nominal squeeze stress, P/A . This assumption appears to be reasonable in the case of protruding head rivets.

In Fig. 49 the computed interference fit, ΔD , (the change in the rivet diameter) is shown as a function of the squeeze force for the same two assumed values of the axial rivet shank stress used in Fig. 48. Since the interference has a direct relation to the beneficial effects gained in improving fatigue life, a plot such as Fig. 49 can be extremely useful. From it, the necessary squeeze force to obtain a given interference can be read directly.

The computed interference for the titanium plate shown in Fig. 49 is compared to some experimental results obtained from the Grumman Corporate Titanium Program. The experimental values were obtained by measuring the diameter of the rivet before squeezing and after the load is removed. To get the final change in diameter, the rivets were cut free of the plate and measured at various locations along their shank. These experimental results cannot be compared directly to the computed curves of Fig. 49 because they were obtained using countersunk rivets that were squeezed into a Ti-6Al-4V annealed plate. Nevertheless, they do indicate a trend similar to that of the analytical results.

3.2.4 Conclusions

The nature of the plate circumferential residual stresses caused by squeeze rivets can change depending on the magnitude of the squeeze force and the type of rivet material used. Interference fit fasteners, however, generally create tensile tangential residual stresses by expanding the hole into which they are applied.

The results shown in Fig. 48 for the titanium plate problem can be very useful in controlling the squeezing force during manufacture. For the particular materials and geometric properties chosen, Fig. 48 gives the range

of expected circumferential and radial residual stresses in the plate at the hole boundary. These stresses can be used in predicting the fatigue life of the joint.

From Fig. 49 it is seen that the method of analysis can be a valuable aid in predicting the amount of interference to be expected. By correlating the interference to the joint fatigue life through testing, the results of Fig. 49 could be used to indicate appropriate values of squeeze force needed to achieve the desired interference fit.

It is concluded from this study that the squeezing technique can be useful in increasing the fatigue life of joints and further correlations with fatigue tests should be made. Of particular interest would be a test program using protruding head rivets to measure the interference fit caused by different values of the squeeze force. The protruding head rivet with a cylindrical shank of constant cross-section corresponds closer to the three dimensional model used in the present analysis. Attempts to evaluate the actual rivet shank stress during the driving operation should also be made.

CHAPTER 4. THREE DIMENSIONAL EFFECTS CAUSED BY FASTENER BENDING AND SHEAR - (LINEAR ANALYSIS)

4.1 General

Because of the extreme complexity of the general nonlinear three dimensional mechanical fastener problem, the preceding analyses were directed towards obtaining strictly planar solutions. These planar solutions might be considered applicable to the plates in a symmetrical double-shear joint, in which the nominal stress is uniform through the plate thickness. The planar solutions might also be assumed to apply to individual thin layers in single-shear joints (in which the nominal stress varies linearly through the plate thickness) provided that the layers are chosen sufficiently thin to put each individual layer in a planar stress condition.

In obtaining the planar solutions the effects of fastener shear and bending deformation and fastener rotation were assumed secondary while the fastener was taken as both rigid and cylindrical. In addition many other effects such as fastener head shape, countersink, clamp-up force, and contact friction were also neglected. In this section, the influence of fastener deformation on stress distribution through the plate thickness and overall joint load deflection characteristics are estimated by means of an engineering-type approximation. In this treatment, the fastener is assumed to act as a short beam with either shear or bending deformations present, but not both existing simultaneously.

The fastener-plate interaction is idealized as a beam resting on a continuous set of mutually uncoupled springs such that the bearing forces per unit of axial length are directly proportional to the fastener's transverse, centerline displacements. The assumption that the springs are not coupled is equivalent to assuming that the plate's transverse shear stiffness may be neglected, and different layers in the plate slide past each other freely.

Solutions for the load distribution of a fastener in both single and double shear are obtained for the linear spring rates obtained in Chapter 3. A more detailed nonlinear analysis of the current beam-on-elastic-foundation idealization is presented in Appendix J. This more detailed analysis outlines how one may include combined shear and bending effects simultaneously, both nonlinear

and nonuniform spring rates, as well as nonuniform beam effects (such as those arising from tapered shanks or countersunk heads).

Numerical results are presented for the simplified linear beam bending and shear theories with clamped and free conditions at the fastener heads. The ratio of maximum bearing stress to nominal bearing stress through the sheet thickness is plotted as a function of fastener-diameter to sheet-thickness ratio for several joint configurations. In addition, experimentally determined joint load-deflection curve data are compared with the analytical predictions based upon the method of this chapter.

4.2 Structural Idealization

For many engineering applications, the problem of a beam interacting with an elastic medium can be treated by adopting the Winkler hypothesis which states that the local transverse beam displacement is directly proportional to the local load intensity per unit of length. The loads and displacements are related through the "foundation modulus", k . The classical treatment of the problem, in which only beam bending effects are included, is discussed in many texts (e.g. Refs. 12 and 13). In a more in-depth treatment of this subject, Hetenyi, Ref. 14, demonstrates that its applicability to foundations with shear continuity, such as the plates comprising our joints, is greater with increasing h_{fastener}/k stiffness ratio and that it yields results which are closer to the more accurate three dimensional case (i.e. beam on a three-dimensional half-space) than does a two-dimensional (i.e. beam on a two-dimensional half-space) elasticity solution.

Timoshenko, Ref. 12, distinguishes among three groups of beams which include bending effects only: "short" ($\bar{B}l < 0.6$), "medium" ($0.6 < \bar{B}l < 5.0$) and "long" ($\bar{B}l > 5.0$) where l is the beam length and

$$\bar{B}^h = \frac{k}{4(EI)_{\text{beam}}}$$

For short beams, which correspond to the present idealization of the fastener, Timoshenko claims that bending can be entirely ignored and that the beam can be considered as absolutely rigid in comparison with the foundation. However, it is well known that shear effects are generally more

important than bending effects for short beams and so a higher order beam theory* should be considered for such applications. Crandall, Ref. 15, Essenburg, Ref. 16, and Hess, Ref. 17, have treated various more accurate beam theories in conjunction with a Winkler foundation. Their results show that for softer foundations, $E/k > 1$ (which corresponds to the fastener problem), shear effects in the beam decrease in importance and the behavior of the beam approaches that predicted by elementary beam theory.

The present section attempts to estimate some of the higher order effects associated with fastener flexibility. To achieve this, two simplified beam theories which separate the bending and shear effects in the fastener, are used. In concluding these preliminary remarks, we note that the remarkably wide range of practicability of the elementary beam formulas stems from the fact that these formulas provide excellent approximations to the elasticity solutions in a large number of problems, Ref. 18.

4.3 Beam Equations

The most elementary and useful beam idealization which includes the effects of both bending and shear is called the Timoshenko beam, Ref. 19. In a manner similar to that used in elementary beam theory, this theory assumes plane sections before loading remain plane after loading. However, unlike simple bending theory, the normal to the beam center line, at any given section, before loading, will differ from the normal to the deflected center-line, after loading, by a shearing angle, β , caused by the local shear, Q . The relationship is expressed by

$$\frac{Q}{\lambda GA} = \beta \quad (14)$$

where G is the material's shear modulus and A is the beam cross-sectional area.

* The various technical beam theories are engineering approximations to the mathematically precise theory of linear elasticity. The technical theories which include shear-deflection, excluded in elementary beam theory, are generally referred to as "higher-order" beam theories.

The parameter λ is dependent on the geometry of the beam cross-section and relates β to the nominal average shearing angle of the cross-section. For circular cross-section beams, Cowper, Ref. 20, gives

$$\lambda = \frac{6(1+\nu)}{7+6\nu} \quad (15)$$

where ν is Poisson's ratio for the beam material.

The linear equilibrium equations for a beam element, regardless of which beam theory is used, (reference Figure 50a)

$$\frac{dM}{dx} + Q = 0 \quad (16)$$

$$\frac{dQ}{dx} + q = 0 \quad (17)$$

where M is the internal moment, q is the external loading per unit of length, and x is the axial coordinate. For a Winkler foundation with spring modulus k we have

$$q = -k y \quad (18)$$

where y is the transverse deflection relative to a rigid portion of the foundation. The usual linear beam hypothesis of plane section behavior yields

$$\frac{M}{EI} = \frac{d\psi}{dx} \quad (19)$$

where ψ is the rotation a normal to the beam center line caused by loading (reference Figure 50b). If β were zero, ψ and $\frac{dy}{dx}$ would coincide.

Hence, from purely kinematic considerations

$$\beta = \frac{dy}{dx} - \psi \quad (20)$$

Combination of Eqs. (14) through (20) yields the appropriate Timoshenko beam equations for interaction of a circular cross-section beam with a Winkler foundation. It should be noted that up to this point no assumptions as to property variation along the beam have been made. Therefore, to further simplify the problem, the assumption is made that EI , k and GA are independent of x (the more general case is discussed in Appendix J). The resulting equation is:

$$\frac{d^4 y}{dx^4} - \frac{k}{\lambda GA} \frac{d^2 y}{dx^2} + \frac{ky}{EI} = 0 \quad (21)$$

Two simplified cases, the first corresponding to the classical beam theory in which shear effects are ignored, and the second corresponding to pure shear theory in which bending effects are ignored, are considered below.

4.3.1 Classical Beam Bending Theory: Ignoring shear deformations (i.e. $\beta = 0$) is equivalent to setting the shear flexibility $(\lambda GA)^{-1}$, to zero in Eq. (14). Once this is done, the classical beam on elastic foundation equations result:

$$\Psi = \frac{dy}{dx} \quad (22a)$$

$$M = EI \frac{d^2 y}{dx^2} \quad (22b)$$

$$\frac{d^4 y}{dx^4} + \frac{k}{EI} y = 0 \quad (22c)$$

The general solution to Eq. (22c) takes the form

$$y = \cosh \bar{\beta} x (A \cos \bar{\beta} x + B \sin \bar{\beta} x) + \sinh \bar{\beta} x (C \cos \bar{\beta} x + D \sin \bar{\beta} x) \quad (22d)$$

where the values of A , B , C and D are determined by boundary conditions on the beam and the solution for Ψ may be obtained by differentiation of the solution as indicated by Eq. (22a).

4.3.2 Shear Beam Theory: In a manner similar to that used in eliminating shear effects, beam bending deflection may be eliminated by setting the bending flexibility, $(EI)^{-1}$, to zero. Eq. (19) gives:

$$\frac{d\Psi}{dx} = 0 \quad (23a)$$

Differentiating Eqs. (14) and (20), and substituting into Eqs. (17) and (18) gives:

$$\frac{d^2 y}{dx^2} - \frac{k}{\lambda GA} y = 0 \quad (23b)$$

The general solution to Eq. (23b) is

$$y = E \sinh \gamma x + F \cosh \gamma x \quad (23c)$$

where

$$\gamma = \sqrt{\frac{k}{\lambda GA}}$$

and E and F may be determined by again satisfying the boundary conditions. The solution for Ψ is readily obtained by integration of Eqs. (16) and (23a) between limits x_1 and x_2 , and substitution of Eqs. (14) and (20) to yield:

$$M_1 - M_2 = \lambda GA ((x_2 - x_1) \Psi - y_2 + y_1) \quad (23d)$$

where,

$$M_1 = M(x = x_1), \quad y_1 = y(x = x_1), \quad 1 = 1, 2$$

Particular solutions of the beam bending and shearing cases are given in the following subsections.

4.4 Fastener in Single Shear

4.4.1 Fastener with Negligible Head Stiffness

To demonstrate typical results from application of the above idealization to a fastener in single shear for an anti-symmetrical structural joint (see Figure 51(a) *, several cases are investigated below.

(1) Bending Theory: The boundary conditions associated with Eqs. (21) are

$$Q(0) = P_0, \quad M(0) = 0, \quad Q(t) = 0, \quad M(t) = 0.$$

The resultant bearing load distribution on the plate (and fastener) is

$$-\frac{ky}{P_0/t} = \frac{qt}{P_0} = 2\beta t \left\{ \frac{\sinh \beta t \cos \beta x \cosh \beta(t-x) - \sin \beta t \cosh \beta x \cos \beta(t-x)}{\sinh^2 \beta t - \sin^2 \beta t} \right\} \quad (24)$$

(2) Shear Theory: Applying the same boundary conditions as for the bending theory to Eqs. (23) yields

$$-\frac{ky}{P_0/t} = \frac{qt}{P_0} = -\gamma t \left\{ \frac{\sinh \gamma x + \sinh \gamma(t-x) - \gamma t \cosh \gamma(t-x)}{\gamma t \sinh \gamma t + 2(1 - \cosh \gamma t)} \right\} \quad (25)$$

(3) Rigid Fastener: For a rigid beam, force and moment equilibrium considerations yield

$$-\frac{ky}{P_0/t} = \frac{qt}{P_0} = 2\left(2 - 3 \frac{x}{t}\right) \quad (26)$$

It should be noted that both Eqs. (24) and (25) can be shown to approach Eq. (26) in the limit as the fastener becomes essentially rigid compared

* Note, that although the springs in Figure 51 are pictured as compression springs acting at the bearing surface of the fastener, the spring constant is actually dependent upon the overall elastic properties of the plate in a complex state of stress involving both tension and compression.

to the foundation. This result may be achieved through consideration of the series-expansions of the cosh and sinh functions, letting γ and \bar{P} approach zero, and performing appropriate limiting processes.

Numerical results, corresponding to Eqs. (24) through (26), for a typical set of parameters are presented in Table 4.

As can be seen from Table 4, for the realistic parameters employed, the fastener performs in an essentially rigid manner and, both shear and bending effects, acting separately, cause only a slight perturbation upon this behavior. However, shear flexibility is the more important factor of the two since its effects are an order of magnitude greater than those due to bending for the parameters used in Table 4. Therefore, in the subsequent solution for the clamped head fastener in single shear, below, principal attention will be given to the solution of the shear beam theory, since it is anticipated that bending effects are negligible.

4.4.2 Fastener With Clamped Head

It should be noted that the assumed highly-flexible-head performance of the previous configuration permits an overall cocking action of the fastener relative to the plate, provided the plate has negligible transverse shear stiffness (as was initially assumed). However, if the fastener has stiff head-attachments which tend to clamp the plate locally, then the fastener is prevented from rotating at the ends relative to the plate. Thus, the limiting case becomes $\Psi = 0$ at the ends of the fastener. In addition, the head is assumed to exert no net shear upon the fastener. Under these conditions, the resultant bearing load distribution on the plate and fastener using shear beam theory is:

$$-\frac{ky}{P_0/t} = \frac{qt}{P_0} = \gamma t \frac{\cosh \gamma(t-x)}{\sinh \gamma t} \quad (27)$$

and the corresponding rigid fastener solution is $qt/P_0 = 1$. Results are presented in Table 5 for the same typical parameters as were used in Table 4.

4.5 Fastener in Double Shear

The idealization considered for the fastener in double shear (Reference Figure 52) is to divide the top half of the fastener into two shorter beams joined by appropriate compatibility conditions. For the symmetrical case, the shear beam theory yields Ψ as constant (from Eq. (19)). Therefore, for the symmetrical case, $\Psi = 0$ regardless of head effects.

The solution for this case is obtained through solution of Eqs. (23) together with the boundary conditions:

$$Q(0) = 0, \quad Q\left(\frac{t_1}{2}\right) = P_0/2, \quad Q\left(\frac{t_1}{2} + t_2\right) = 0,$$

$$\text{and is given as} \quad \frac{q t_1}{P_0} = \gamma_1 t_1 \frac{\cosh \gamma_1 x}{2 \sinh\left(\frac{\gamma_1 t_1}{2}\right)} \quad 0 \leq x \leq \frac{t_1}{2} \quad (28a)$$

$$\frac{q(2t_2)}{P_0} = -\lambda_2 t_2 \frac{\cosh \gamma_2 \left(\frac{t_1}{2} + t_2 - x\right)}{\sinh \gamma_2 t_2} \quad \frac{t_1}{2} \leq x \leq \frac{t_1}{2} + t_2 \quad (28b)$$

$$\text{where} \quad \gamma_1 = \sqrt{\frac{k_1}{\gamma GA}}, \quad \gamma_2 = \sqrt{\frac{k_2}{\gamma GA}}$$

The corresponding bending solution with free and clamped heads is obtained from the general solutions

$$y = C_1 \cos \bar{\beta}_1 x \cosh \bar{\beta}_1 x + C_2 \sin \bar{\beta}_1 x \sinh \bar{\beta}_1 x \quad 0 \leq x \leq \frac{t_1}{2} \quad (29a)$$

and

$$y = \cos \bar{\beta}_2 \bar{x} (C_3 \cosh \bar{\beta}_2 \bar{x} + C_4 \sinh \bar{\beta}_2 \bar{x}) + \sin \bar{\beta}_2 \bar{x} (C_5 \cosh \bar{\beta}_2 \bar{x} + C_6 \sinh \bar{\beta}_2 \bar{x}) \quad \frac{t_1}{2} \leq x \leq \frac{t_1}{2} + t_2 \quad (29b)$$

$$\text{where} \quad \bar{x} = \frac{t_1}{2} + t_2 - x;$$

The constants C_1 and C_2 may be determined through the boundary conditions

$$M(0) = \frac{P_0(t_1 + t_2)}{4} \quad \text{and} \quad Q\left(\frac{t_1}{2}\right) = \frac{P_0}{2}$$

The constants C_3 through C_6 may be determined by the boundary conditions $Q(\bar{x}=0) \neq 0$, and $M(\bar{x}=0) = 0$ (for a very flexible fastener) or $\Psi(\bar{x}=0)=0$ (for a rigidly clamped fastener head), and compatibility conditions on Q and Ψ at $x = \frac{t_1}{2}$.

Typical numerical results using Eqs. (28) and (29) are presented in Table 6. It should be noted that although the shear theory produces greater fastener flexibility effects, the difference from the bending theory results are far less pronounced than was the case for the fastener in single shear. This indicates that fastener bending is as important a factor as shear is for such configurations.

4.6 Parametric Study:

The ratio of peak bearing stress for a flexible fastener versus peak bearing stress for a rigid fastener may be employed as a stress concentration factor to account for variations in bearing load through the plate thickness. A series of bearing stress ratio curves as functions of fastener diameter to plate thickness ratio are presented in Figures 53 and 54. Results for two values of E/k , which represent approximate practical limits for this parameter, are presented. As can be expected, the stiffer fastener to plate ratios (i.e., higher E/k) and larger D/t ratios yield results which approach those for a rigid fastener.

It should be noted that the bearing stress in each plate lamina is a measure of the two dimensional stress state level in the lamina, as given in Chapters 2 and 3. Thus, the bearing stress ratios presented in Figures 53 and 54 may be viewed as stress concentration factors, caused by fastener flexibility, over and above the plate stress concentrations resulting from the purely two dimensional plate-fastener considerations of Chapter 3.

4.7 Comparison of Tests with Theory

Overall joint flexibilities for single fasteners in double shear, for a given gage length, were determined both experimentally and by the methods of this section. A series of 15 tests were performed on aluminum-plate, symmetrical, double-shear lap joints using titanium fasteners. Twelve (12) of these specimens were joined by loose fitting fasteners and the remaining three (3) by interference-fit fasteners. A more complete description of the tests and parameters investigated, such as plate friction and fastener clamp-up, is presented in Chapter 7.

The analytical model used for comparison purposes is the shear beam of Section 4.5 (Eqs. (28) and Figure 52). However, since the elastic foundation supports for the center and outside plates are assumed fixed in that model, we must add rigid body displacements to each segment idealization to obtain the total joint deflection, δ_T . This is equivalent to obtaining the differences in displacements, relative to the fastener, of the foundations in the center and outside plates, in the planes common to both (i.e. $x = \frac{t_1}{2}$). Thus,

$$\delta_T = y_1 \left(x = \frac{t_1}{2} \right) - y_2 \left(x = \frac{t_1}{2} \right)$$

which by virtue of Equations (28) yields

$$\delta_T = \frac{P_0}{2\lambda GA} \left(\frac{1}{\gamma_1 \tanh \frac{\gamma_1 t_1}{2}} + \frac{1}{\gamma_2 \tanh \gamma_2 t_2} \right) \quad (30)$$

Since the plates joined in the tests were of the same material and the thicknesses satisfied the relationship

$$t_1 = 2t_2$$

Equation (30) reduces to

$$\delta_T = \frac{P_0 \gamma}{k \tanh \frac{\gamma t_2}{2}}$$

Thus, the effective joint stiffness, k_{eff} , becomes

$$k_{eff} = \frac{P_0}{\delta_T} = \frac{k \tanh \frac{\gamma t_1}{\delta}}{\gamma} \quad (31)$$

which, in the limit, as the ratio of plate stiffness (k) to fastener stiffness (γ) approaches zero, becomes

$$\lim_{\gamma \rightarrow 0} k_{eff} = \frac{kt_1}{2} \quad (32)$$

Equations (31) and (32) were used to generate effective stiffnesses for both titanium and steel fasteners joining aluminum plates. The plate stiffness was varied to account for various conditions of fit; i.e. loose, neat, and interference. Numerical results for these cases are presented in Table 7 and the corresponding stiffness for the linear portion of the experimental results (summarized from Appendix K) are presented in Table 8.

The last three columns of Table 7 show the effect of increasing the fastener to plate stiffness ratio, as well as the relative importance of including the three dimensional effect of fastener flexibility upon the overall joint stiffness. However, it should be noted that the effects of fastener-to-hole fit, which can be observed by comparing the loose, neat, and interference fit results, are more significant in establishing the joint's net compliance.

A comparison of results between Tables 7 and 8 reveals excellent agreement for the loose fitting titanium fasteners and good agreement for the interference fit titanium fasteners. However, it should be noted that the variation of measured results from test to test, for similar conditions of fit, varied significantly and, that Table 8 gives only average values. In addition, the definition for the elastic plate modulus, k_e , is somewhat arbitrary in the case of the interference fit fasteners (Fig. 75) because of the highly non-linear nature of the virgin load-deflection curve for all levels of loading. (The method used in obtaining the k_e values for these types of curves was to define k_e to be the secant modulus to an arbitrary value of the load level equal to 1/3 of the ultimate load, P_u , sustained by the joint.)

CHAPTER 5 APPLICATION OF RESULTS TO FATIGUE ANALYSIS OF JOINTS

5.1 Introduction

The fatigue strength of a discretely fastened joint is dependent to different degrees on various parameters within the joint. Of major importance to fatigue life is the magnitude and range of stress and strain at critical points within the joint. The stress distribution is largely related to joint geometry, but is influenced to significant degrees by various other parameters, such as type and order of loading, clamp-up forces, variation in fit between different fasteners in a multi-fastener joint and clearance or interference between fastener and hole. Fatigue strength is also influenced by fretting, which can occur as a result of relative movement between fastener shank and hole surface or between the inner and outer members of a joint. This phenomenon is difficult to include in a fatigue strength calculation, except by the use of empirically determined fatigue strength reduction factors. The effects of in-plane loads induced by frictional forces due to clamp-up are also difficult to assess except by experimental procedures.

Of the many factors which affect the fatigue life of a mechanically fastened joint, clearance or interference between fastener and hole is one of the most important. In addition, it is also a joint parameter that can be controlled in design and manufacture. Fatigue tests have shown that fatigue life is generally reduced as a result of increasing clearance, and improved when there is an interference fit. If the degree of reduction or improvement in fatigue life is reasonably well established it is possible to use various fits to advantage. For example, increased hole tolerances in certain areas may result in a saving in production costs. Also, the necessity of otherwise using interference fit fasteners to increase fatigue life can be evaluated.

The present investigation is, therefore, largely concerned with the effect of changes in stress distribution due to geometry and fit, as they influence the calculation of fatigue strength. Except as inherent properties of the

test results, fretting and clamp-up are neglected.

Some examples of test data demonstrating the effect of clearance and interference are presented in References 21 and 22 for constant amplitude loading. However, the number of systematically run test programs to evaluate the effect of clearance and interference fits on fatigue life is limited, and such data as are available are usually confined to constant amplitude tests where residual stress effects are not present to complicate the results. In spectrum fatigue loading, the effects of residual stresses induced as a result of high loads in the spectrum must be considered.

In the application of the results of the present study to the fatigue analysis of joints, two specific joint configurations have been considered. The fatigue life of these joints has been investigated for conditions of clearance and neat fit, for typical aircraft maneuver spectrum loadings. In addition, constant amplitude fatigue life predictions have been made for conditions of neat fit and interference on one joint configuration. The methods of fatigue analysis are described in the following section.

5.2 Method of Fatigue Analysis for Joints with Neat Fits and Clearance Fits

5.2.1 Constant Amplitude Loading

The fatigue life prediction method of Reference 23 which has been used in this analysis is based on the strain cycling concepts of Manson, Peterson and others (References 24 and 25), who employed constant strain amplitude, fully reversed strain cycling data to establish relationships between total strain amplitude and cycles to failure, for unnotched specimens tested in fatigue at zero mean stress. The method of Reference 23 gives a procedure for extending the approach to cover cases other than zero mean stress. In using the method, the basic assumption is made that the material at the edge of a hole or other stress raiser will fail in fatigue at the same life as an unnotched specimen subjected to the same strains. It is therefore necessary to establish the strain range at the critical section of the notch. For a given load cycle, stress-strain variation at the critical fatigue section (generally the region of maximum stress concentration) is established using several approximations. The procedure is illustrated in Figure 55 , for the condition in which the applied maximum stress is tensile, and the minimum stress is zero in the fatigue cycle.

For values of minimum applied stress other than zero, a modification of this procedure is given in Reference 23. The nominal applied stress in the loading cycle varies from 0 to f_{\max} (Figure 55), Point A. The corresponding stress at the notch root is assumed to vary from 0 to σ_{\max} , (point B), where σ_{\max} is given by the expression

$$\sigma_{\max} = K_{fp} f_{\max} \quad (33)$$

K_{fp} is a plastic stress concentration factor calculated by the Stowell formula (Reference 26). The strain, ϵ_{\max} , corresponding to σ_{\max} , is obtained from a cyclic stress-strain curve similar to Figure 56. When the section is unloaded, the material at the edge of the notch is assumed to unload in two stages. In calculating the first stage from the point B to the point C, the elastic part of the unload cycle, use is made of the Neuber factor, K_N , which is given for a circular hole by the formula:

$$K_N = 1 + \frac{K_T - 1}{1 + \frac{s}{\sqrt{p}}} \quad (34)$$

where K_T = elastic stress concentration factor
 s = material constant
 p = radius of hole or notch

The elastic unload stress increment is given by the expression:

$$\Delta\sigma_1 = \frac{K_N f_{\max}}{2} \quad (35)$$

with a corresponding unload strain of: $\Delta\epsilon_1 = \frac{\Delta\sigma_1}{E}$. (36)

To determine the remaining part of the unloading cycle at the notch (to point D), it is assumed that the material follows the same cyclic stress-strain curve as during initial loading, and that the strain at the notch will decrease by an additional amount:

$$\Delta\epsilon_2 = \frac{\epsilon_{\max}}{2} \quad (37)$$

with a corresponding stress increment, $\Delta\sigma_2$, obtained from the cyclic stress-strain curve.

The strain amplitude at the notch will be: $\epsilon_s = \frac{\epsilon_1 + \epsilon_2}{2}$ (38)

The minimum stress at the notch, at point D, is given by:

$$\sigma_{\min} = \sigma_{\max} - (\Delta\sigma_1 + \Delta\sigma_2) \quad (39)$$

The mean stress at the notch will be:

$$\sigma_{\text{mean}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \quad (40)$$

Subsequent applications of the same load cycle are assumed to follow the same loop, i.e., D,B,C,D shown in Figure 55. At the end of the loading cycle (at point D), there will be a residual compressive stress at the notch, which is equal to σ_{min} (Eq. 39). This residual stress may have a significant beneficial effect on fatigue under spectrum loading, as discussed in Section 5.2.2.

In calculating fatigue damage, use is made of constant amplitude, fully reversed strain cycling data. The calculated strain amplitude at the notch, ϵ_a , is therefore modified by the use of a Goodman type correction, to obtain an equivalent fully reversed strain amplitude.

$$\epsilon_t = \frac{\epsilon_a}{1 - \frac{\sigma_m}{\sigma_f}} \quad (41)$$

where

ϵ_t = equivalent fully reversed strain amplitude; ϵ_a = strain amplitude;
 σ_m = mean stress in cycle and σ_f = stress at fracture for the material

The life is then obtained by entering the strain-life curve for the material. Such a curve for aluminum alloy, 7075-T6, is shown in Figure 57. The life of the element is then determined by applying Miner's linear damage rule:

$$\sum \frac{n}{N} = 1 \quad (42)$$

5.2.2 Spectrum Loading

The method for predicting the fatigue life of a joint subjected to spectrum loading is basically similar to that used for predicting constant amplitude fatigue life. The cyclic strain amplitude corresponding to any particular load level in the spectrum is determined exactly as for the case of constant amplitude loading, independent of the previous load levels that were applied. The cyclic mean stress corresponding to any given load level is also independent of the previous loading history, provided that the previous applied loads were all equal to or lower than the subsequent loads. If, however, any of the previous applied loads were higher than the subsequent applied load, then relatively large residual compression stresses remaining from the previous higher loads would tend to reduce the cyclic mean stress in the subsequent load cycles. The amount of this reduction is directly related to differences in residual stress

levels, as explained in Reference 23.

In general, the types of assumptions made in the empirical fatigue analysis procedures described above, both for constant amplitude loading and for spectrum loading, were based on qualitative, physical reasoning. The specific details, however, were determined so as to obtain a best fit with available fatigue test data. Further discussion is given in Reference 23.

5.3 Method of Fatigue Analysis for Joints with Interference Fits

In the case of joints with neat fit pins or with pins having positive clearance the fatigue analysis method described in Section 5.2 gives generally satisfactory results. One of the principal reasons for this is that for initial loading into the plastic range, the use of the Stowell formula for determining the plastic strains and stresses at the edge of a hole with a known elastic stress concentration factor, will give sufficiently accurate results in such joints. Subsequent unloading is largely elastic and can, therefore, also be determined with good accuracy.

In general, the Stowell formula applies quite well to cases where there is no initial self-balancing system of stresses, such as those resulting from the use of a pin with an interference fit in a joint. However, for joints with interference fit pins the plastic stresses and strains that result from the combined effects of interference fit loads and applied loads cannot be accurately predicted with the use of the Stowell formula, especially for large interferences which cause local plasticity even before any external load is applied.

Because a simple, reasonably accurate procedure has not yet been developed for approximating the plastic stresses and strains around the holes in a joint with interference fit pins, a much simpler version of the fatigue analysis procedure described in Section 5.2 has been used to determine, at least qualitatively, if not quantitatively, the effects of different interference fits on the fatigue life of a joint. This simplified procedure makes use of the elastic strain distributions shown in Tables 9 through 12, in conjunction with the cyclic stress-strain and strain-life curves shown in Figures 56 and 57. This procedure can be illustrated by considering the elastic stresses (Table 12) at $\theta = 81^\circ$ (Fig. 58) for a point on the hole circumference in the aluminum alloy joint of Fig. 68.

Assuming an elastic modulus of $E = 10 \times 10^6$ psi for simplicity, the elastic strain amplitude at $\theta = 81^\circ$ when cycling from no load to an 8000 lb load is 0.00375. This elastic strain amplitude is then modified to account for mean stress effect by the use of the Goodman type correction discussed in Section 5.2.1, thereby obtaining the equivalent fully reversed strain amplitude. The value of σ_m is obtained by determining the average of stresses corresponding to the maximum and minimum elastic stress in the cycle from the cyclic stress-strain curve for 7075-T6 aluminum alloy (Fig. 56). In this case, the elastic strains are 0.01087 and 0.01837, the corresponding stresses are 60 ksi and 71 ksi, and the mean stress is 65.5 ksi. The resulting value of ϵ_t from Equation 41, is 0.00935. The fatigue life at $\theta = 81^\circ$ obtained by entering the fully-reversed strain-life curve (Fig. 57) with this value of ϵ_t is 1200 cycles.

It should be emphasized that this simplified fatigue analysis procedure can be regarded as giving only qualitative results, and it may be concluded that improved methods for determining plastic stresses and strains in joints with interference fits are needed to obtain good quantitative results.

As will be discussed later in Section 5.4, interference fits result in non-linear variations of edge-of-hole stresses with applied load (the actual variation approaches a bi-linear variation). Also, the initial interference stresses may vary considerably around the hole circumference, depending on the joint geometry. The result is that, especially under spectrum loading conditions, it may be necessary to examine several locations around the circumference to determine which is fatigue critical.

5.4 Discussion of Results

5.4.1 Joints with Clearance Fits

Figure 59 shows the calculated fatigue life for clearance fits in an aluminum alloy joint similar to that shown in Fig. 68 with a 3/4" Dia. fastener (See also Fig. 24). The fatigue analysis has been performed for a typical maneuver loading spectrum, to a zero 'g' base, using the method described in Section 5.3. Four conditions of fastener-hole fit are considered, i.e. neat fit, 0.4% dia 2.0% dia and 4.0% dia. The curves show the variation of service

life with limit stress level in the spectrum. For a limit stress level of 35 ksi (a typical design stress level for a 7075-T6 aluminum alloy wing), results indicate that fatigue life decreases as a result of going from a neat fit to a clearance fit. Much of the reduction in life occurs in the range between a neat fit and a clearance equal to about 2% of the pin diameter as indicated in Figure 60 where service life is plotted against % clearance for a constant limit stress level. Similar indications are evident in the constant amplitude test data of Reference 21.

Figure 61 shows a similar calculation for a 6Al-6V-2Sn titanium alloy joint (Figs. 179 and 39). together with a limited quantity of test data. The joint represents a configuration used in a particular design investigation, in which a number of specimens were tested to a simplified fighter aircraft maneuver spectrum, (somewhat more severe than that used for obtaining the curves in Figure 59). Test data for clearance of 1.2% and 2.4% diameter are presented. Considerable scatter is evident in the test results. Fatigue predictions show a significant reduction in life in going from a neat fit to a 2.4% dia clearance. At a limit stress level of 90ksi, for example, the calculated life is reduced from 18700 hours to 12000 hours. The reduction is less than that calculated for the case of the aluminum alloy joint previously discussed (the curves in Figure 59 indicate that at a limit stress of 29 ksi, the life of the aluminum joint would be reduced from 19000 hours for a neat fit condition to 3300 hours for a 2.4% clearance condition), but this is mainly due to the joint configuration, as reflected in the stress concentration factors shown in Figures 33 and 40. The fatigue predictions for the titanium joint show good general correlation with test data.

5.4.2 Joints with Interference Fits

As discussed in Section 5.3, a simplified approach has been used in fatigue predictions for the interference fit cases. The aluminum alloy joint discussed in Section 5.4.1 has been analyzed for constant amplitude cyclic loading. The neat fit, 0.4% and 2.0% interferences have been considered, and the results are presented in Figure 62. The calculations show the fatigue strength at a point on the hole circumference at 81° from the axis of applied load.

It should be noted that the curves for 0.4% interference and 2.0% interference show increased fatigue life over the neat fit for the entire range of net section stresses considered. However, a comparison of the degree of improvement in fatigue life between the 0.4% and 2% curves indicates that larger increases in fatigue life are obtained with the smaller interference when the applied stresses are less than about 47ksi. At this stress level, the two curves cross at a life of about 3000 cycles. Below 47 ksi the 0.4% interference

joint shows somewhat longer lives. This behavior would indicate that at any given stress level in this particular joint, there may be a limiting degree of interference, beyond which fatigue life would decrease with increased interference.

In order to further examine this result, Figures 63 and 64 have been plotted. Figure 63 shows the variation of elastic strain at the edge of the hole with the applied net section stress, for two points on the hole circumference, at 45° and 99° to the axis of applied load. Curves are plotted for the 0.4% D and 2.0% D interference conditions. These plots are obtained directly from the data given in Tables 9 through 12. Figure 64 shows edge elastic strain variation with nominal stress at the edge of a hole at a point that is 81° from the applied load axis. This location was the one considered in calculating the constant amplitude fatigue curves shown on Figure 62.

An indication of the effects of interference fit on the elastic behavior at the edge of the hole is obtained from an examination of the curves in Figs. 63 and 64. It can be seen from these figures that, for each value of θ , the initial slope of the curves (a measure of the rate of change in the elastic strain at the hole with change in the elastic stress on the net section) is the same for the 2% D interference case as for the 0.4% D interference case. However, as the applied stress is increased, the initial interference condition is eventually overcome and the slope of the curves approaches the slope that would be obtained in a neat fit (0% interference) case. The applied stress level at which the initial interference is overcome increases with increasing interference. The significance of this behavior,

as it affects fatigue life, may be better understood by examining in detail some of the calculations that were made in obtaining the constant amplitude fatigue curves in Fig. 62.

Consider, for example, the elastic strain ranges at the edge of the hole for 2.0% and 0.4% interference when the net section stress varies from 0 to 47 ksi (the cross over point for these two interferences on the constant amplitude fatigue curves in Fig. 62). The elastic strain ranges at the edge of the hole at $\theta = 81^\circ$, obtained directly from Tables 9 through 12, are as follows:

Interference % Dia.	ϵ_{el_r}
0.0	0.014
0.4	0.0096
2.0	0.00625

As would be expected, the strain range at the edge of the hole in the joint decreases with increasing interference. However, as discussed in Section 5.2 the fatigue life of either joint depends not only on the strain range but also on the mean stress. Therefore, when the values of ϵ_{el_r} are corrected for mean stress effects using Equation 41 (which can be applied to strain range or strain amplitude), the resulting "equivalent" fully reversed cyclic strain ranges are:

Interference % D	ϵ_{el_r}
0.0	0.0200
0.4	0.01564
2.0	0.01558

Because the "equivalent" strain ranges for the two interference fits are essentially equal, the corresponding fatigue lives, (obtained by entering the strain-life curve in Fig. 57 with the strain amplitude $\epsilon_t = \epsilon_{t_r}/2$),

are also equal, and are approximately three times the life of the neat fit joint.

At net section applied stresses greater than 47 ksi, the "equivalent" fully reversed strain ranges are lower for the 2.0% D interference than for the 0.4% D, resulting in longer fatigue lives for the larger interference. For example, at a net section applied stress of 55 ksi, the 0.4% D interference strain range is 0.01205, while the 2.0% D interference strain range is 0.0073. The corresponding "equivalent" strain ranges, when the effect of mean stress has been accounted for, are 0.0203 and 0.0184 resulting in a longer fatigue life for the 2.0% D interference case (Fig. 62). The strain range for the neat fit joint, at the same applied stress level is 0.0164, but when the mean stress correction is applied, the "equivalent" fully reversed strain range becomes 0.0242 and the corresponding fatigue life is lower than for either of the interference fit cases.

At lower values of applied stress, 30 ksi for example, both the 0.4% D and 2.0% D interferences show improvement in fatigue life. However, the considerably larger mean stress correction for the 2% interference case predominates so that the improvement in it's fatigue life is consequently less than that shown by the 0.4% D interference case.

From an examination of the curves in Fig. 64 and the above calculations it becomes obvious that when a load is applied to a joint, the resulting increase in stress and strain at the edge of the hole is smaller in the case of an interference fit than in the case of a neat fit. This tends to increase the fatigue life of joints with interference fits. However, when the interference fit becomes large, the mean tensile stress at any point on the hole periphery also becomes large, and this tends to decrease the fatigue life. There would, therefore, appear to be optimum amounts of interference for different applied stress levels. This appears to be partly born out by the test data shown in Fig. 7 of Ref. 27, which showed that the constant amplitude fatigue life of a small lug with a taper pin increased with increasing pin interference up to a certain point, and then showed a significant reduction at higher interferences.

5.5 Conclusions

1. The fatigue life of a discretely fastened joint is reduced if there is a loose fit between fasteners and holes. This reduction increases significantly up to a point where the clearance is between 1% and 2% of the fastener diameter, after which the rate of reduction in fatigue life becomes noticeably less. Limited test data tend to substantiate this.

2. Increases in local stress due to clearance, and resulting reduction in fatigue life may vary significantly with joint geometry. A well designed joint with standard proportions could be less affected than one with undesirable geometrical characteristics, such as extremely short edge distance.

3. Variations in fit between different fasteners in the same group will affect fatigue life. The analysis presented in this report can account for such variations in fit where these are known.

4. The degree to which interference fits between fastener and hole will affect the fatigue life of a joint is a function of the amount of interference, the joint geometry and the applied stress levels in the fatigue loading cycle. In general, interference fits will improve fatigue life, but the improvement is related to the joint geometric parameters mentioned above, and there would appear to be optimum amounts of interference for different applied stress levels, (as discussed in Section 5.3). It is possible that excessive interference may be detrimental to fatigue life.

5. The trends shown by the fatigue analysis in general are of considerable interest. The availability of the very detailed elastic stress data generated in this report, if utilized to the fullest extent, offers an attractive prospect of extensive parametric study. Extension of the present work to include plasticity effects would result in a greatly increased capacity for the analytical prediction of joint fatigue life.

6. Existing methods of fatigue analysis are not fully adequate for the condition of interference fit. However, a more accurate knowledge of the stress-strain behavior at the edge of a notch would undoubtedly permit a better fatigue analysis to be made.

CHAPTER 6. FASTENER-SHEET LOAD-DEFLECTION DATA

6.1 Introduction

Of considerable importance to the analysis of multi-fastener joints is the determination of the "spring constants" or the slopes to the load-deflection curve of the individual fasteners in combination with the local sheet. This has been pointed out by previous investigators (Refs. 1, 2 and 28) who have treated the joint as a set of springs and attempted to determine experimentally the particular spring constants needed in their analyses. In Ref. 2 the fastener experimental load-deflection curve was approximated by a "Ramberg-Osgood" type expression and in Refs. 1 and 28 an incremental piecewise linear approximation to the curve was used in the analysis.

The lack of a tabulated set of data for a variety of fastener-sheet combinations was considered a major shortcoming in the analysis of load distribution in multi-fastener joints of any type. In this chapter a method for approximating the load-deflection curve of individual fasteners is presented. Comparisons using this approximation show good correlation with experimental data. Tabulations of several hundred load-deflection curves found in the literature and covering a large variety of fastener types and plate combinations are also presented.

6.2 Parametric Representation of Fastener-Sheet Load-Deflection Curves

A typical load-deflection curve is shown in Fig. 65. The initial portion of the curve denoted by the letters OA is usually linear and rises to only a small percentage of the total load carrying capacity of the fastener. For fasteners with small tolerances and a small clamping action, the portion of the curve OA represents the frictional effects in the joint. This aspect has been studied experimentally and comparisons of its effect on the load-deflection curve are presented in Chapter 7. The portion of the curve denoted by AB represents the effect of initial clearance or "slop".

A modification to the initial portion of the generally complex curve (Fig. 65) is proposed in order to simplify the application of load-deflection data to load distribution analyses.

This modification consists of extending the linear portion of the curve to cut the δ axis at zero load as shown in Fig. 65 at point O'. The offset is designated δ_o and depending on the nature of the curve can be positive or negative. An expression for the displacement at any load level can be obtained after the modification to the curve has been made. Writing the total displacement across the fastener, δ , as a combination of an elastic and a plastic part we have:

$$\delta = \delta_e + \delta_p = \frac{P}{k_e} + A \left(\frac{P}{P_j} \right)^B \quad (43)$$

where, $\delta_e = \frac{P}{k_e} \quad (44)$

and $\delta_p = A \left(\frac{P}{P_j} \right)^B \quad (45)$

and A and B are constants.

The quantities k_e , P_1 and P_j are defined in Fig. 65. The value of k_e is easily obtained from an automatic load-deflection trace. By choosing appropriate offsets δ_1 and δ_j from the initial corrected linear part of the curve we define the corresponding values of P_1 and P_j . The two constants A and B in Eq. 43 can then be determined from the following expressions:

$$A = -\delta_e + \delta_j \quad (46)$$

and

$$B = \frac{\ln \left(\frac{-\delta_o + \delta_1}{-\delta_o + \delta_j} \right)}{\ln \left(\frac{P_1}{P_j} \right)} \quad (47)$$

The main difference between the method proposed here and the "Ramberg-Osgood" type curve proposed in Ref. 2 is the method of obtaining the values P_1 and P_3 at the "knee" of the curve. Gehring and Maines (Ref. 2) recommend P_1 to be chosen at a secant modulus of $0.7 k_e$ to be consistent with the yield defined by Ramberg - Osgood (Ref. 29) and the point P_3 is selected "as far out on the curve as practicable". It appears preferable and much easier construction-wise to use direct offsets parallel to the linear slope of the modified curve at finite displacements as shown in Fig. 65 to accomplish the same thing. Because we are more interested in the region near the knee of the curve (moderate plasticity) rather than at failure (large plasticity) this appeared to be a good technique.

6.3 Evaluation of the Accuracy of the Proposed Parametric Load-Deflection

Curve

Before the proposed method of parametric representation of load-deflection curves was applied to existing experimental data, an evaluation of its accuracy was made. Fig. 66 shows in solid line an experimentally obtained automatic load-deflection curve of a single Hi-Lok fastener in double shear. Using the method described in Section 7.2 two parametric approximations to the curve were obtained using two different sets of offset points. In the first curve (designated by crosses) offsets at 0.002 and 0.005 inches were used. The second curve (circles) was obtained using offsets at 0.005 and 0.012 inches. It appears from the comparison shown in Fig. 66 that both parametric approximations are good representations of the experimental curve over the useful range. However, as would be expected, the second approximation shows better correlation to the experimental curve in the inelastic range.

Another comparison is shown in Fig. 67 which correlates three different parametric curves to the original experimental curve. It is seen from the equations of the three curves that as the constant B increases from 5.65 to 9.76 the knee of the curves becomes sharper and the parametric curve deviates from the test curve. The two curves with the lower values of B appear better approximations to the particular experimental curve. It is suggested that in tabulating data by the parametric procedure an attempt be made to see which offsets give a better correlation for each group of fasteners analyzed. In

the results given in Appendix K this procedure was followed.

6.4 Summary of Existing Load-Deflection Data

A tabulation of several hundred load-deflection curves are presented in Appendix K. Data published in Refs. 1 and 30 are presented as well as Grumman unpublished test data (Ref. 31). Use of new commercially available fastening systems, so widespread in the aircraft industry at the present time, necessitates a large accumulation of such data as those presented in Appendix K. However, due to the unavailability of test results in the open literature, the scope of the tabulation shown is somewhat limited. It is hoped that future tabulations and additions of this type can be made available to other interested users in the near future.

The format used in presenting the data of Appendix K is shown on pg. 255 together with the definition of the various symbols. The table gives the following: information concerning the plates making up the joint (Cols. 1 to 7); information concerning the fastener data (Cols. 8 to 13); measured interference or clearance (Cols. 14 to 16); the computed load-deflection parameters (Cols. 17 to 22) and test information and identification (Cols. 23 to 28). Pages 255 and 256 summarize the data presented by Mc Combs et al. (Ref. 1) for single and double shear single fastener joints. On page 257 is given a summary of the single fastener double shear tests of the present study. Pages 258 to 264 summarize the results of Grumman data obtained in a Corporate titanium study (Ref. 31). The code used in describing each test is shown in Cols. 27 and 28 and on top of page 258.

CHAPTER 7 EXPERIMENTAL PROGRAM

7.1 Introduction

The main purpose of the experimental program was the verification of the analytical methods described in Chapters 2 to 5. In addition, the experimental results pointed out the relative importance of some of the parameters which influence the structural behavior of mechanically fastened joints. Two types of tests are described: static strength and fatigue. The static strength tests were performed on single fastener and multi-fastener joints and the fatigue tests were performed on multi-fastener joints. Some of the results presented herein were obtained under other Grumman Independent Research and Development programs and were not part of the present contract. However, the results are pertinent to the present analytical study and are presented for illustrative purposes.

7.2 Static Strength Tests

7.2.1 Single Fastener Load Deflection Curves

(1) Description of the Tests

The configuration for the double shear test specimen chosen is shown in Fig. 68. A view of the test set up prior to testing is shown in Fig. 69. Note the extensometer device used for obtaining the automatic trace of the load-deflection curve across the joint. Table 13 gives a summary of the 15 static strength tests which were performed. All fasteners had a nominal diameter of 0.25 inches.

The method of testing followed closely the Grumman procedures used in obtaining fastener design allowables and is described in Ref. 32. The specimen is given a very small initial load of the order of 100 to 300 lbs. prior to start of test. The specimen is then loaded slightly beyond the beginning of its inelastic behavior and then unloaded. Reloading of the specimen then follows and a second unloading is made at a deflection of approximately 0.010 inches greater than the first. These two unloading cycles are chosen with the help of visual observation of the continuous load-deflection trace and are intended to bracket the 0.012 inch offset point which is used to obtain the allowable yield load of the fastener.

A typical failed specimen is shown in Fig. 70 which shows the large local deformation causing ovalization of the holes and the excessive bending deformations in the broken titanium Hi-Lok fastener

(2) Results and Discussion

An attempt was made to determine the influence of gage length on the load-deflection curve of the single flush Hi-Lok fastener joint shown in Fig. 69. Three "identical" specimens having a single Hi-Lok fastener in double shear were tested by the procedure described above. The only variable in these three specimens was the length over which the deformations across the fastener were recorded by the extensometer. Three lengths were used in the study 2, 4 and 6 inches. The results are shown in Fig. 71. Although no definite trend is indicated the results do point to the fact that a fair amount of scatter can be expected in the load-deflection characteristics of seemingly identical specimens installed under normal production conditions.

The results of a limited study of the effect of friction coefficient on the faying surfaces of a joint are shown in Figs. 72 to 74. Fig. 72 shows the results of tests on joints made from plates as received with no special finish. The flush Hi-Lok installation torque is of the order of 65-80 in.-lbs. causing a small bi-linear effect in the elastic portion of the load-deflection curve with the change in slope occurring at an applied load of approximately 500 lbs. By finishing the plates with a standard protective alodine treatment used at Grumman the point at which the bi-linear effects occurs is shown to be about 1000 lbs. in the three tests shown in Fig. 73. This indicates an increase in the friction coefficient over the natural finish of Fig. 72 since the same Hi-Lok fasteners were used in both cases.

By Teflon coating the faying surfaces, the friction coefficient is reduced to practically zero, hence the break in the initial portion of the bi-linear curve occurs at a much lower load (approximately 100 lbs.) as shown in Fig. 74. It should be noted however, that for all three types of surface finish, the effect of a relatively small clamp-up (65-80 in.-lbs) has a very small effect on the over-all load deflection characteristics of the fastener.

The results of three tests using a single $\frac{1}{4}$ " interference Hi-Lok-Hi-Tigue fastener in double shear are shown in Fig. 75. Comparing these results

with the tests using normal flush Hi-Lok fasteners having a small clearance due to the manufacturing tolerances, two things stand out. The load-deflection curve is nonlinear almost from the start of loading and the elastic modulus measured by the value of k_e is also much higher.

7.2.2 Multi-Fastener Joints

An example of a multi-fastener joint is shown in Figure 76. The configuration is representative of a structural element fatigue test on a wing center line splice. The material of the outer plates is aluminum alloy, 7075-T6 clad and the inner plate is aluminum alloy 2024-T3511. The fasteners are $\frac{1}{8}$ " dia. Hi-Loks. Three conditions of fastener-hole fit were tested, neat fit, 0.002" clearance and 0.004" clearance. In addition, the effects of clamp-up as applied to the fasteners due to installation torque, were studied. Tests were conducted with the fasteners lightly torqued by hand as against the normal installation torque used in manufacturing assembly.

The static behavior of the joint when assembled with the normal clamp-up is clearly demonstrated in Fig. 77, where the load-deflection curve for the joint over a 4 inch gage length shows a marked breaking point. The joints with clearance, as distinct from the neat fit, show the break in slope more clearly. In all three cases, the break occurs at a significant percentage of the joint ultimate strength. The relative effect of clamp-up and clearance on the fatigue strength of the joint is illustrated in Fig. 78. The fatigue life tends to decrease as clearance is increased, for the specimens without clamp-up. A considerable increase in fatigue life is obtained when normal fastener clamp-up is present.

A second double shear joint specimen is shown in Fig. 79. This configuration has two fasteners on a line, perpendicular to the applied load line. The material is titanium alloy, 6Al-6V-2 Sn annealed sheet. Joints assembled with $\frac{1}{4}$ " dia. Taper-Lok fasteners were tested with interference fits of 0.0018" and 0.0042". Joints with $\frac{1}{4}$ " Hi-Lok fasteners were tested with clearances of 0.003" and 0.006". A typical aircraft maneuver load spectrum was used in all the tests.

Test results are presented for the interference fit specimens in Fig. 80. The data show substantial increases in fatigue life for the 0.0042" interference case over the 0.0018" interference case. Several specimens were tested after a 14 day exposure to a 3.5% NaCl solution, but the effect on

fatigue did not appear to be significant, based on very limited data.

Test results for the clearance fit specimens are presented in Fig. 81. The data show considerable scatter, and the results for the 0.003" clearance and the 0.006" clearance overlap to the extent that a separate scatter band for each condition has not been defined. These test results are also discussed briefly in Section 4 of Chapter 5, where fatigue life predictions for the joint are presented.

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Problem No.	Diam. D_1 & D_2 (in.)	Width W (in.)	Spacing S (in.)	e Edge Dist. (in.)	Thick- ness t (in.)	$\frac{S}{W}$	$\frac{W}{D}$	$\frac{e}{D}$	$\frac{D}{t}$	$\frac{S}{D}$
1	2	3	4	5	6	7	8	9	10	11
P 1.1	.25	2	4	1	.125	2	8	4	2	16
P 1.3	.50	2	4	1	.125	2	4	2	4	8
P 1.6	.75	2	4	1	.125	2	2.67	1.33	6	5.33
P 1.2	.25	2	2	1	.125	1	8	4	2	8
P 1.4	.50	2	2	1	.125	1	4	2	4	4
P 1.5	.75	2	2	1	.125	1	2.67	1.33	6	2.67

Table 1 Summary of Plate Problems With Two Unloaded Holes

Problem No.	Diam. D_1 to D_6 (in.)	Taper	Spacing Along Width S_W (in.)	Spacing Along Length S_L (in.)	Edge Dist. Plate Holes e_1 (in.)	Edge Dist. End Holes e_2 (in.)	Thick-ness t (in.)	S_W/D	S_L/D	e_1/D	e_2/D	D/t	Q Width W_1 (in.)	Width Across Edge Holes W_2 (in.)	W_1/D	W_2/D
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
3.1A	.250	1:4	.5	1	.75	.5	.125	2	4	3	2	2	2	1.5	8	6
3.1B	.125	1:4	.5	1	.75	.5	.125	4	8	6	4	1	2	1.5	16	12
3.2B	.125	1:8	.5	1	.5	.375	.125	4	8	4	3	1	1.5	1.25	12	10
3.3B	.125	1:16	.5	1	.375	.3125	.125	4	8	3	2.5	1	1.25	1.125	10	9

Table 2 Summary of Tapered Plate Problems with Six Unloaded Holes

Problem Number	Diameter D (in.)	Width W (in.)	Edge Distance e (in.)	Thickness t (in.)	$\frac{W}{D}$	$\frac{e}{D}$	$\frac{D}{t}$
P2.1	.25	.9375	.625	.125	7.5	2.5	2
P2.2	.50	.9375	.625	.125	3.75	1.25	4
P2.3	.75	.9375	.625	.125	2.5	.835	6
P2.4	.75	.9375	1.6	.3	2.5	2.13	2.5

Table 3 - Summary of Geometric Parameters of Aluminum Plates with Single Rigid Fastener

x/t	Flex. Beam (Rigid in Shear) Eq. (24)	Shear Beam (Rigid in Bending) Eq. (25)	Completely Rigid Beam Eq. (26)
0	4.003	4.032	4.000
.1	3.401	3.410	3.400
.2	2.800	2.796	2.800
.3	2.199	2.190	2.200
.4	1.599	1.588	1.600
.5	0.998	0.990	1.000
.6	0.399	0.395	0.400
.7	-0.201	-0.200	-0.200
.8	-0.800	-0.795	-0.800
.9	-1.399	-1.392	-1.400
1.0	-1.998	-1.992	-2.000

Table 4 - Load Interaction Parameter $\frac{qt}{P_0}$ for a Fastener with Negligible Head

Stiffness in Single Shear (Ref. Figure 51a) ($E = 30 \times 10^6$ psi, $\nu = 0.3$,
 $k = 7.68 \times 10^6$, $t = 0.125$ in, $d = 0.25$ in.)


x/t	Shear Beam (Rigid in Bending)	Completely Rigid Beam
0	1.078	1.000
.1	1.056	
.2	1.036	
.3	1.018	
.4	1.009	
.5	0.990	
.6	0.980	
.7	0.972	
.8	0.966	
.9	0.962	
1.0	0.961	

Table 5 - Load Interaction Parameter $\frac{qt}{P_0}$ for a Rigid-Head

Fastener in Single Shear (Ref. Fig. 51 b)

($E = 30 \times 10^6$, $\nu = 0.3$, $k = 7.68 \times 10^6$ psi, $t = 0.125$ in, $d = 0.25$ in)

	x/t_1	Completely Rigid Beam	Free & Clamped Heads Shear Beam (Rigid in Bending)	Clamped Head Bent Beam (Rigid in Shear)	Free Head Bent Beam (Rigid in Shear)
inner plate	0	1.000 ↓	0.990	0.995	0.995
	.1		0.991	0.996	0.996
	.2		0.995	0.997	0.997
	.3		1.001	1.000	1.000
	.4		1.009	1.005	1.005
	.5	1.000	1.020	1.010	1.010
outer plates	.5	1.000 ↓	1.020	1.009	1.015
	.6		1.009	1.004	1.009
	.7		1.001	1.000	1.003
	.8		0.995	0.997	0.997
	.9		0.991	0.995	0.991
	1.0	1.000	0.990	0.995	0.985

TABLE 6 - Load Interaction Parameter, $\frac{qt_1}{P_0}$ or $\frac{q(2t_2)}{P_0}$,
for a Fastener in Double Shear (Reference Fig. 52)

($E = 30 \times 10^6$ psi, $\nu = 0.3$, $k_1 = 7.68 \times 10^6$ psi, $k_2 = 7.68 \times 10^6$ psi,
 $t_1 = 0.125$ in, $t_2 = 0.0625$ in, $d = 0.25$ in.)

Type Fit	Plate Stiffness (Refer to Fig. 38) $k \times 10^{-6} (\text{psi})$	Joint Stiffness $k_{\text{eff}} \times 10^{-6} (\frac{\text{lb}}{\text{in}})$		
		Titanium Fastener	Steel Fastener	Rigid Fastener
Loose	4.255	.416	.512	.535
Neat	5.330	.610	.635	.670
Interference	6.966	.778	.816	.876

Table 7 - Analytical Results for the Double Lap Joints Tested

Type Fit	Linearized Joint Stiffness $k_{\text{eff}} \times 10^{-6} (\frac{\text{lb}}{\text{in}})$
	Titanium fastener
Loose	.420 (avg. of 12 tests)
Interference	.803 (avg. of 3 tests)

Table 8 - Summary of Experimental Results Presented in Appendix K

Load Level (lbs)	Net Section Applied Stress (psi)	Maximum Tangential Stress at Edge of Hole in psi			
		$\theta = 9^\circ$	$\theta = 45^\circ$	$\theta = 31^\circ$	$\theta = 99^\circ$
2	14.2	25.62	28.34	42.60	45.56
500	3555	6404.9	7092	10,669	11,390
1000	7111	12,810	14,185	21,338	22,781
2000	14,222	25,620	28,369	42,676	45,562
4000	28,444	51,239	56,739	85,351	91,123
8000	56,888	102,479	113,477	170,702	182,247
9000	63,999	115,288	127,662	192,040	205,027
10000	71,111	128,098	141,847	213,378	227,809

Table 9 Elastic Stresses for Neat Fit Case

Load Level (lbs)	Net Sect. Appl. Str. (psi)	Maximum Tangential Stress at Edge of Hole in psi					
		$\theta = 9^\circ$	$\theta = 27^\circ$	$\theta = 45^\circ$	$\theta = 63^\circ$	$\theta = 81^\circ$	$\theta = 99^\circ$
2	14.232	50,615	36,062	28,281	23,732	21,764	22,330
500	3,555	49,842	39,140	32,934	28,759	26,433	25,973
1000	7,111	29,066	42,231	37,606	33,807	31,121	29,631
2000	14,222	48,993	47,791	45,456	42,165	38,956	35,890
4000	28,444	68,526	67,655	66,120	64,410	64,831	73,709
8000	56,888	119,532	120,634	122,547	127,133	149,354	154,939
9000	63,999	132,342	133,946	136,731	142,912	170,692	187,720
10000	71,111	145,152	147,257	150,916	158,690	192,030	210,500

Table 10 Elastic Stresses for the Case of $\Delta D/D = 0.4\%$ Interference

Load Level (lbs)	Net Section Applied Stress (psi)	Maximum Tangential Stress at Edge of Hole in psi			
		$\theta = 9^\circ$	$\theta = 45^\circ$	$\theta = 81^\circ$	$\theta = 99^\circ$
2	14.2	253,087	141,330	108,744	111,593
500	3555	252,315	145,984	113,413	115,236
1000	7111	251,538	150,656	118,101	118,893
2000	14,222	249,986	159,999	127,477	126,209
4000	28,444	246,882	178,686	146,229	140,839
8000	56,888	240,673	216,059	183,733	170,100
9000	63,999	240,752	223,206	190,762	175,591
10000	71,111	244,964	227,282	194,780	179,448

Table 11 Elastic Stresses for the Case
of $\Delta D/D = 2.0\%$ Interference

Load Level (lbs)	Net Section Applied Stress (psi)	Maximum Tangential Stress at Edge of Hole in psi			
		$\theta = 9^\circ$	$\theta = 45^\circ$	$\theta = 81^\circ$	$\theta = 99^\circ$
2	14.2	506,178	282,644	217,469	223,172
500	3555	505,404	287,296	222,138	226,815
1000	7111	504,629	291,968	226,827	230,472
2000	14,222	503,077	301,312	236,203	237,767
4000	28,444	499,972	319,998	254,954	252,418
8000	56,888	493,764	357,371	292,458	281,679
9000	63,999	492,211	366,715	301,835	288,994
10000	71,111	490,659	376,058	311,210	296,309

Table 12 Elastic Stresses for the Case
of $\Delta D/D = 4\%$ Interference

Specimen No.	Finish on Faying Surfaces	Hole			Diameters			Fastener Diam. in.	Fastener Installation Torque in.-lbs.	Component Part Designation
		Outer Plate A in.	Middle Plate B in.	Outer Plate C in.						
HT-1	None	—	.2514	—	—	—	—	—	65-80	GB510A-4-9 -GN512H-4
-2			.2515							
-3	None	—	.2514	—	—	—	—	—		
-4	Alodine 823-011	—	.2511	—	—	—	—	—		
-5			.2510							
-6	Alodine 823-011	—	.2510	—	—	—	—	—		
-7	Teflon	—	.2512	—	—	—	—	—		
-8			.2513							
-9	Teflon	—	.2510	—	—	—	—	—		
-10	None	.2509	.2509	.2510	.2510	.2492	.2492			
-11	None	.2509	.2509	.2510	.2510	.2492	.2492			GB510A-4-9 -GN512H-4
-12	Teflon	.2509	.2509	.2510	.2510	.2491	.2491			GB510A-4-9 -GN512H-4
HT-1	None	—	.2518	—	—	—	—	—	65-80	HLT-1170-8-9
-2	None	—	.2514	—	—	—	—	—		
-3	None	—	.2515	—	—	—	—	—		HLT-1170-8-9

Table 13 Single Fastener Tests - Physical Description

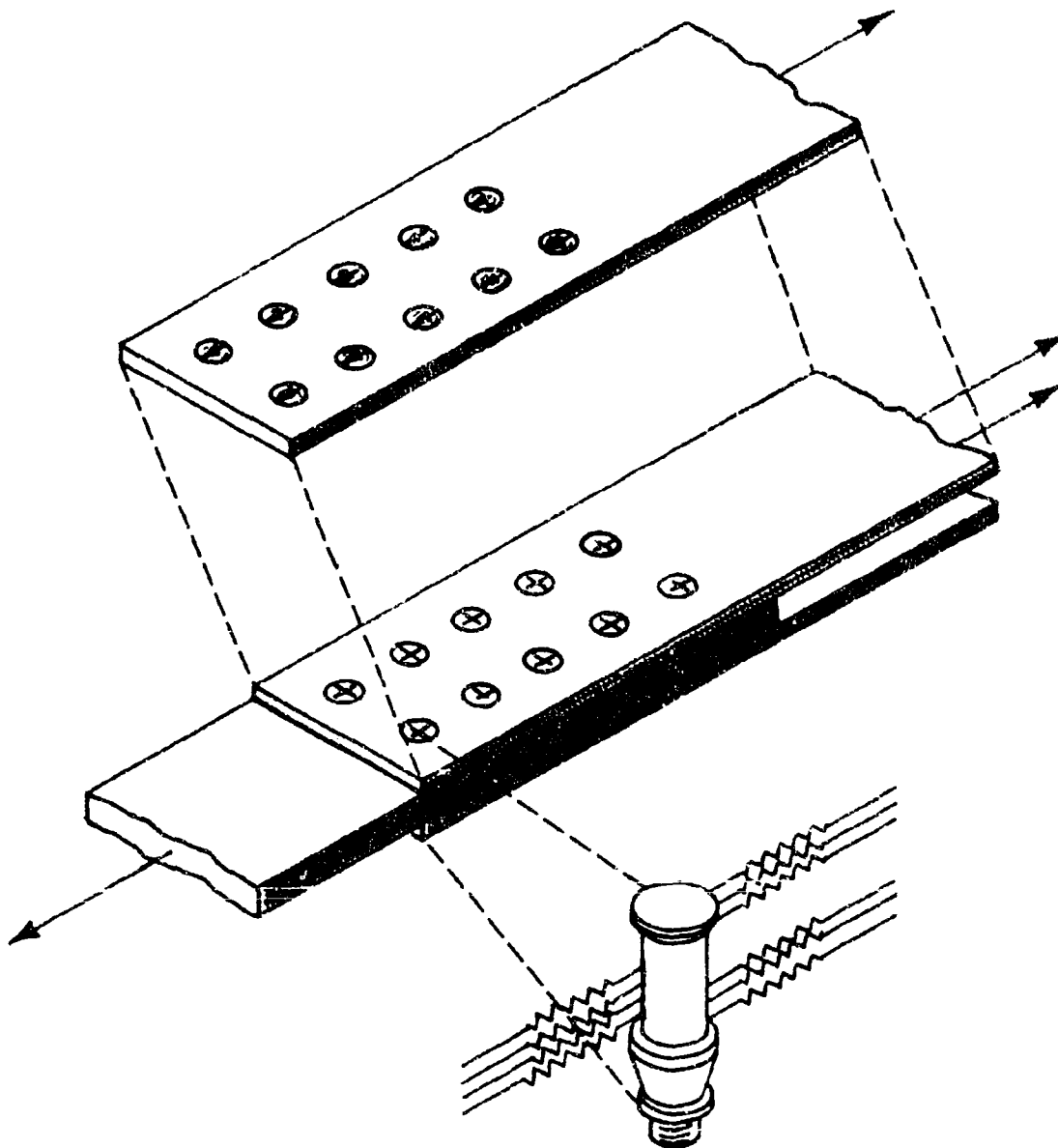


Fig. 1 Schematic Representation of Mechanically Fastened Structural Joint
Showing the Method Used to Decompose the Structure for Analysis Purposes

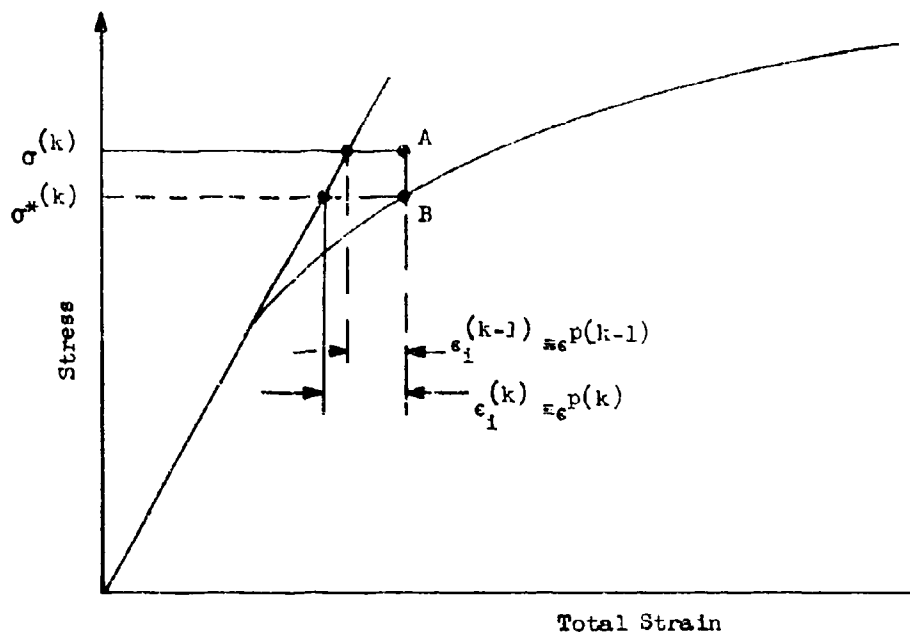


Fig. 2 Illustration of the Constant Strain Method for a Uniaxial Stress Case

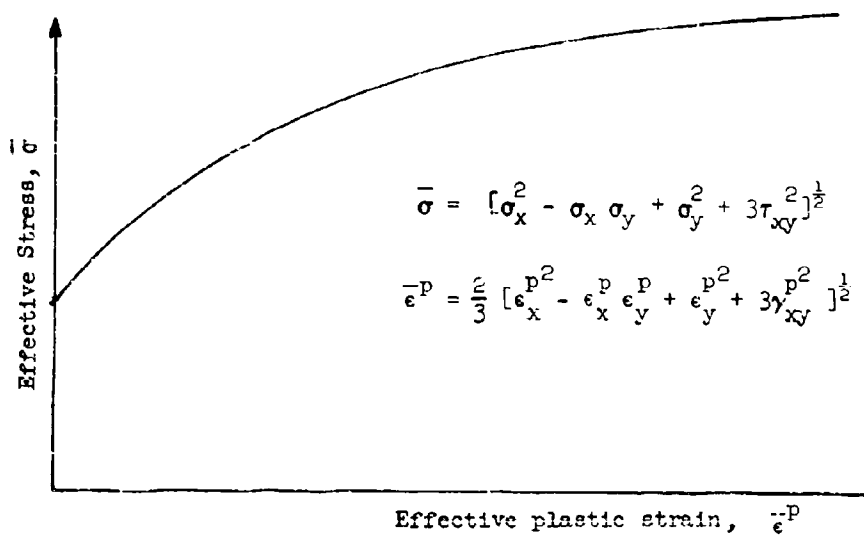


Fig. 3 Effective Stress-Strain Curve

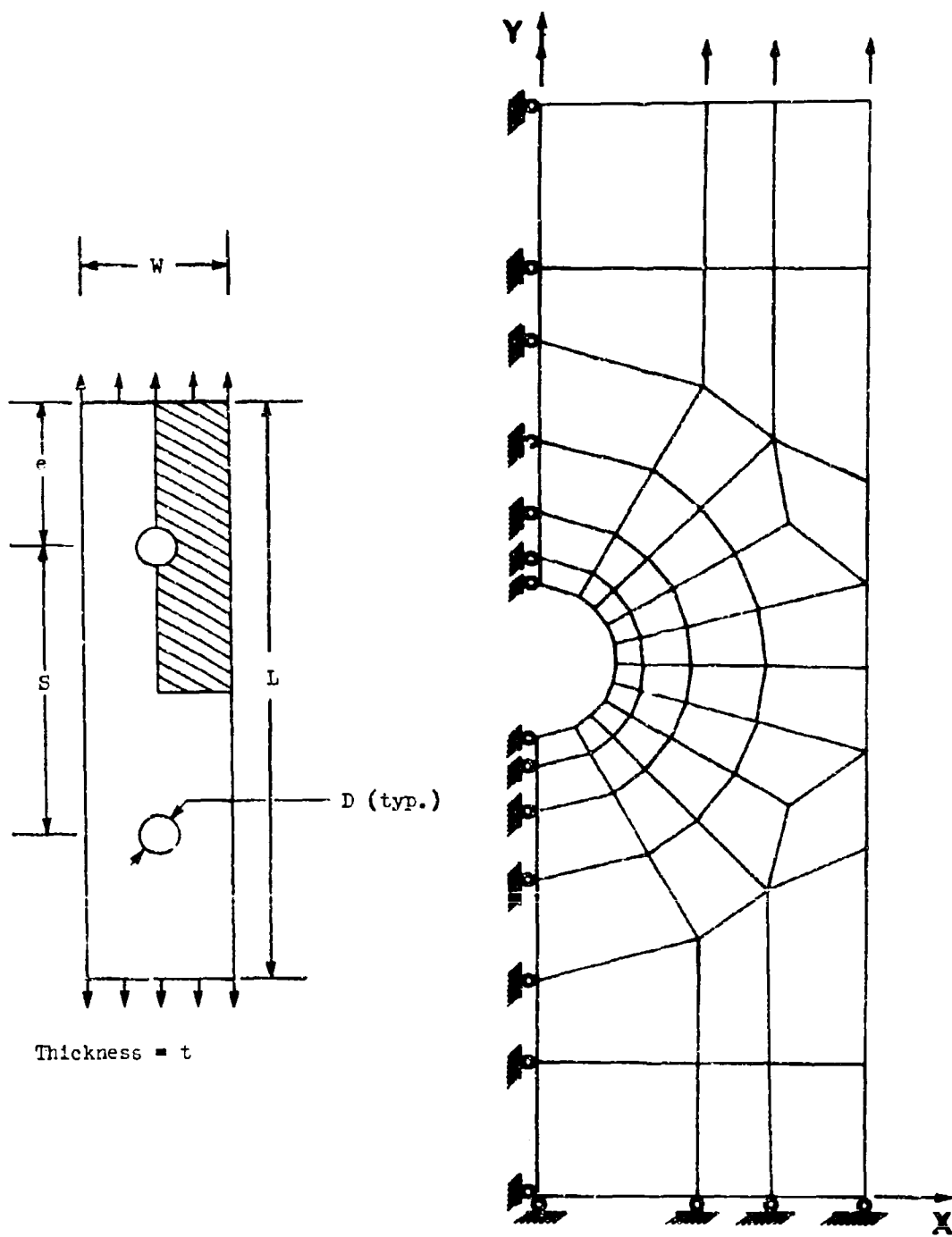
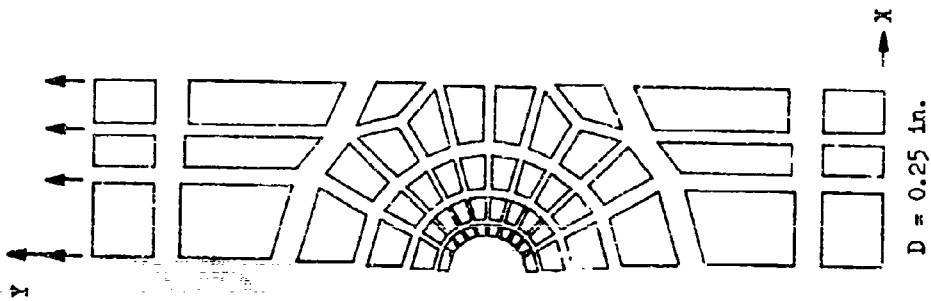
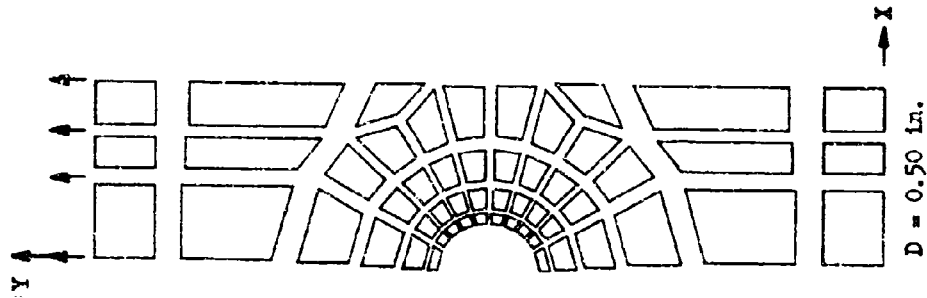


Fig. 4 Model for Determination of Effective Width and Effects of Fastener Spacing

Problem No. P 1.1



Problem No. P 1.3



Problem No. P 1.6

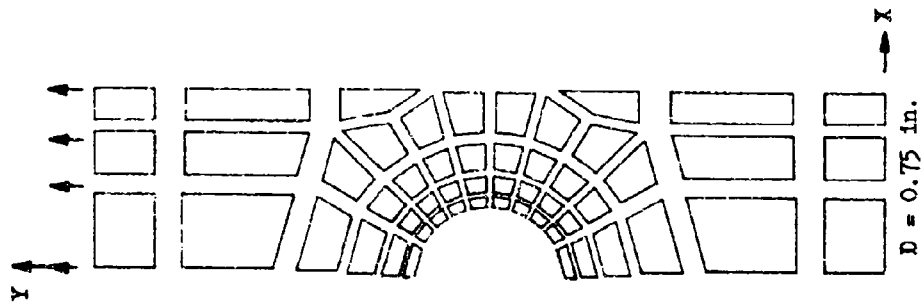


Fig. 5 Idealizations of Two Hole Plates in 2024-T4 Aluminum with $S/W = 2$

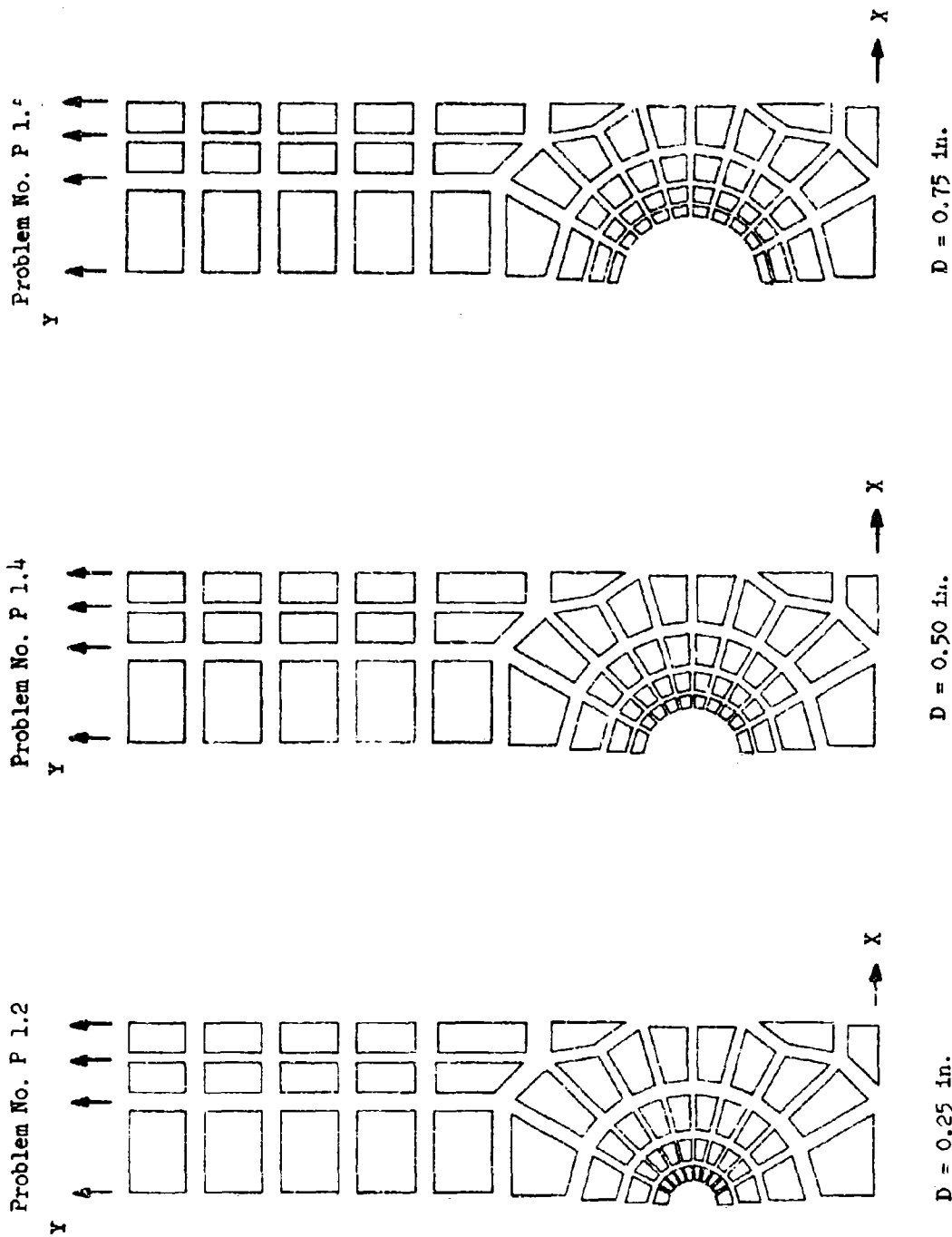


Fig. 6 Idealizations of Two Hole Plates in 2024-T4 Aluminum with $S/W = 1$

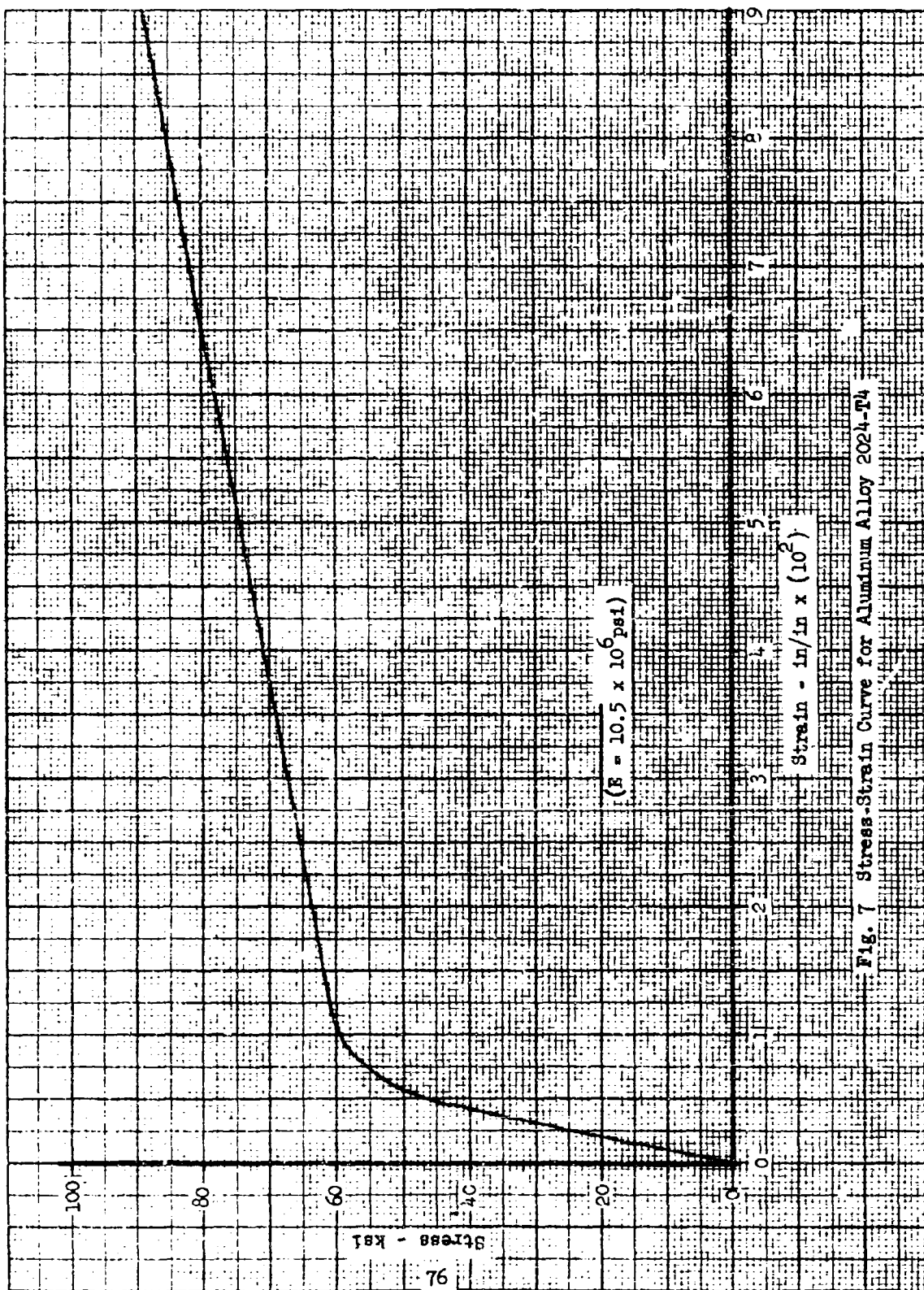


Fig. 7 Stress-Strain Curve for Aluminum Alloy 2024-T4

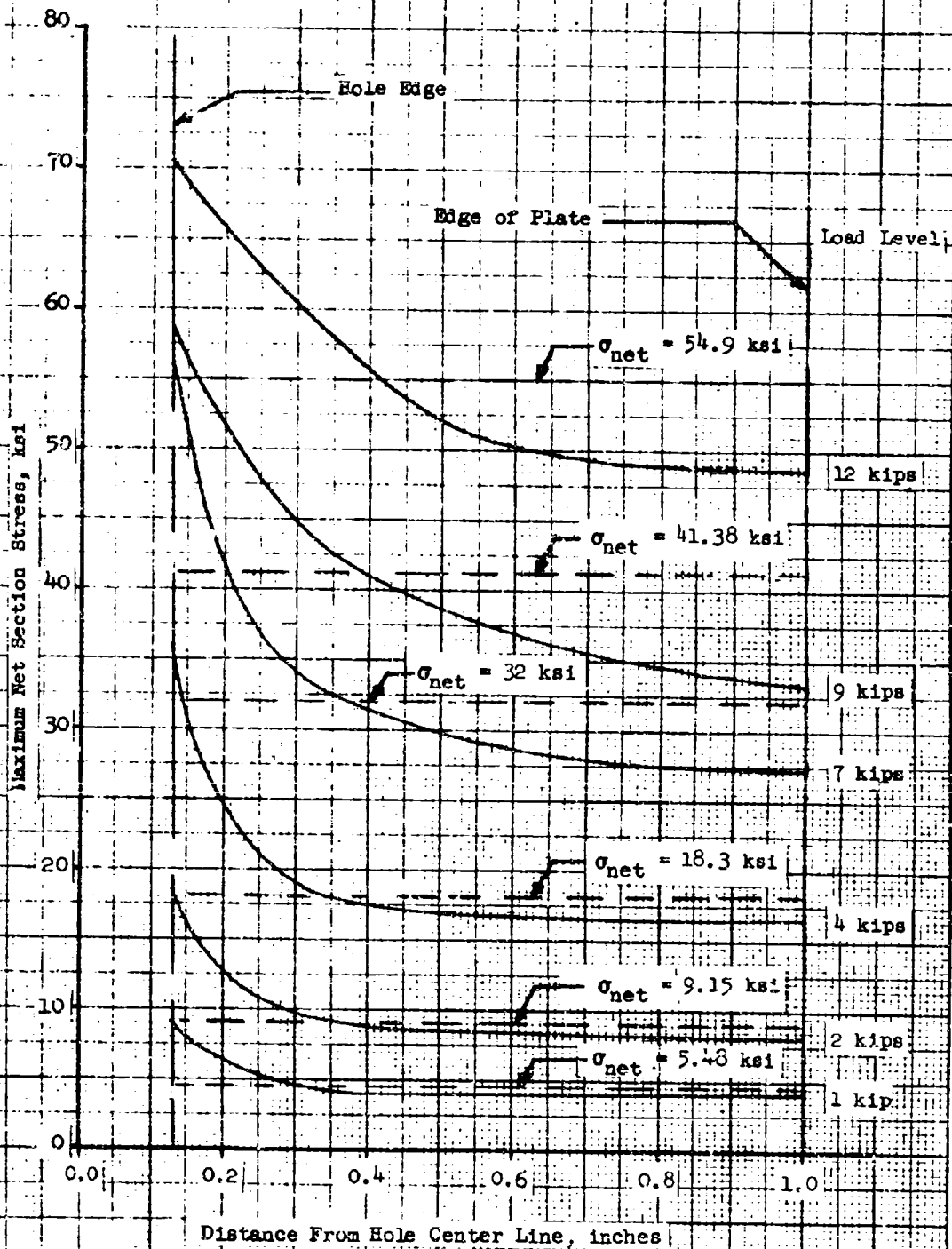
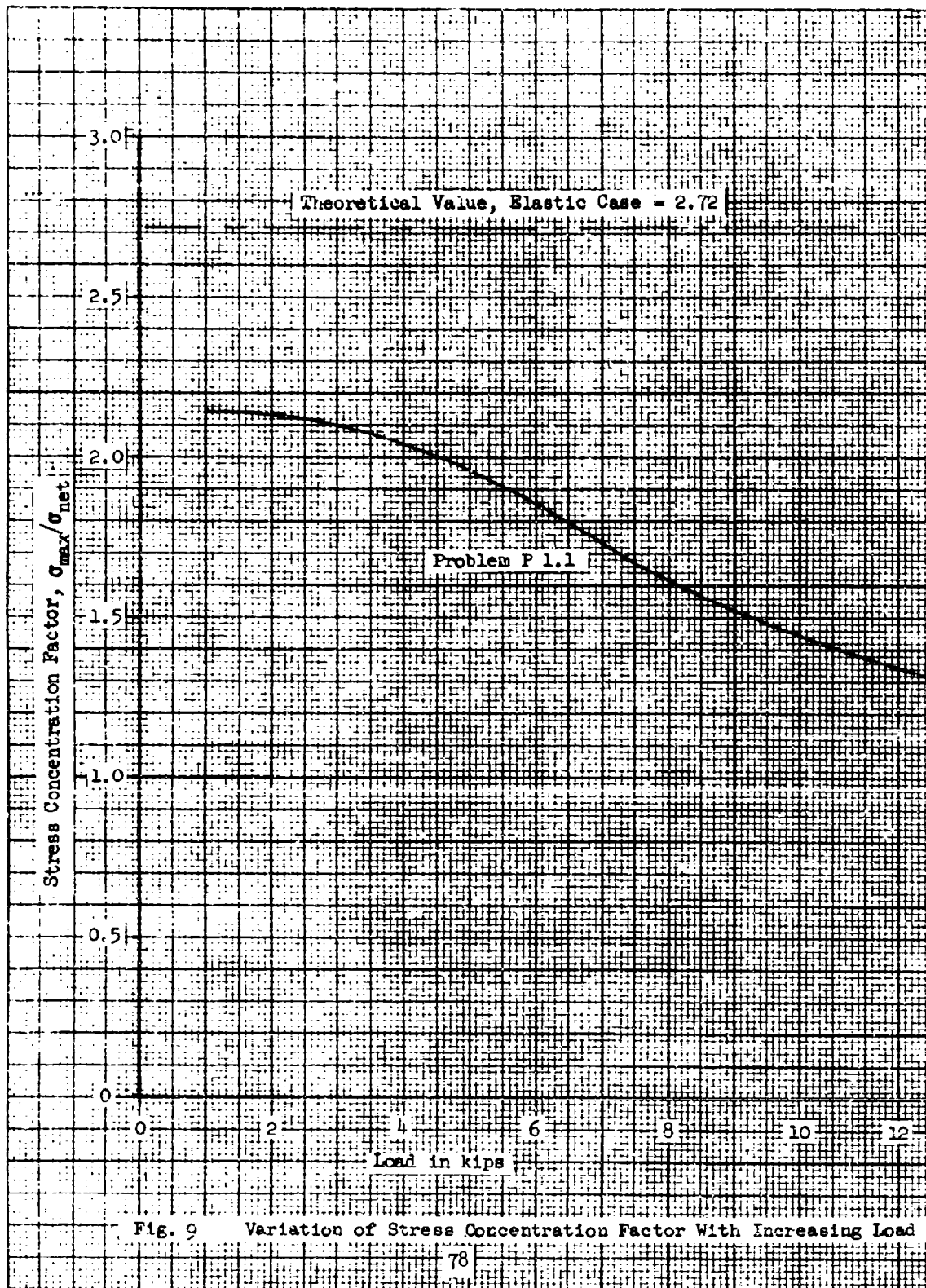
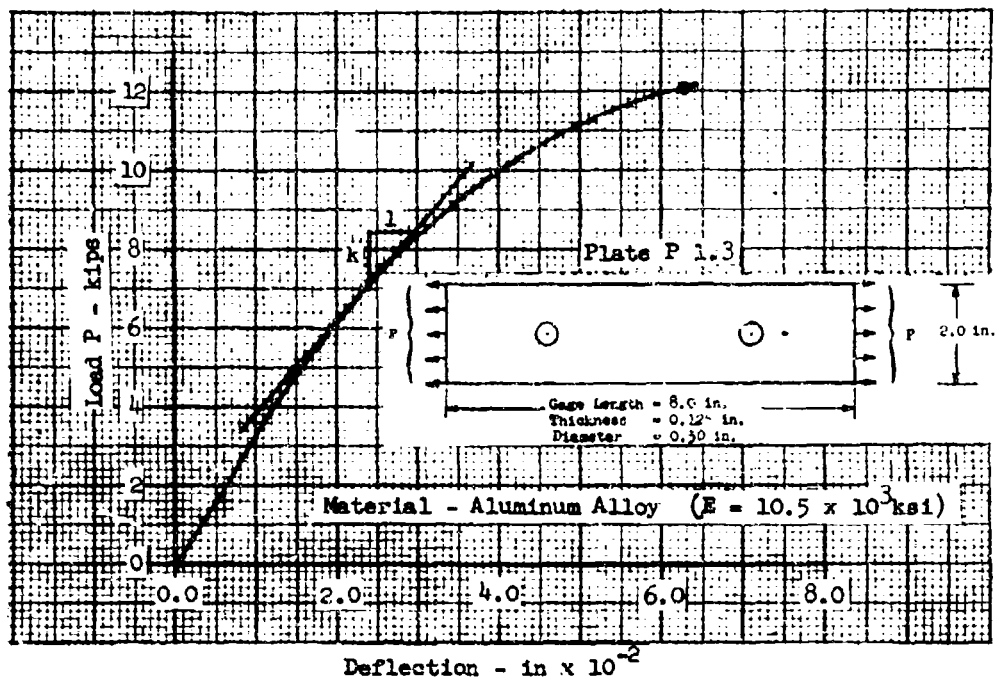
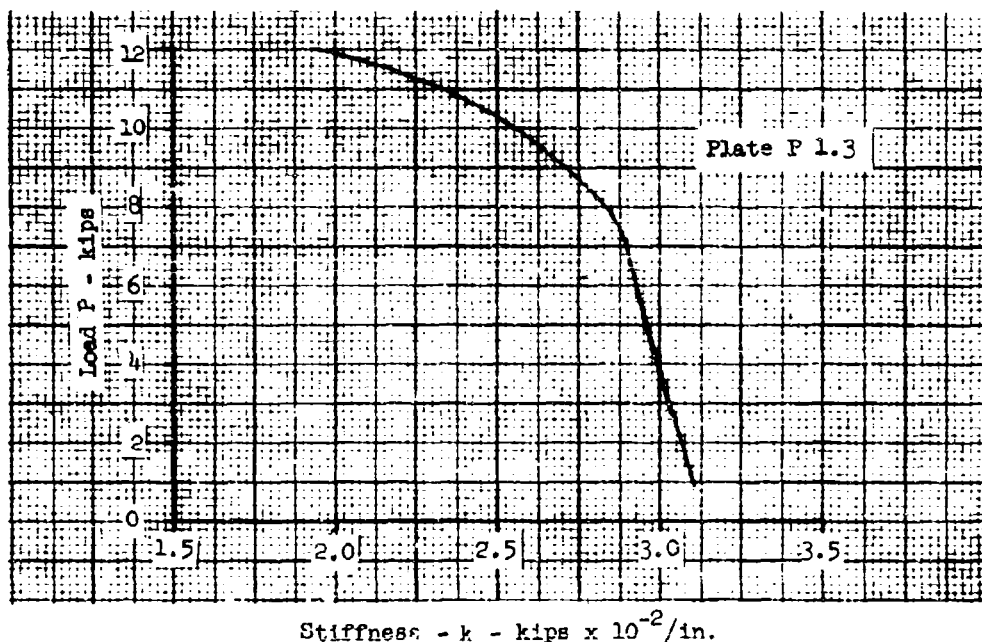


Fig. 8 Stress Distribution Across Net Section of Plate No. P 1.1





(a) Load-Deflection Curve



(b) Load vs. Tangent Modulus

Fig. 10 Variation in Stiffness or Slope of the Load-Deflection Curve with Increasing Load.

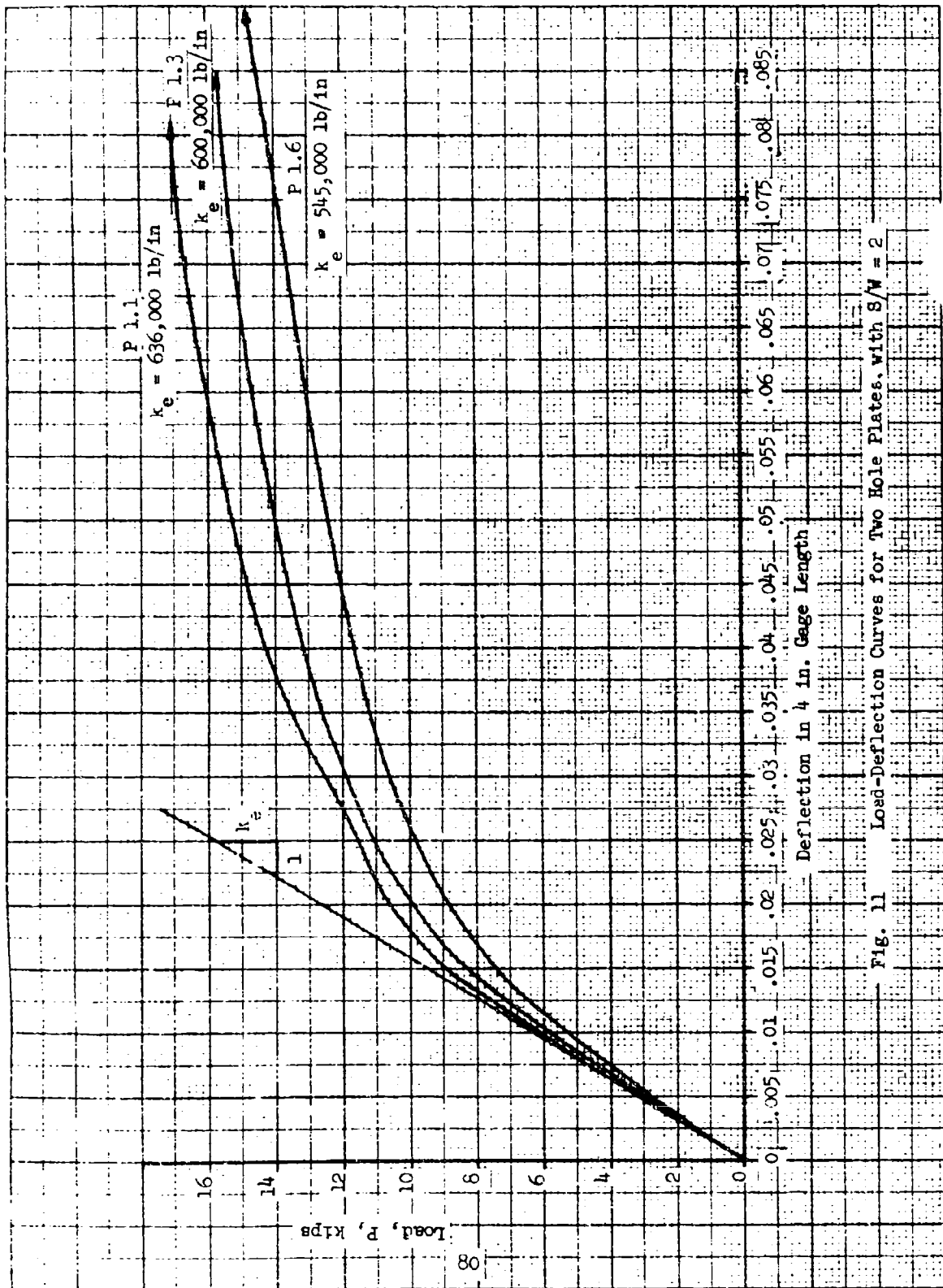


Fig. 11 Load-Deflection Curves for Two Hole Plates, with $S/W = 2$

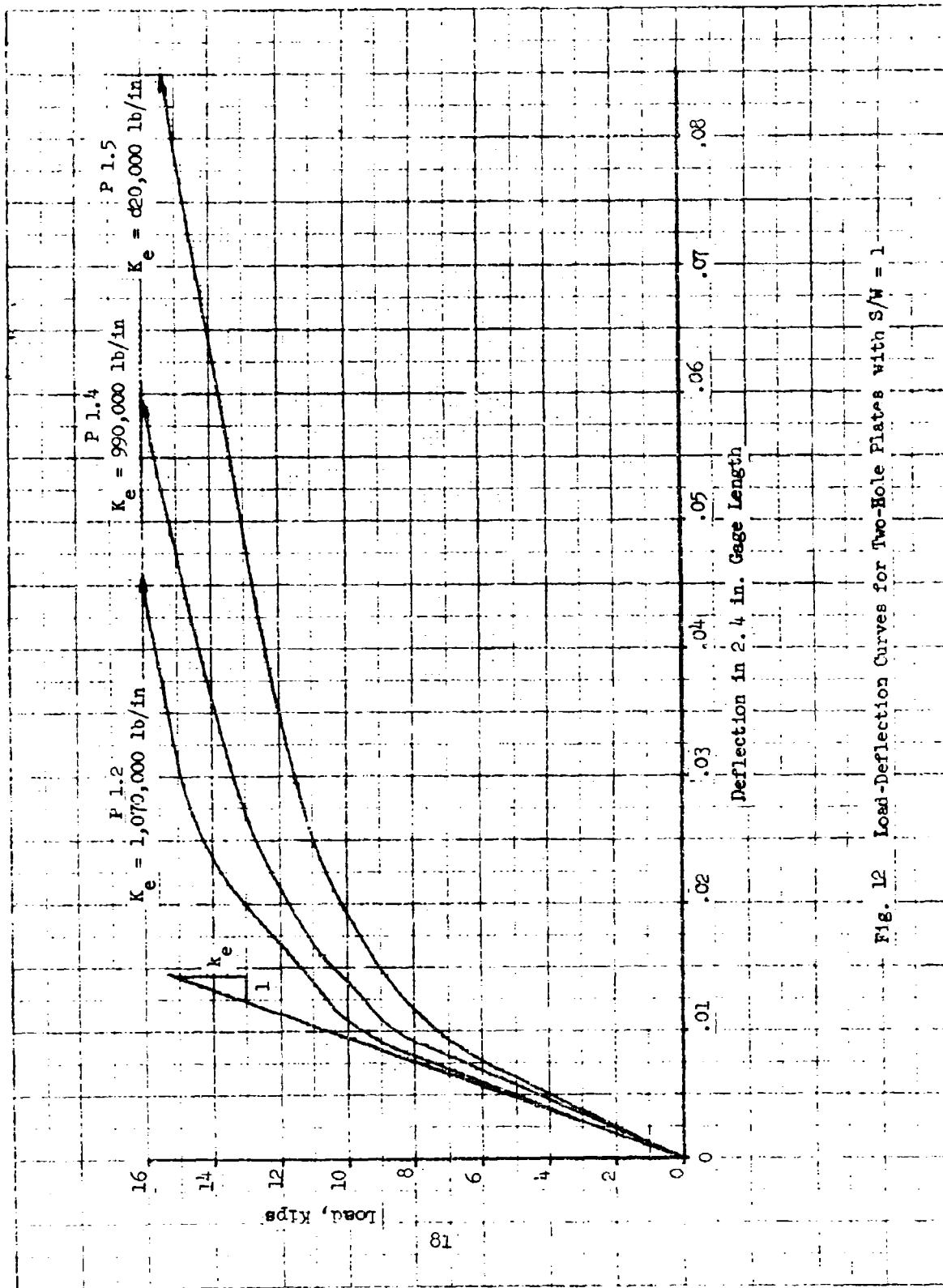


Fig. 12 Load-Deflection Curves for Two-Hole Plates with $S/W = 1$

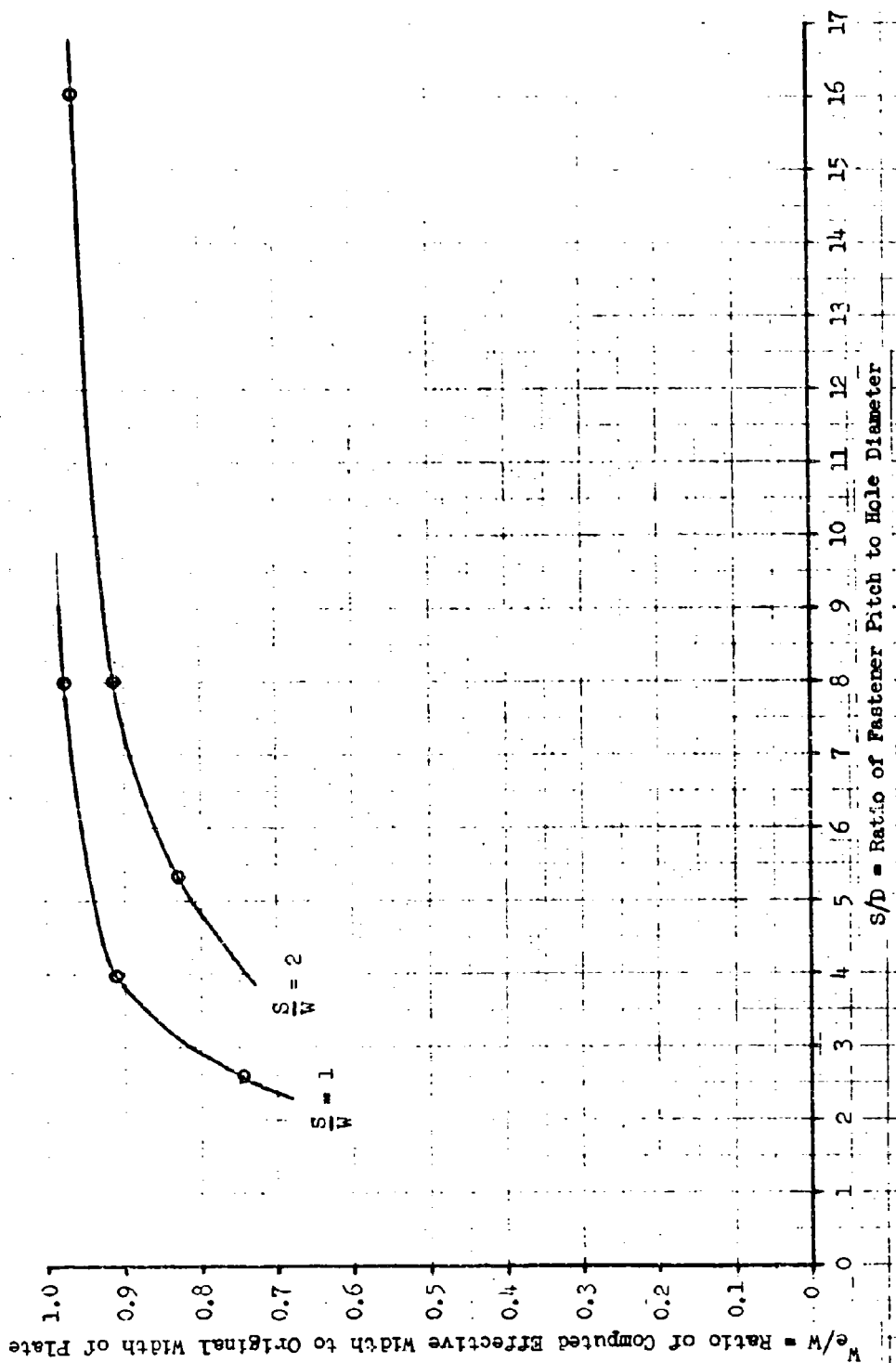


Fig. 13 Plate Effective Width as Function of the Pitch to Diameter Ratio, S/W

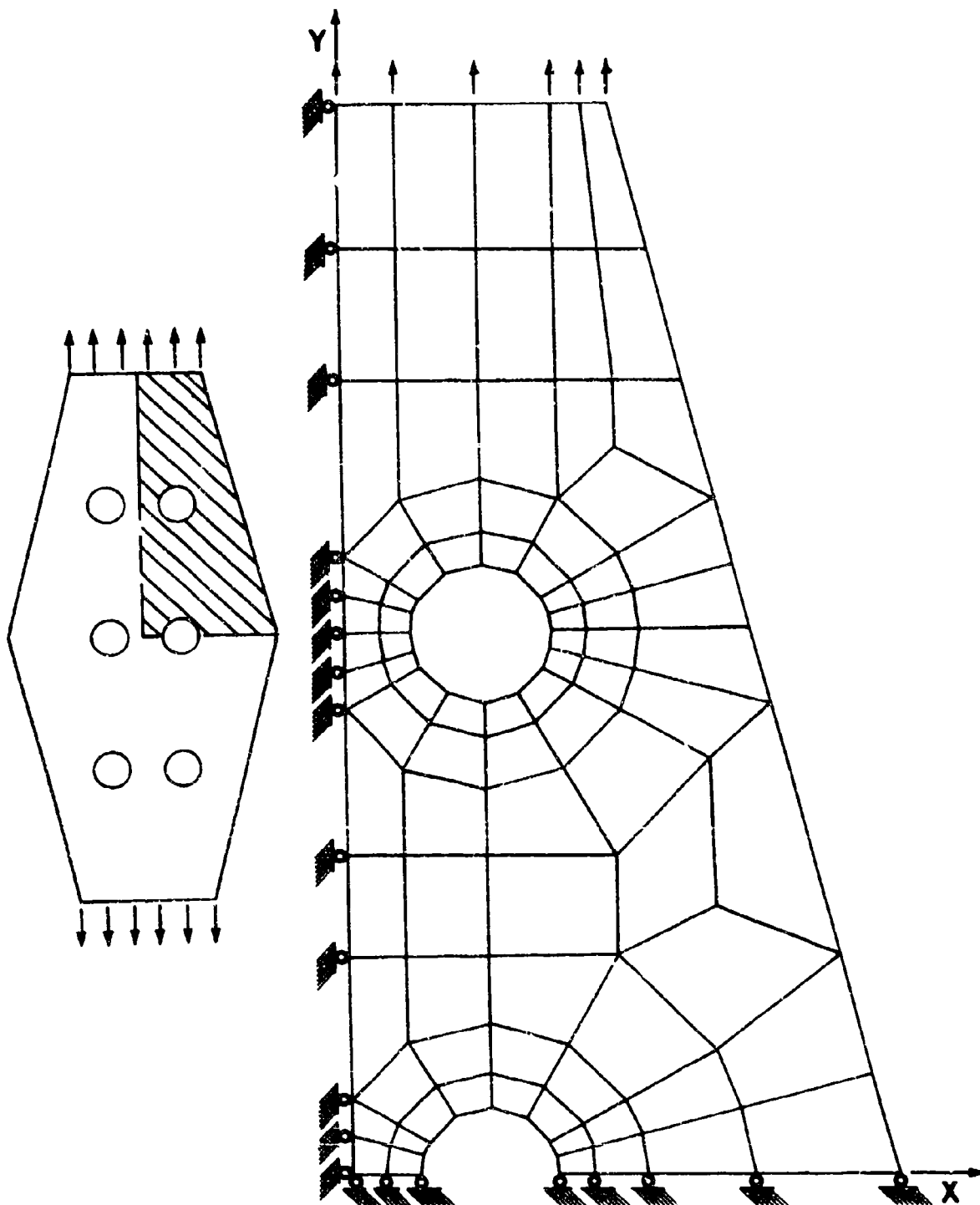


Fig. 14 Idealized Model for Evaluation of Effects of Width Taper

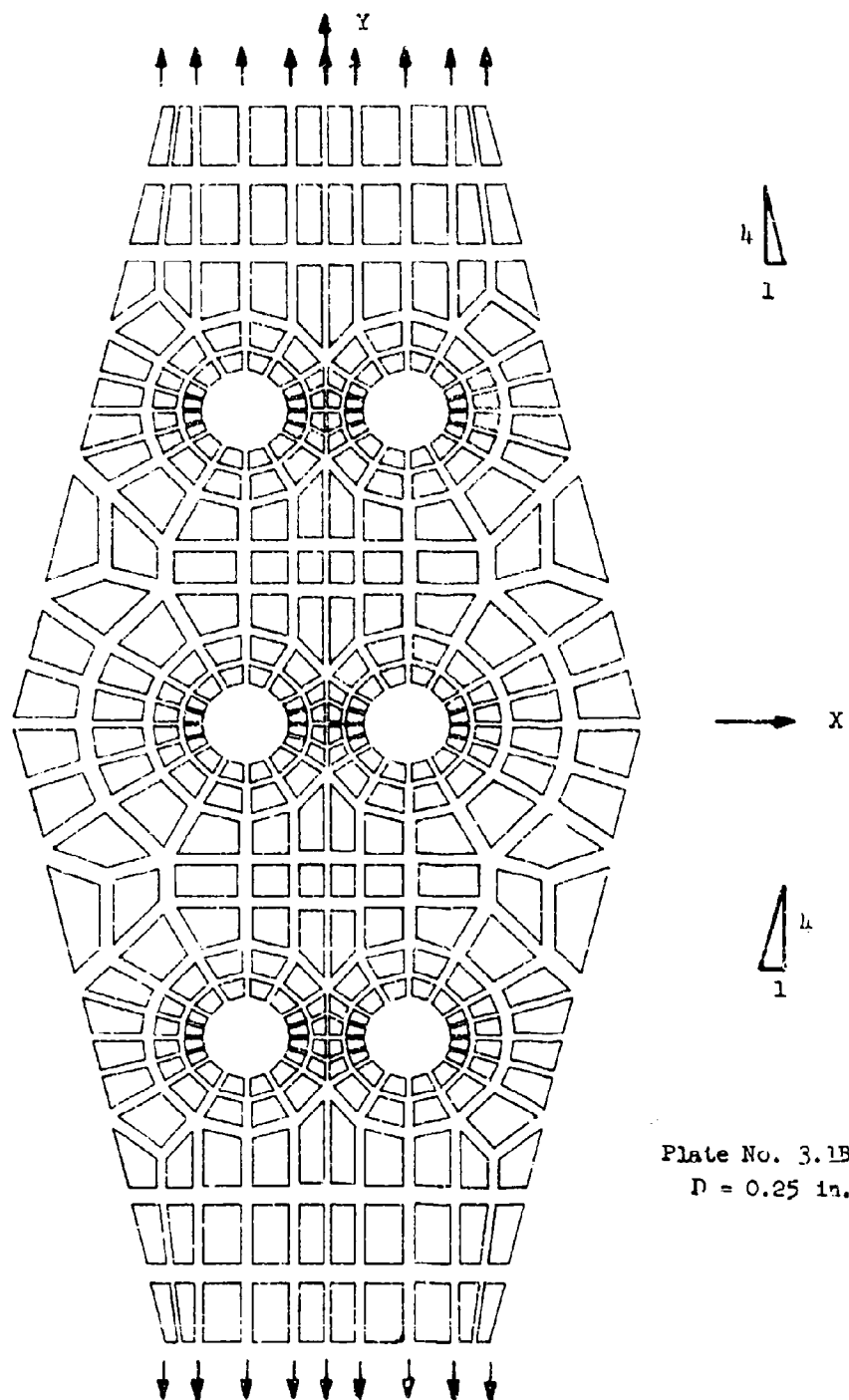


Plate No. 3.15
 $D = 0.25$ in.

Fig. 3.15 Idealization of Tapered Plate with Taper of 1:4 and $D = 0.25$ in.

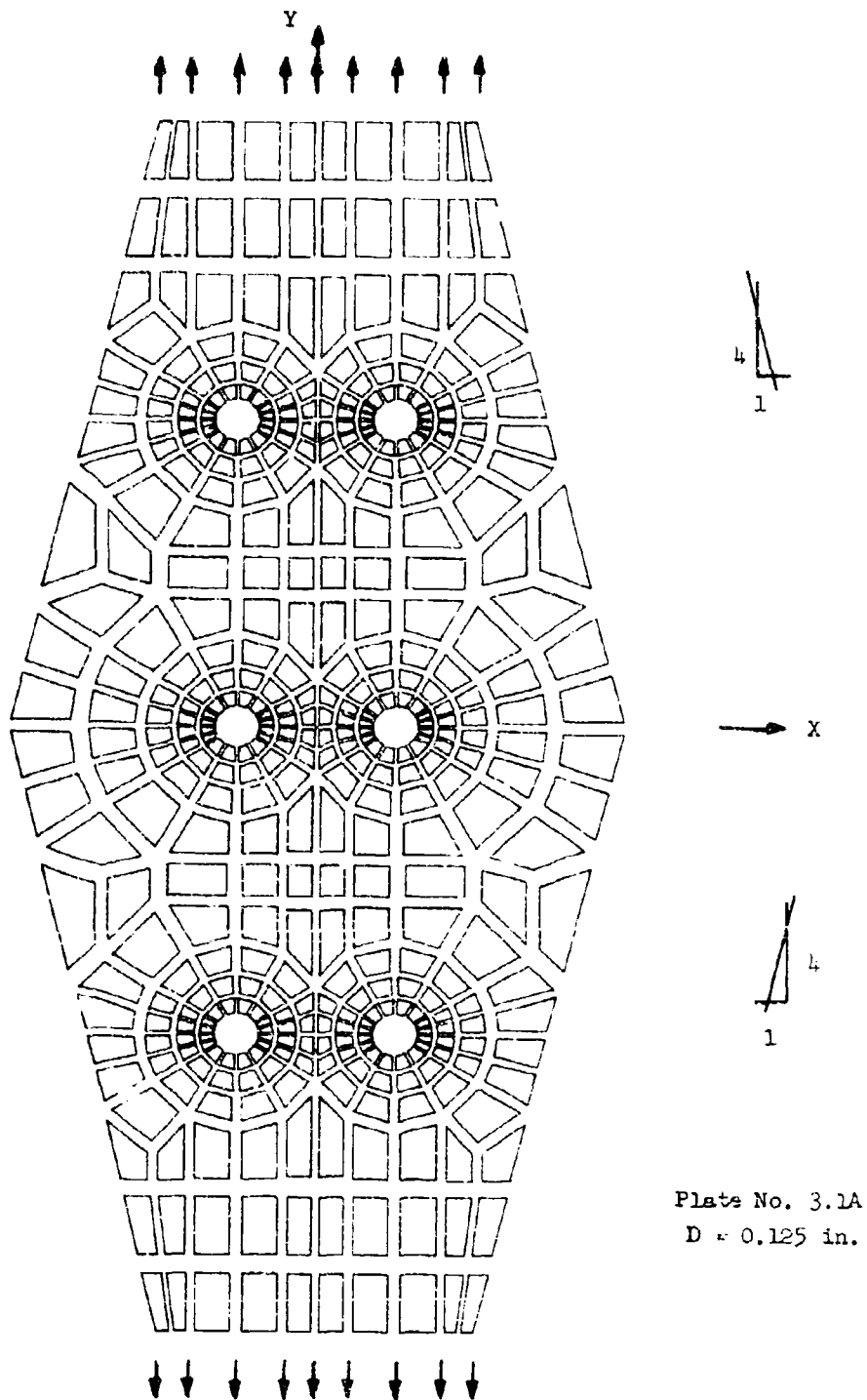


Fig. 16 Idealization of Tapered Plate with Taper of 1:4 and D = 0.125 in.

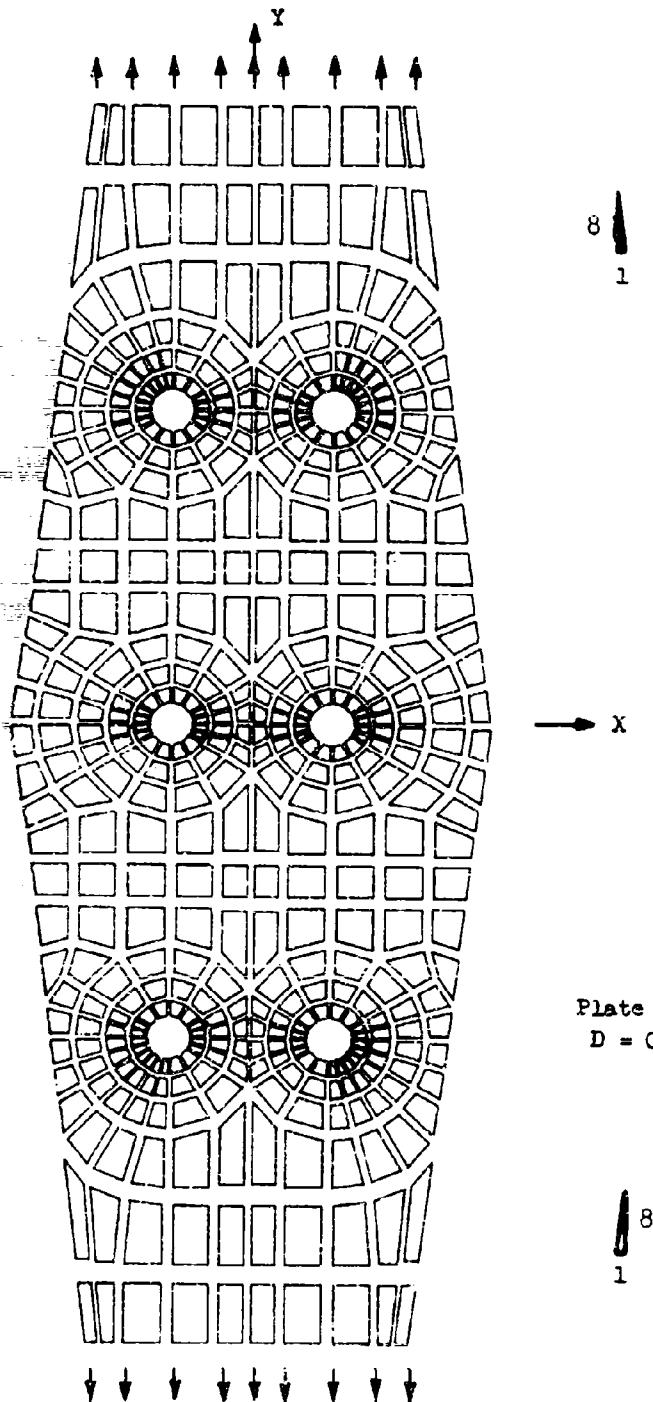


Plate No. 3.23
 $D = 0.125$ in.

Fig. 17 Idealization of Tapered Plate with Taper of 1% and $D = 0.125$ in.

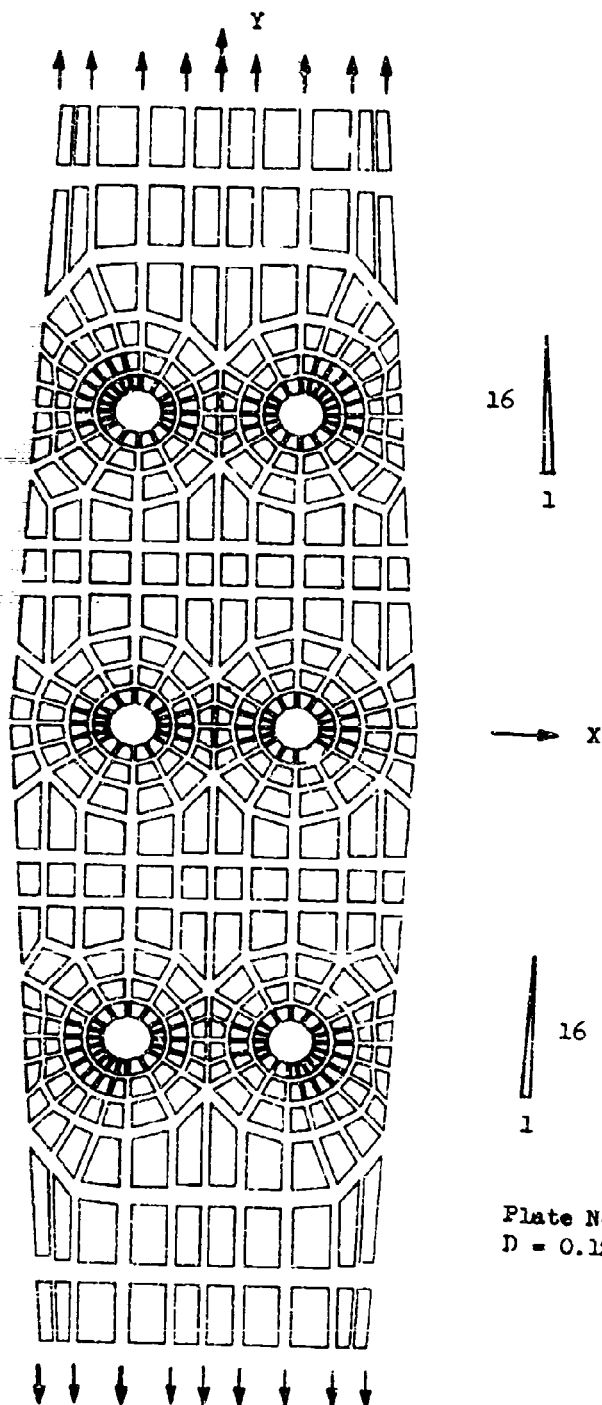
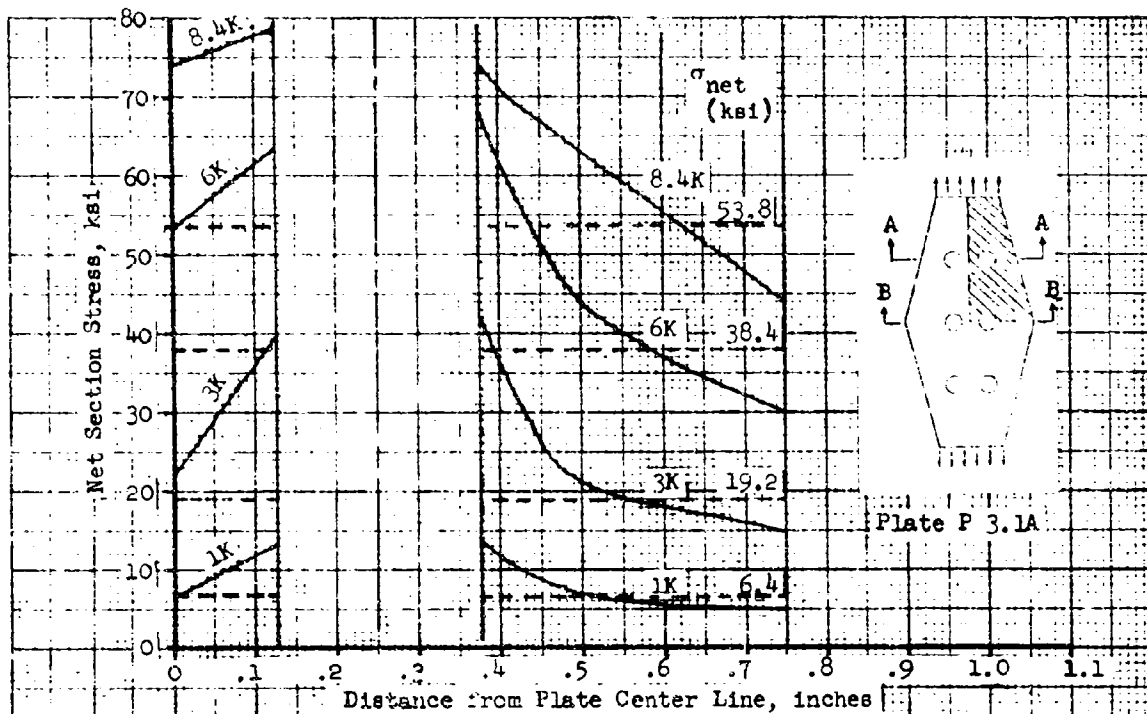
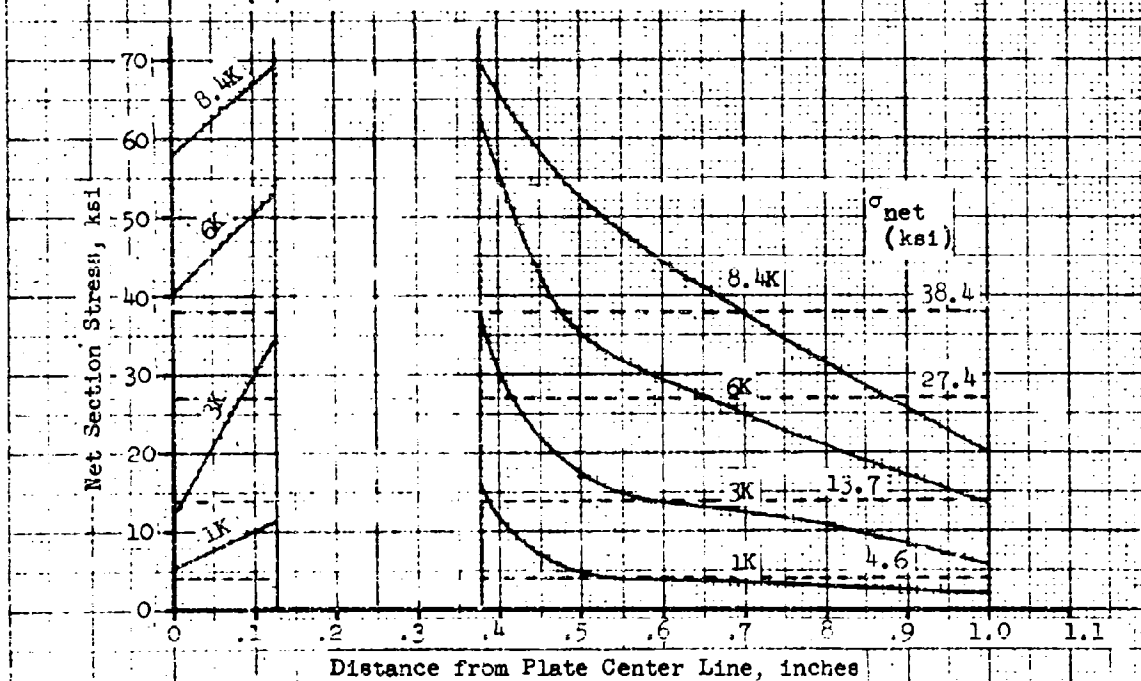


Plate No. 3.3B
 $D = 0.125$ in.

Fig. 18 Idealization of Tapered Plate with Taper of 1:16 and $D = 0.125$ in.



(a) Stress Distribution at Section A-A



(b) Stress Distribution at Section B-B

Fig. 19 Variation of Longitudinal Stress, σ_y , Across the Net Sections of Plate No. P 3.1A

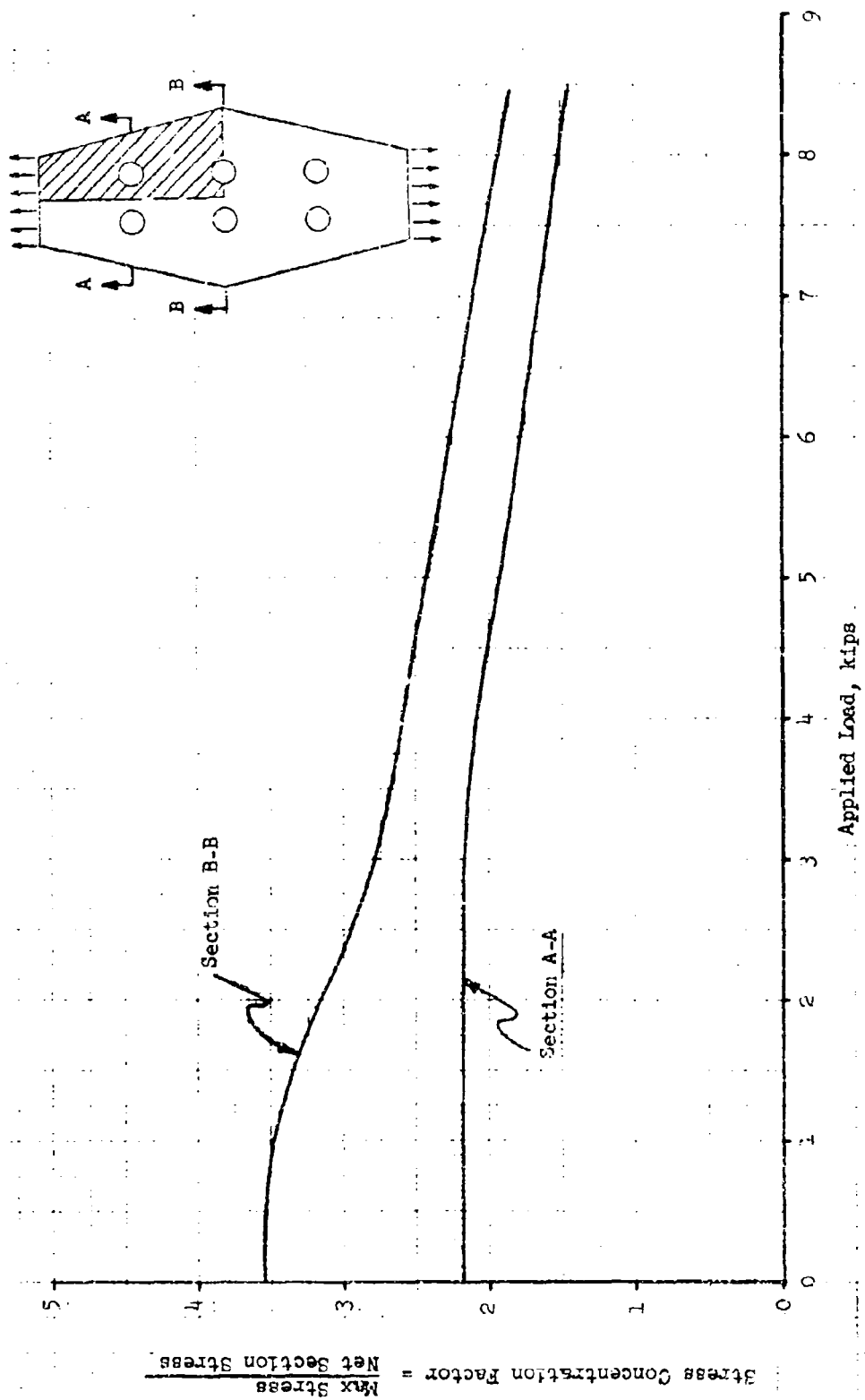


Fig. 20 Variation of Stress Concentration Factors with Load for Tapered Plate P 3.1A

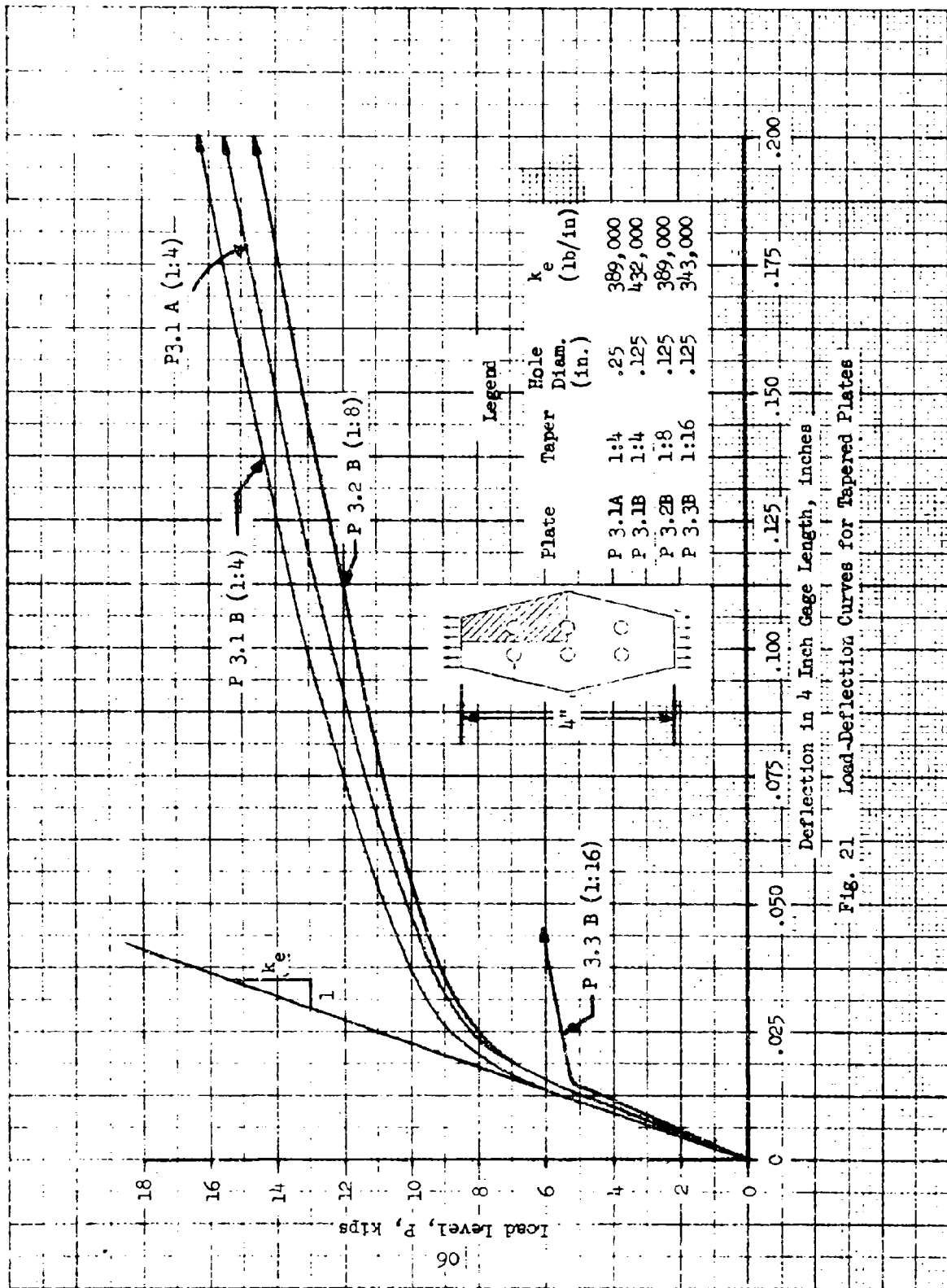


Fig. 21 Load-Deflection Curves for Tapered Plates

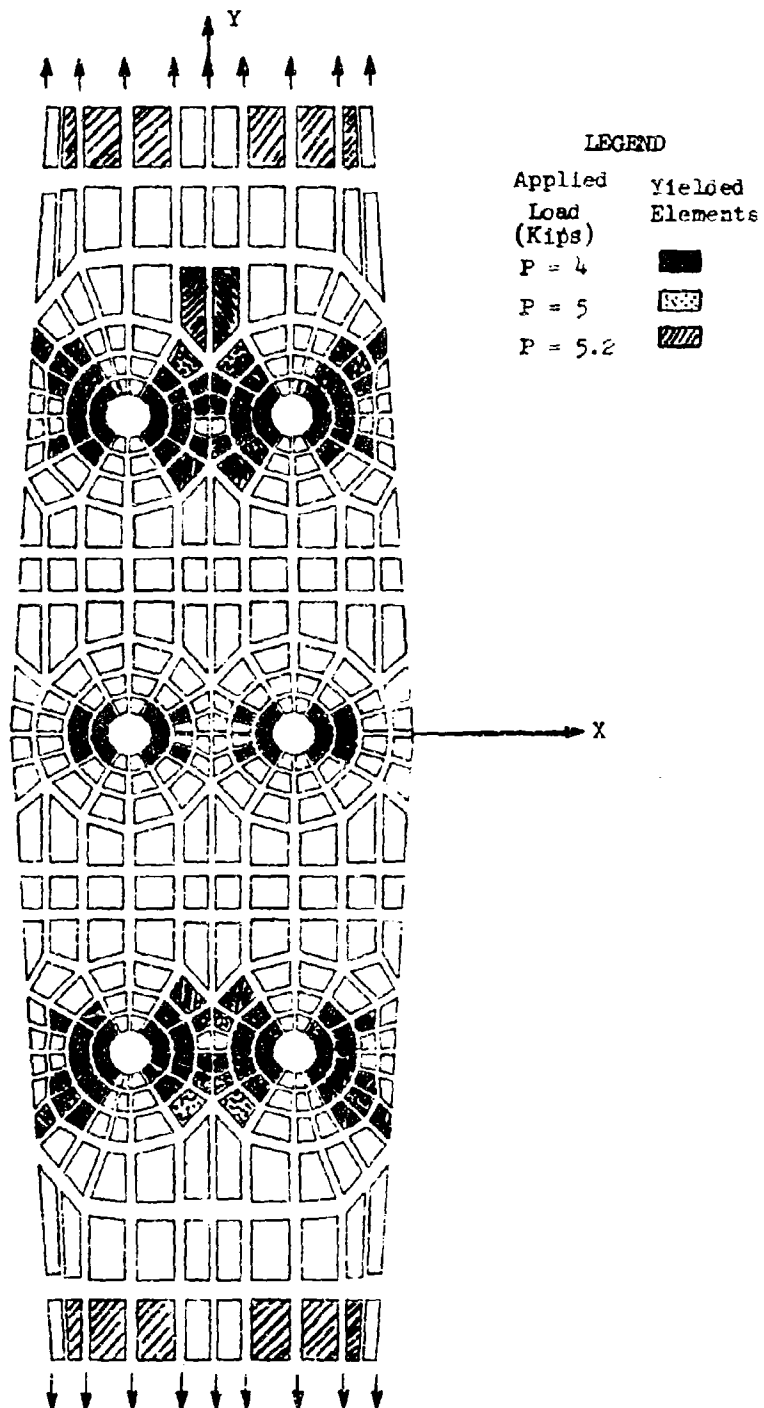
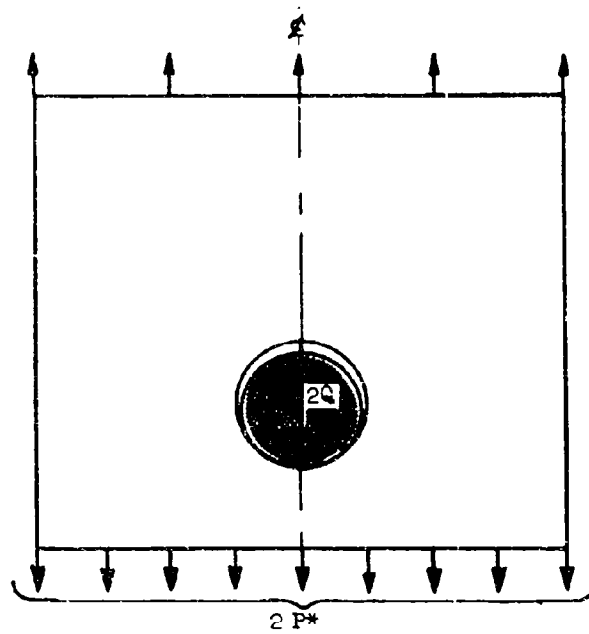
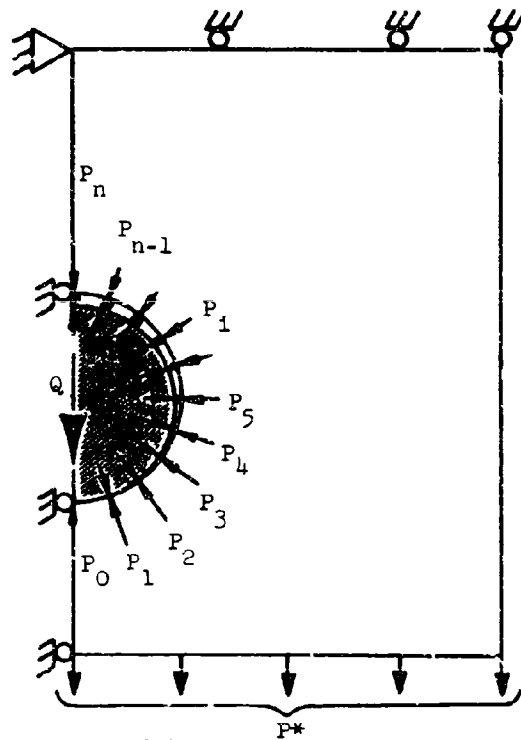


Fig. 22 Progression of Yield Zones with Increasing Applied Load, Plate P 3.3B



(a) Region Around a Typical Fastener



Q = Transfer Load
 P^* = By-pass Load

(b) Idealization Used

Fig. 23 Mathematical Model of Plate Loaded Through a Fastener

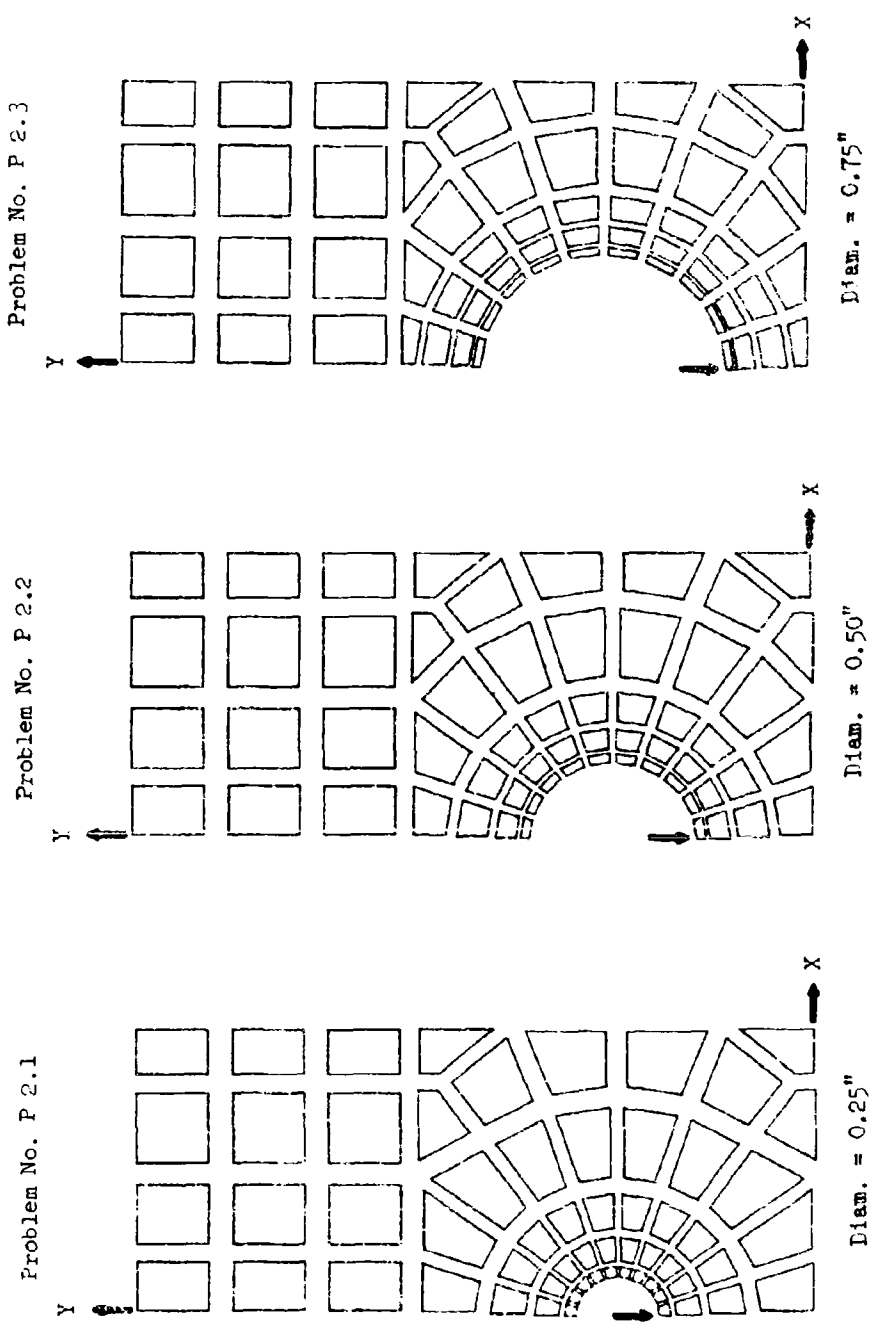


Fig. 24 Finite Element Idealizations of Aluminum Joints With $e = 0.625$ Inches

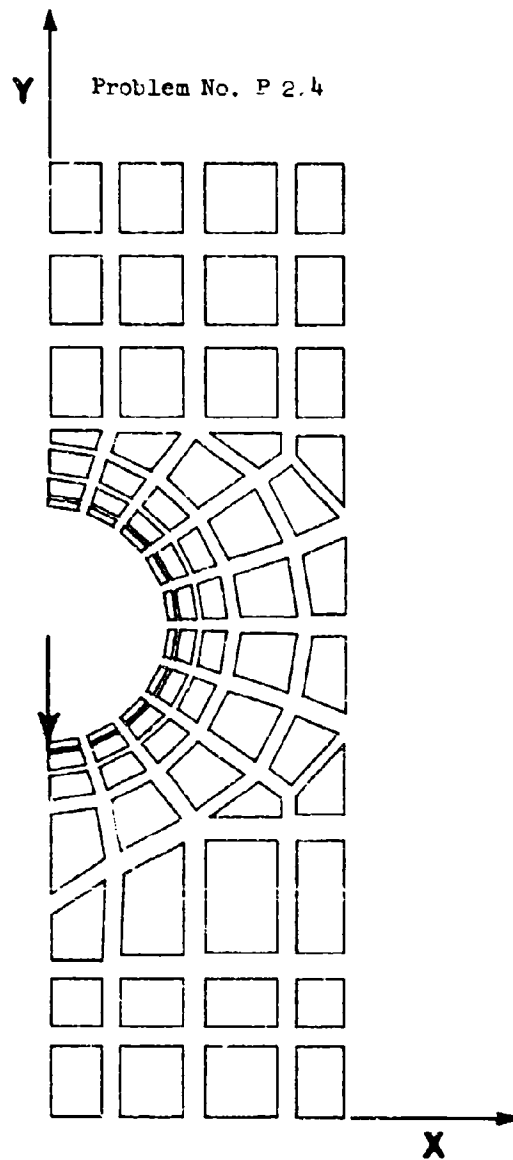


Fig. 25 Finite Element Idealization of Aluminum Joint With $e = 1.6$ Inches

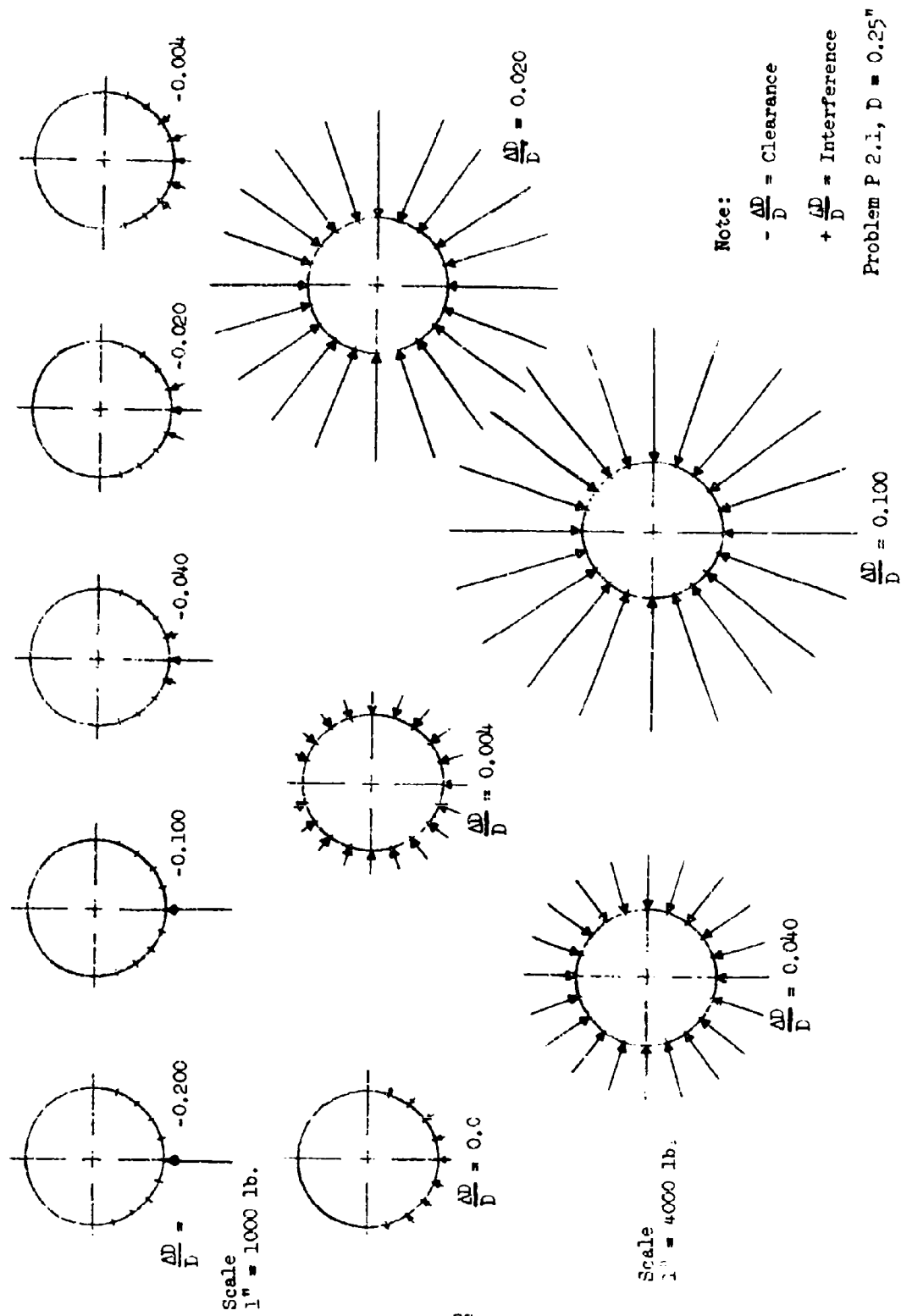


Fig. 26 Load Distribution Around Fastener at 500 lb. Applied Load

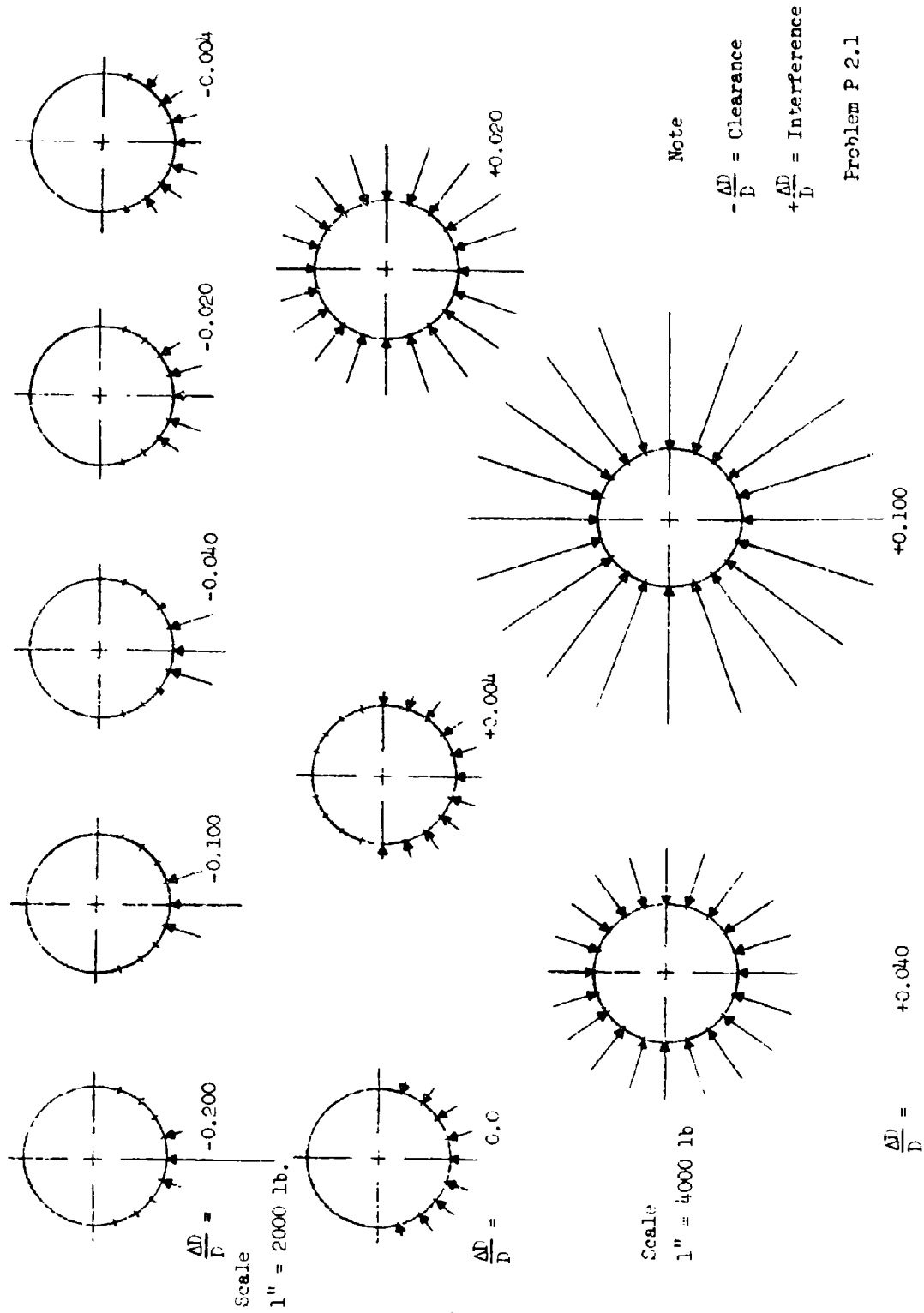


Fig. 27 Load Distribution Around Fastener at 2000 lb Applied Load

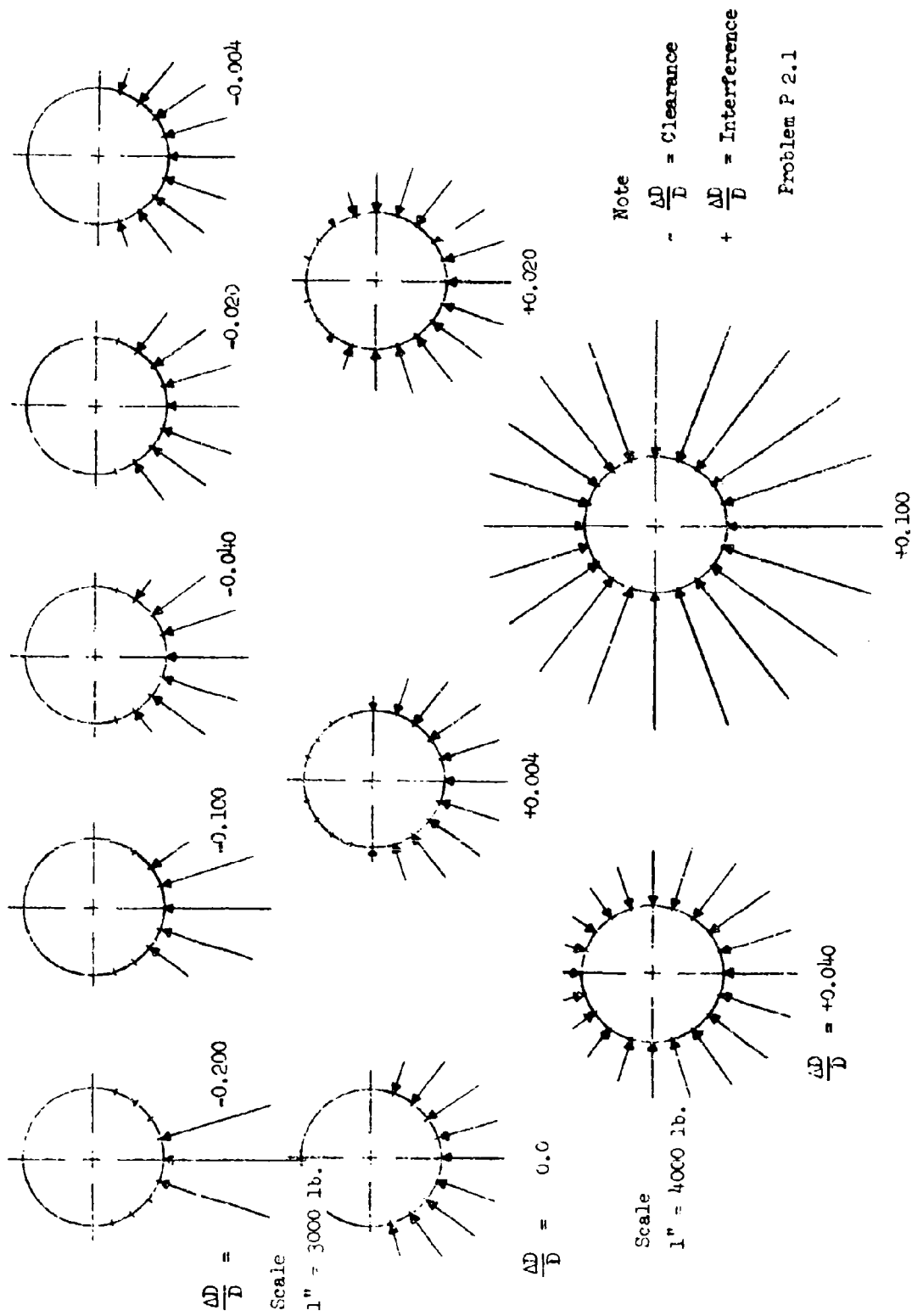
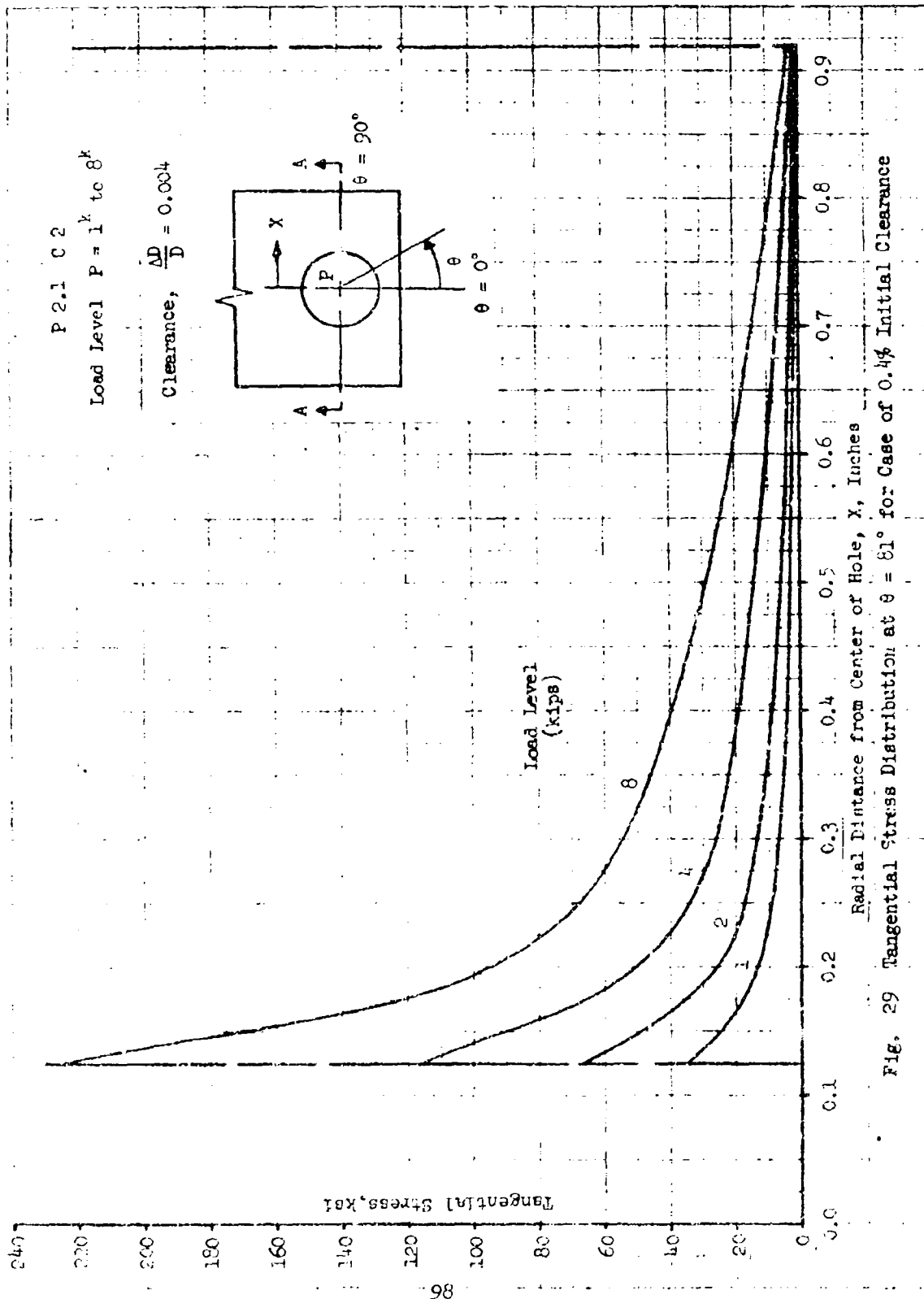


Fig. 28 Load Distribution Around Fastener at 8000 lbs. Applied Load



P 2.1 C 3

Load Level $P = 1k$ to $8k$

Clearance, $\frac{\Delta D}{D} = 0.02$

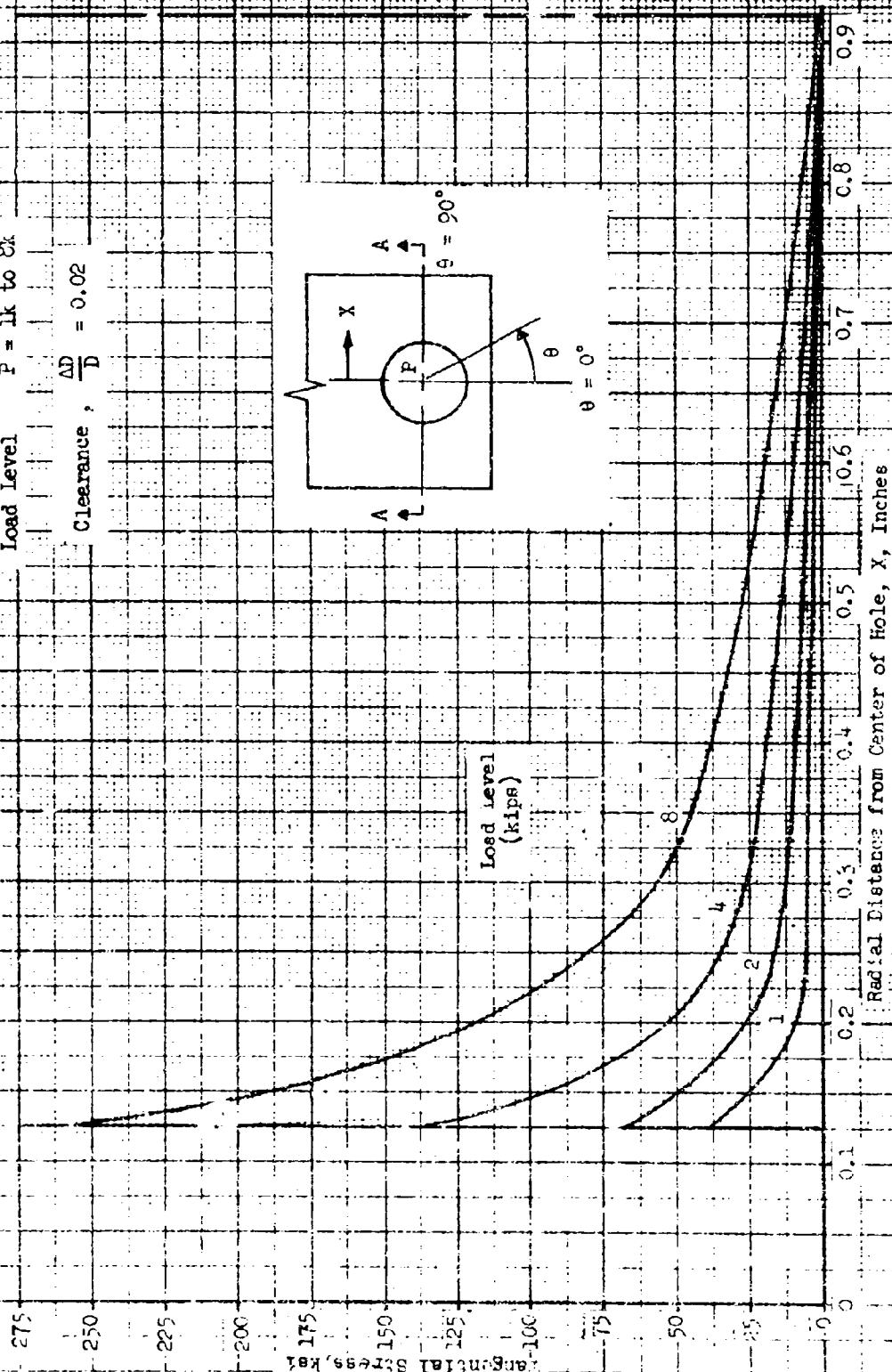
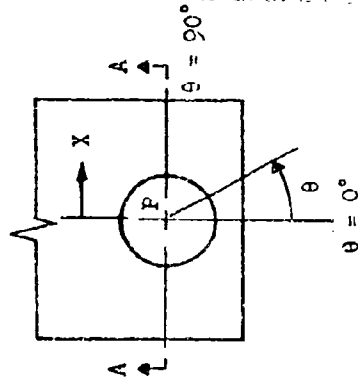


Fig. 30 Tangential Stress Distribution at $\theta = 91^\circ$ For Case of 2% Initial Clearance

P 2.1 C 7 to C 10

Load Level $P = 500$ lbs.

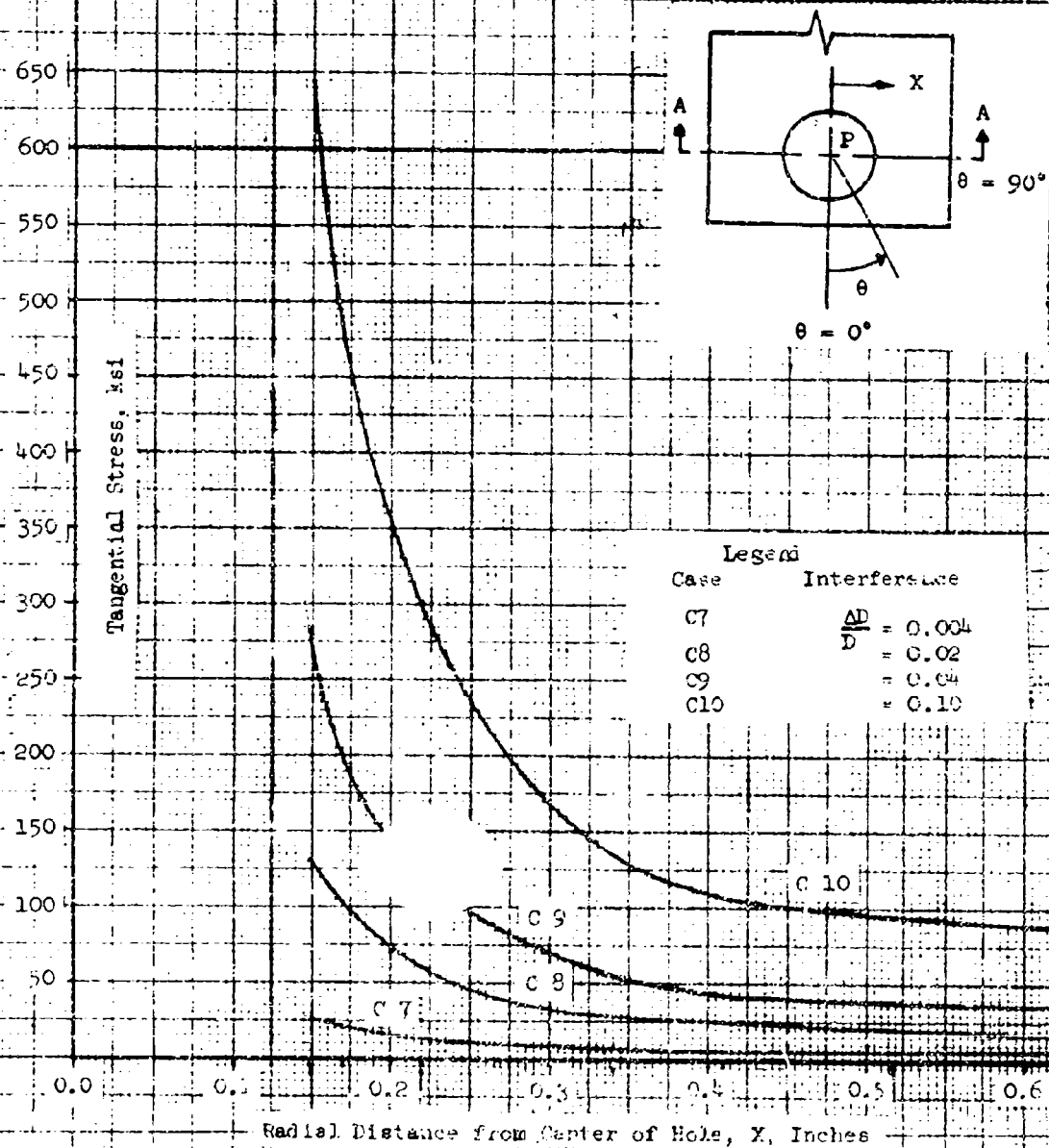


Fig. 31 Maximum Tangential Stress Variation for Different Interference Fits.

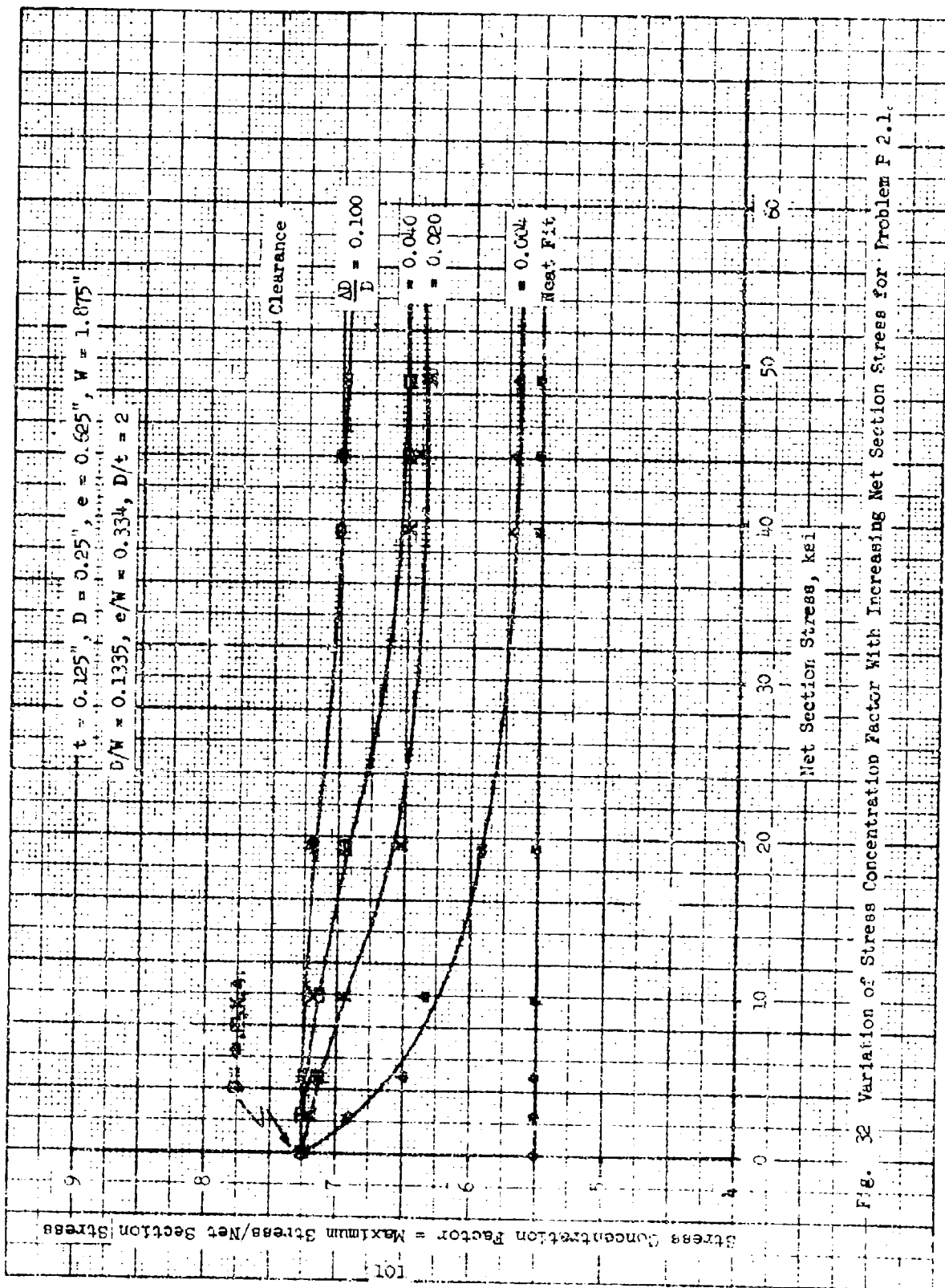


Fig. 32 Variation of Stress Concentration Factor With Increasing Net Section Stress for Problem P 2.1.

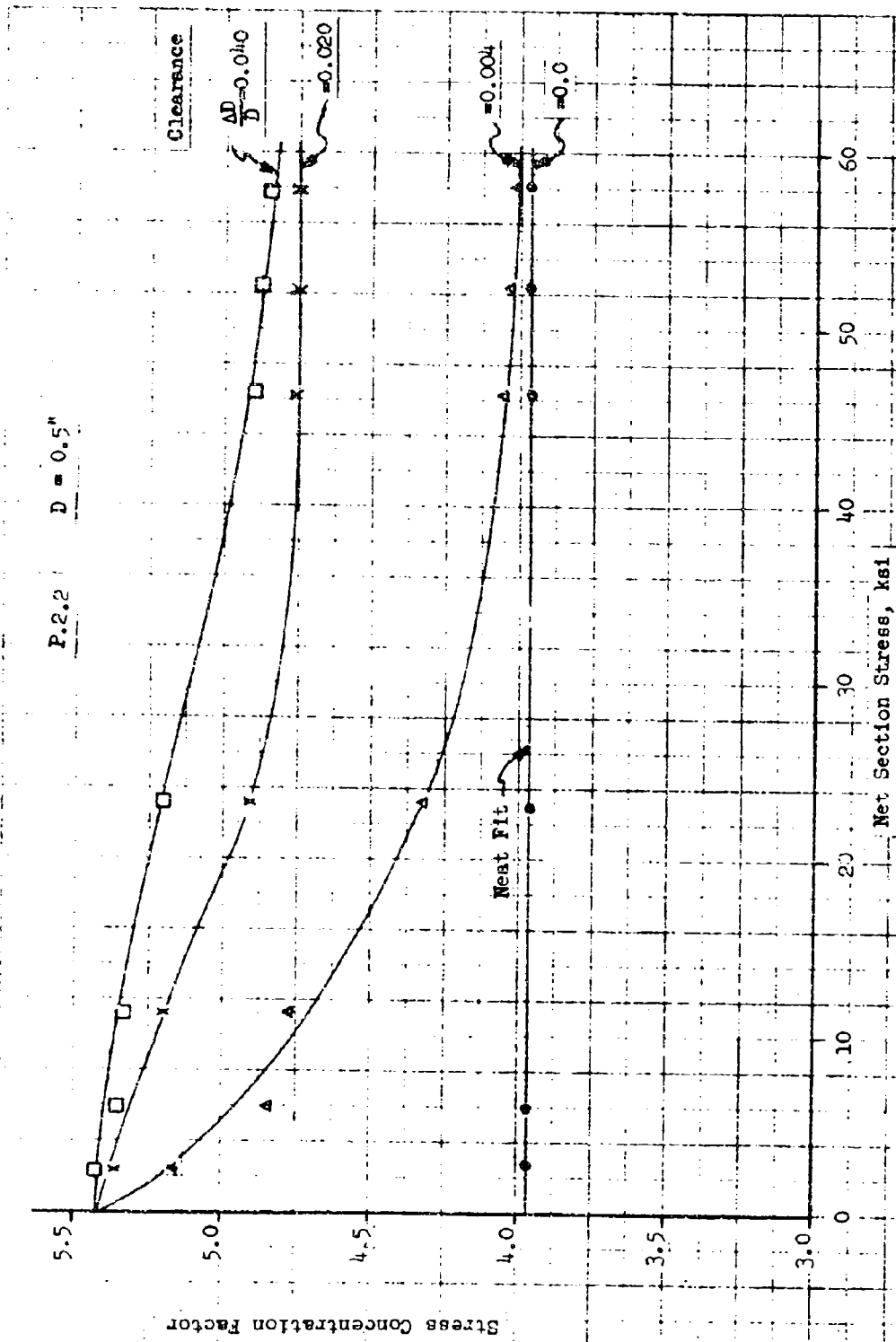


Fig. 33 Variation of Stress Concentration Factor With Increasing Net Section Stress For Problem P.2.2

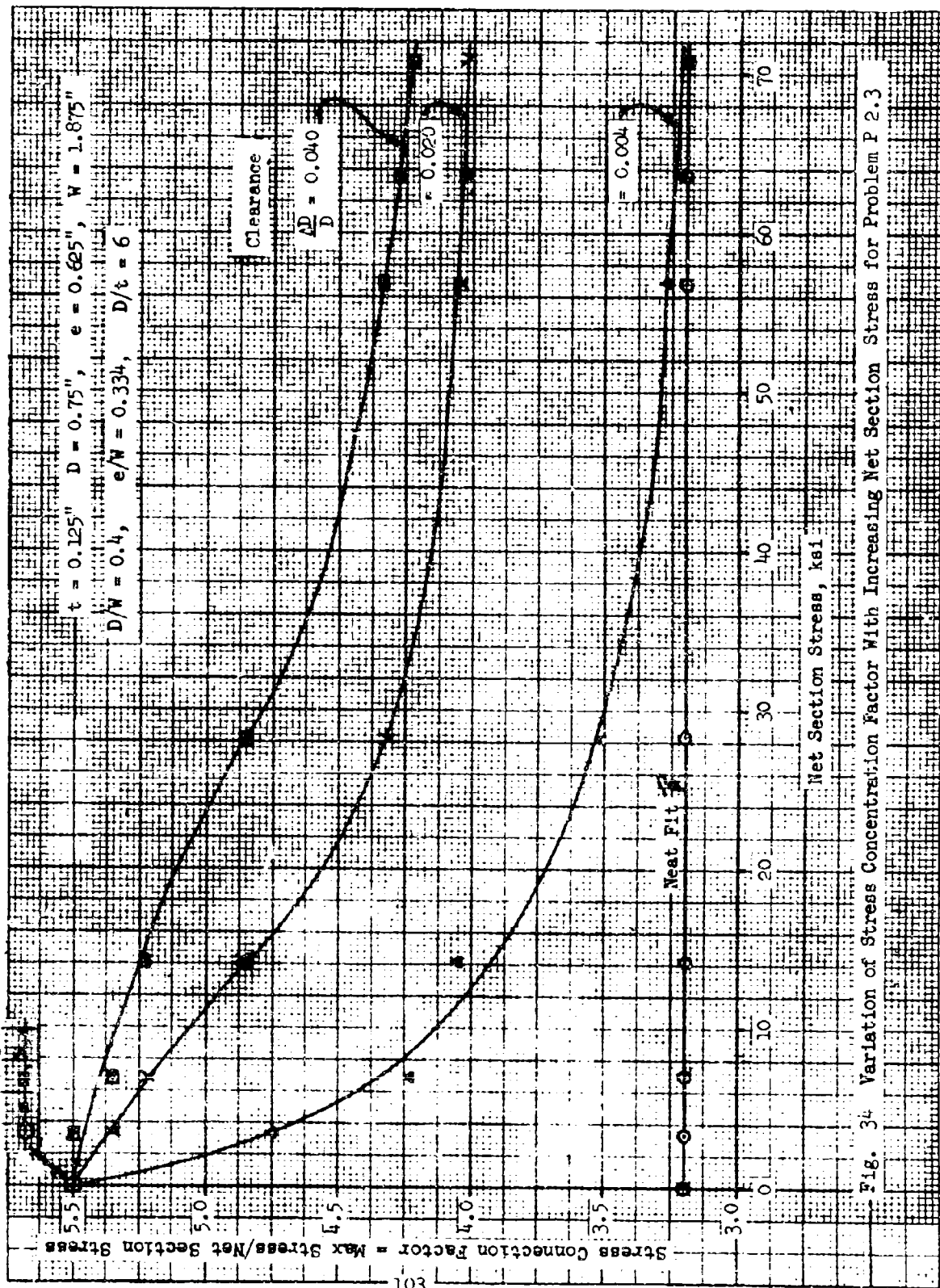


Fig. 34 Variation of Stress Concentration Factor With Increasing Net Section Stress for Problem P 2.3

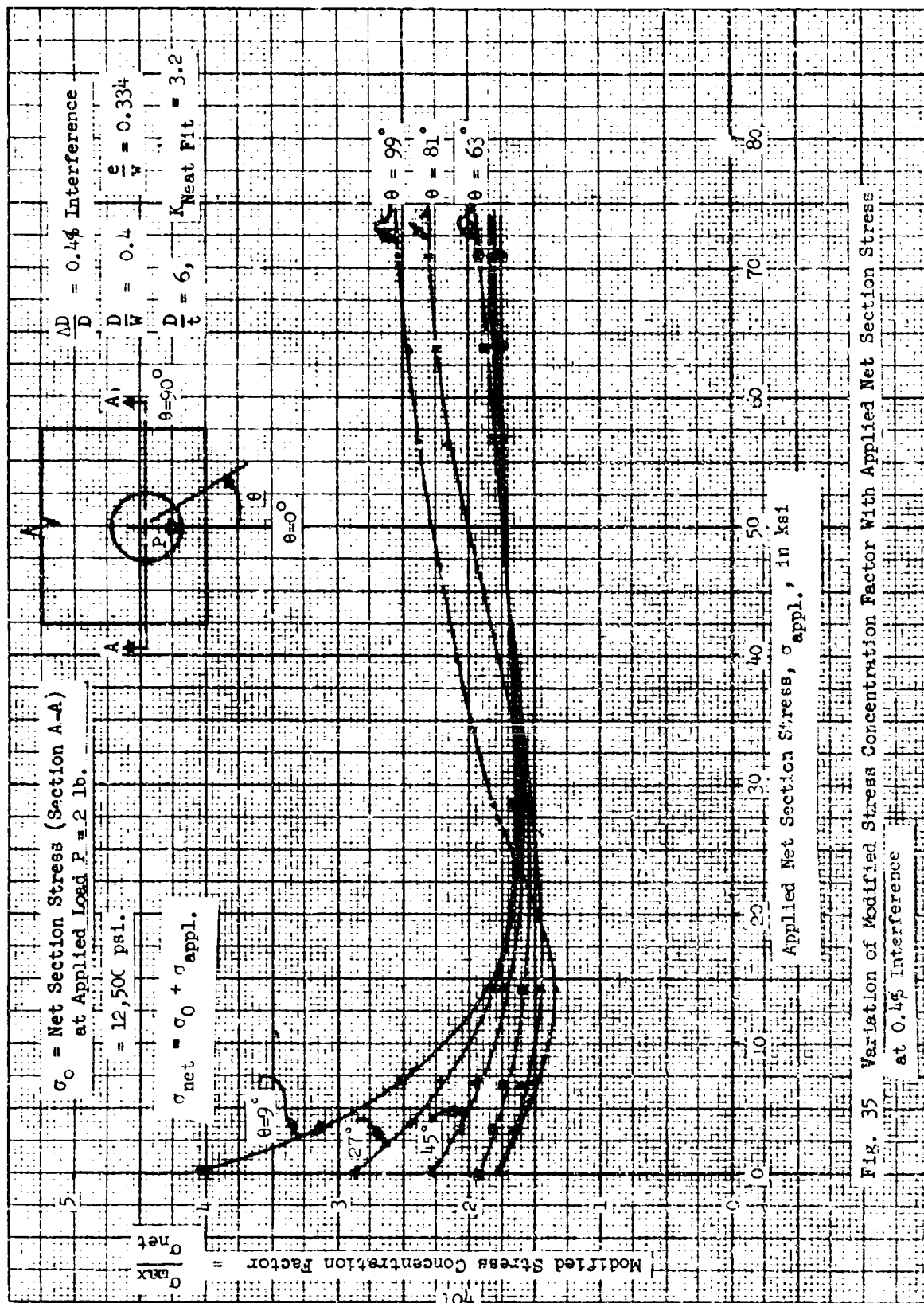
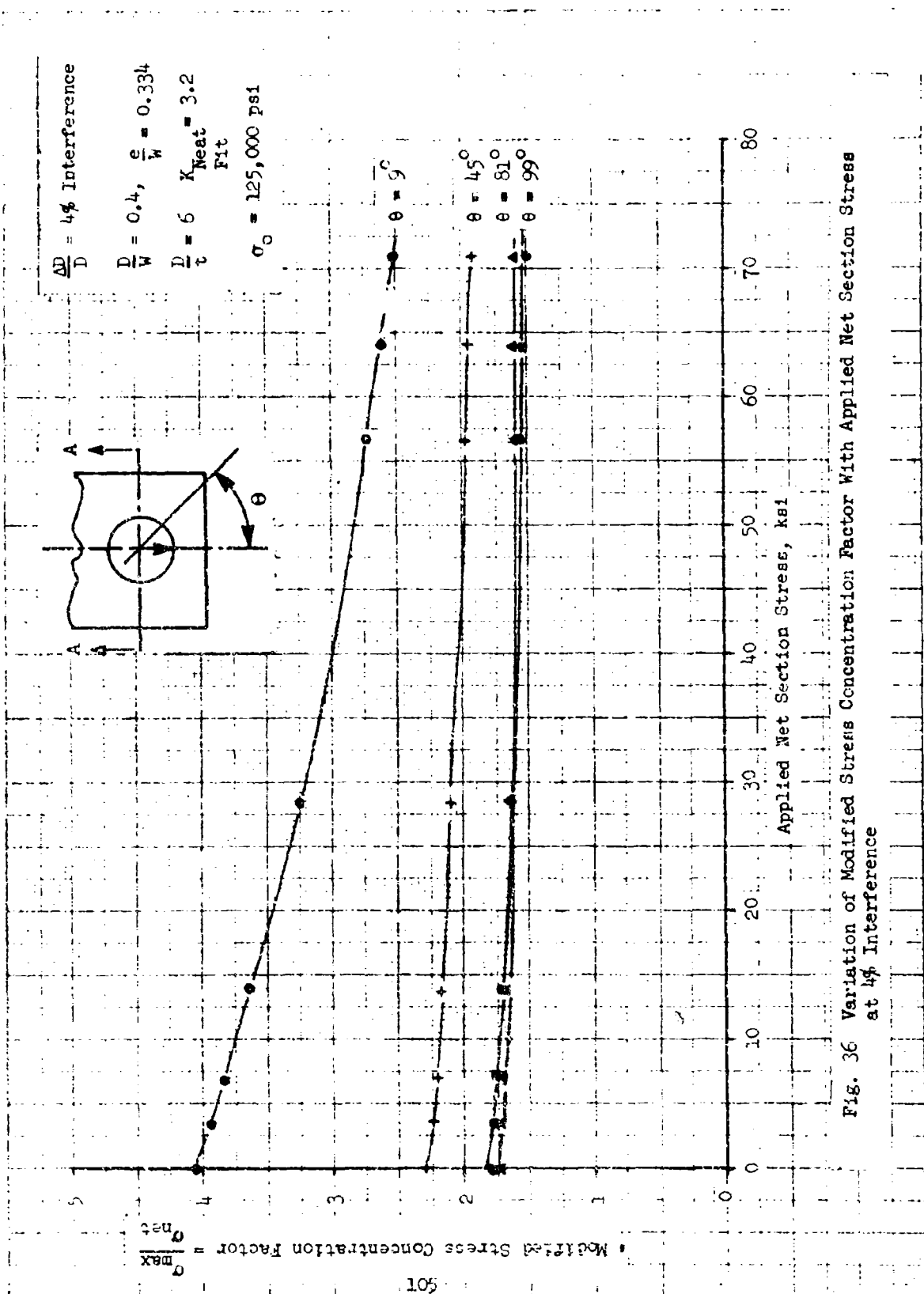
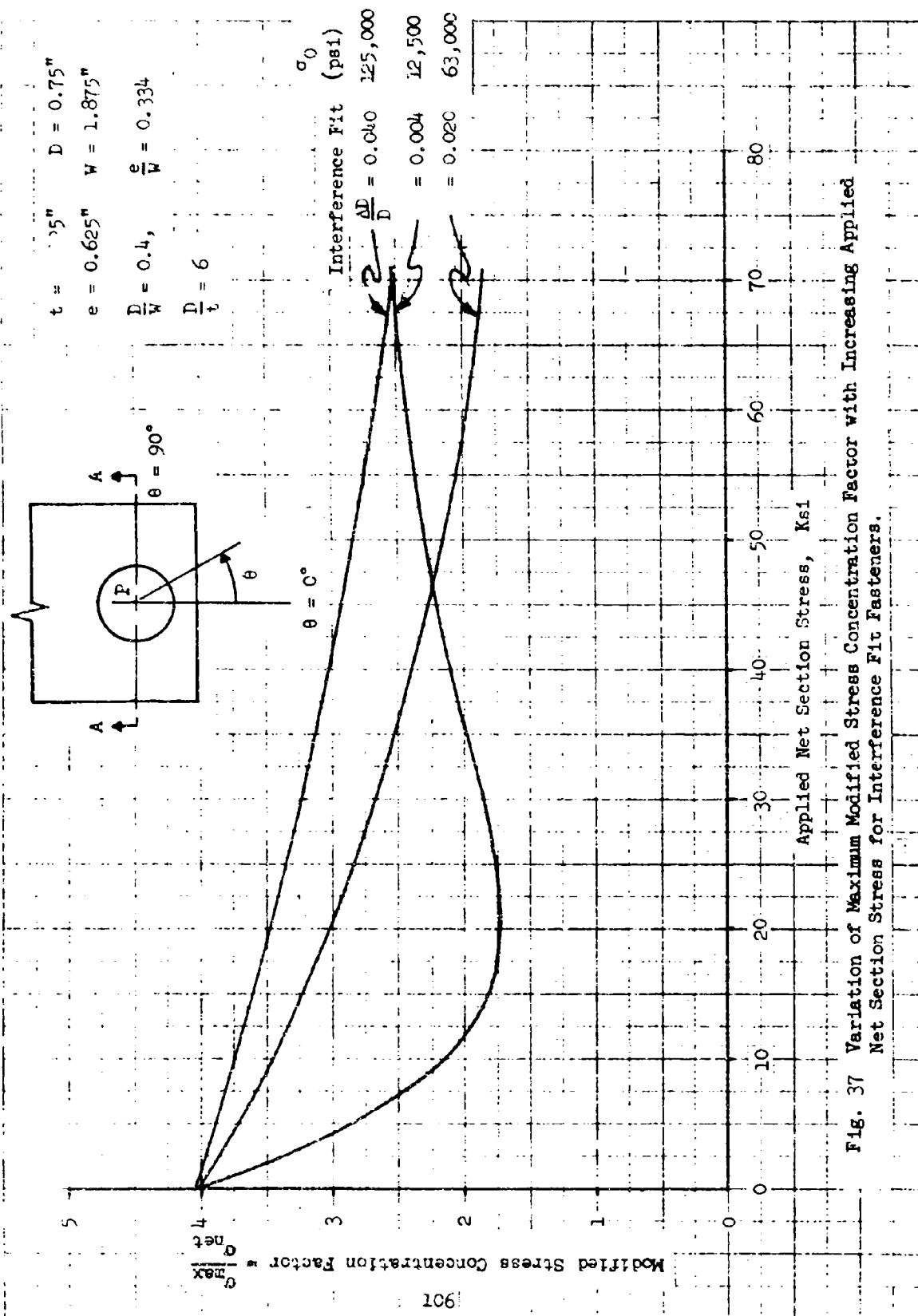


Fig. 35 Variation of Modified Stress Concentration Factor With Applied Net Section Stress at 0.4% Interference





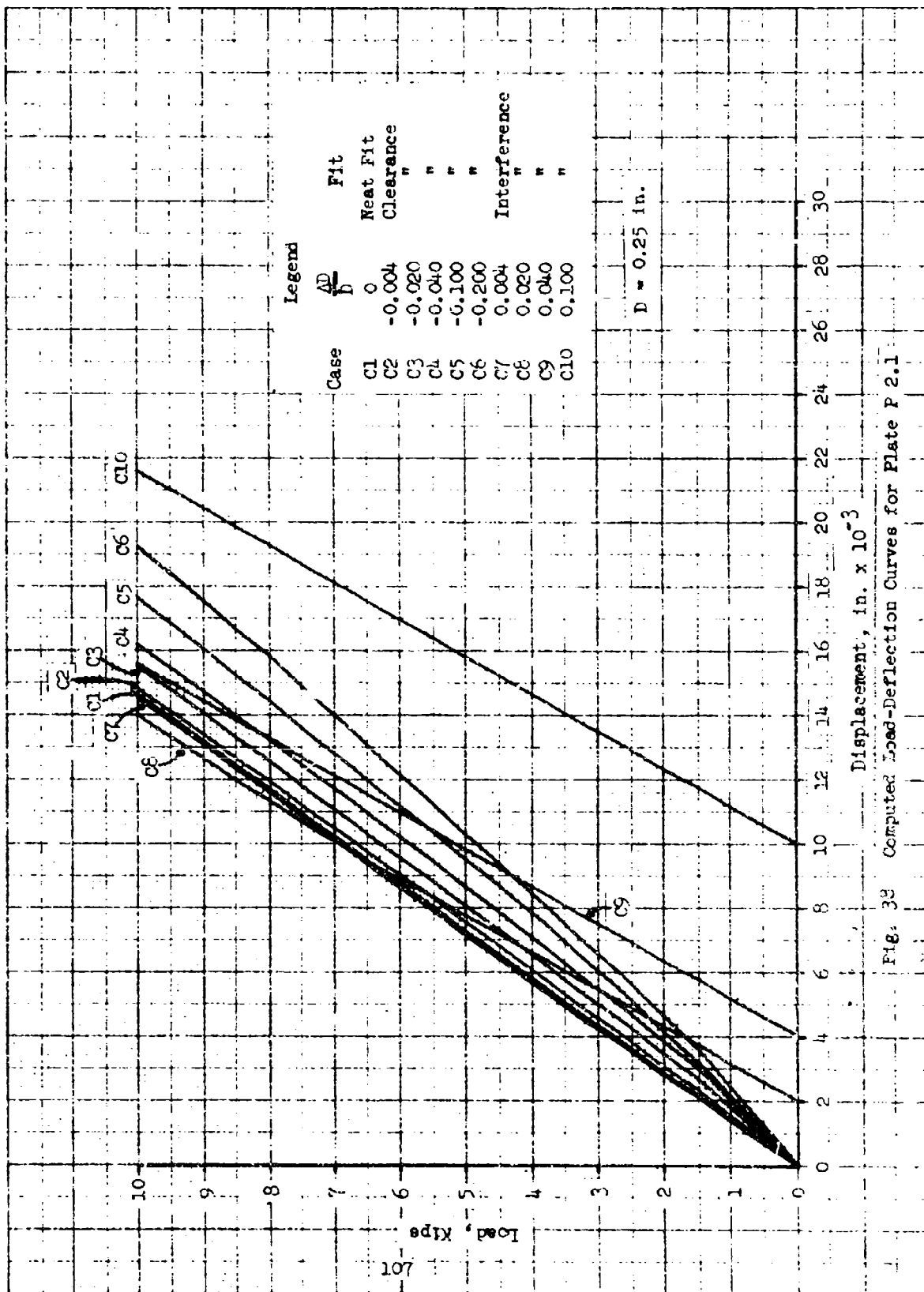


Fig. 38 Computed Load-Deflection Curves for Plate P 2.1

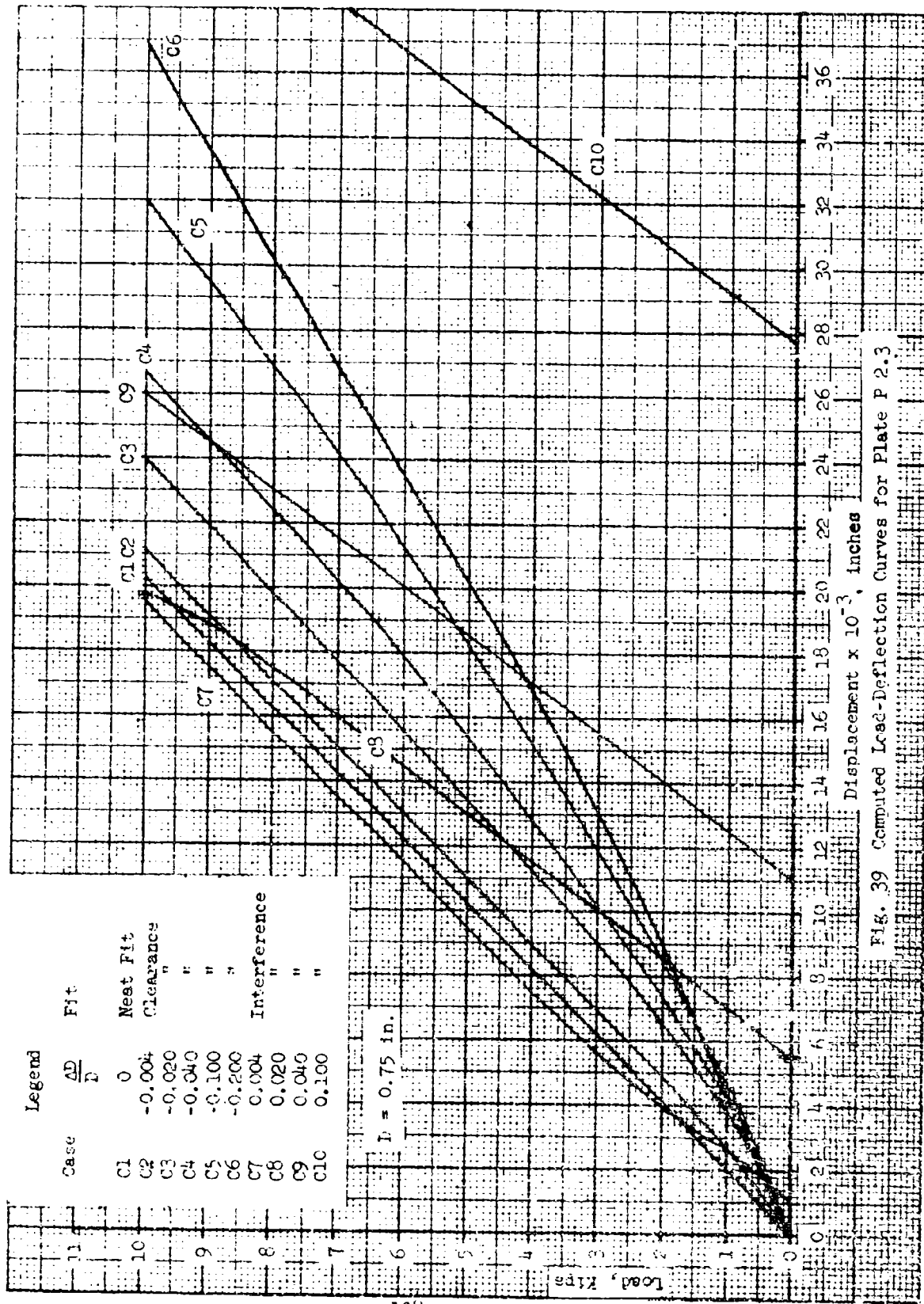


Fig. 39 Computed Load-Deflection Curves for Plate P 2.3

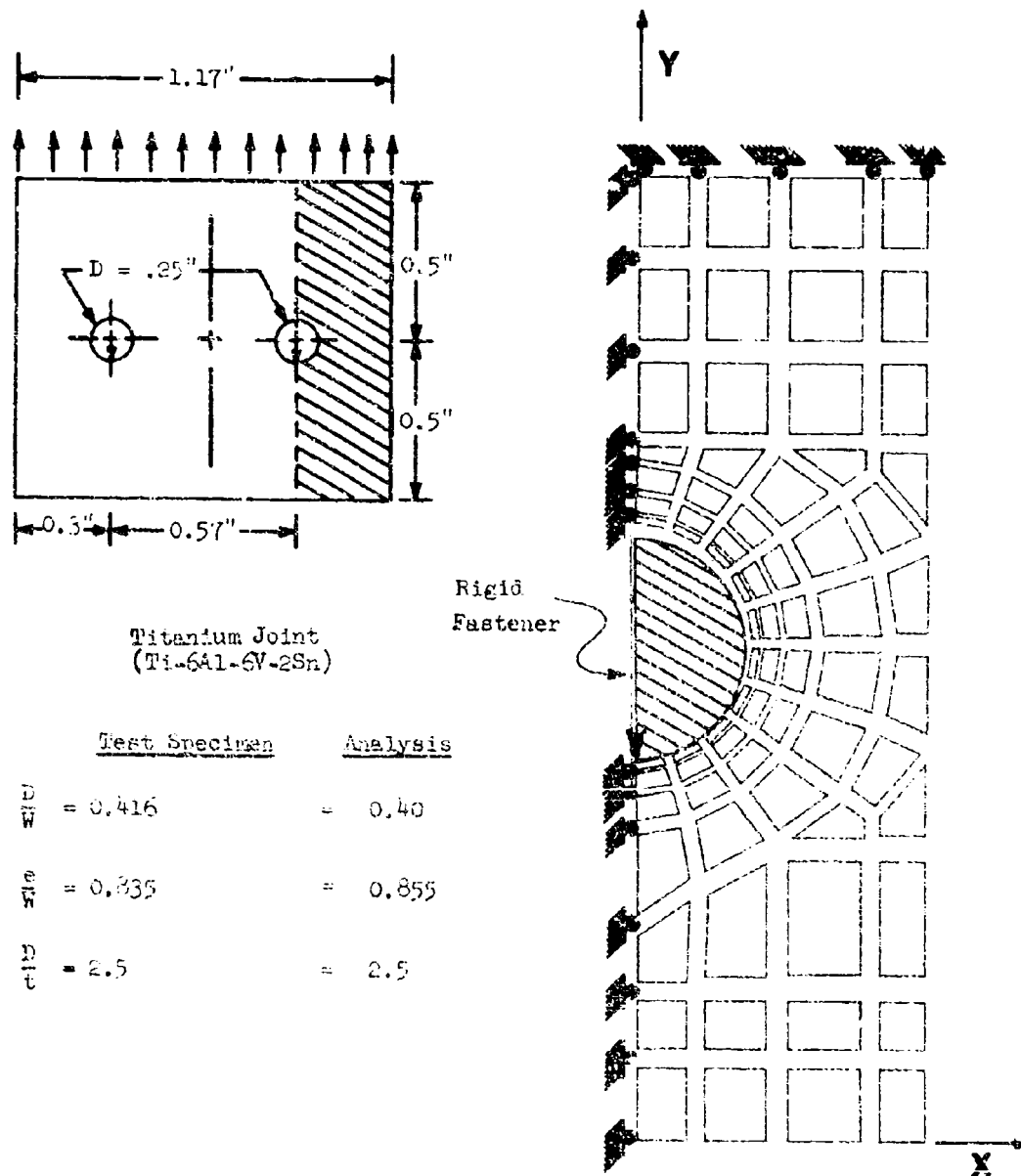


FIG. 40 Dimensions of Tested Titanium Joint and Idealization Used in the Analysis

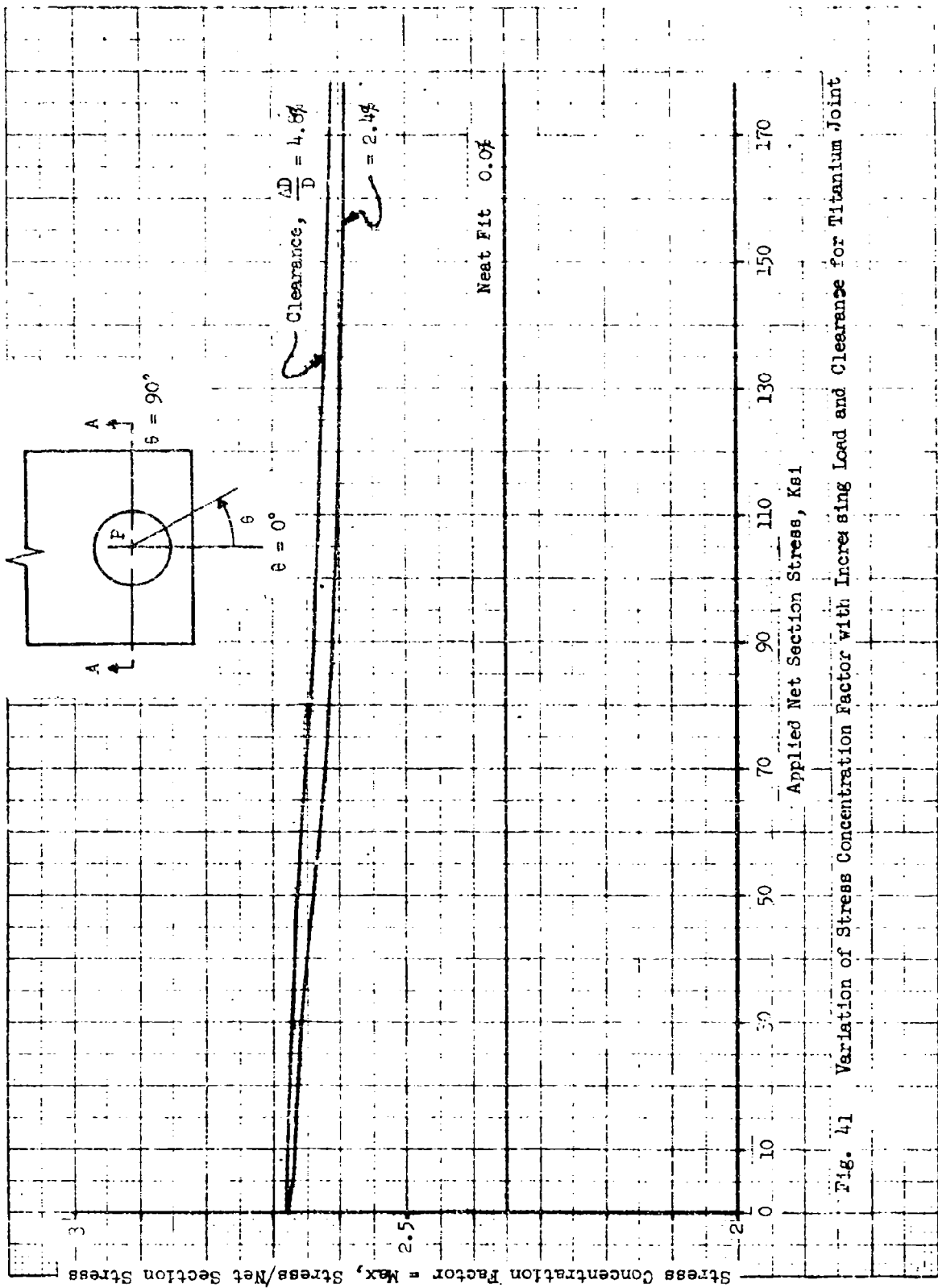


Fig. 41 Variation of Stress Concentration Factor with Increasing Load and Clearance for Titanium Joint

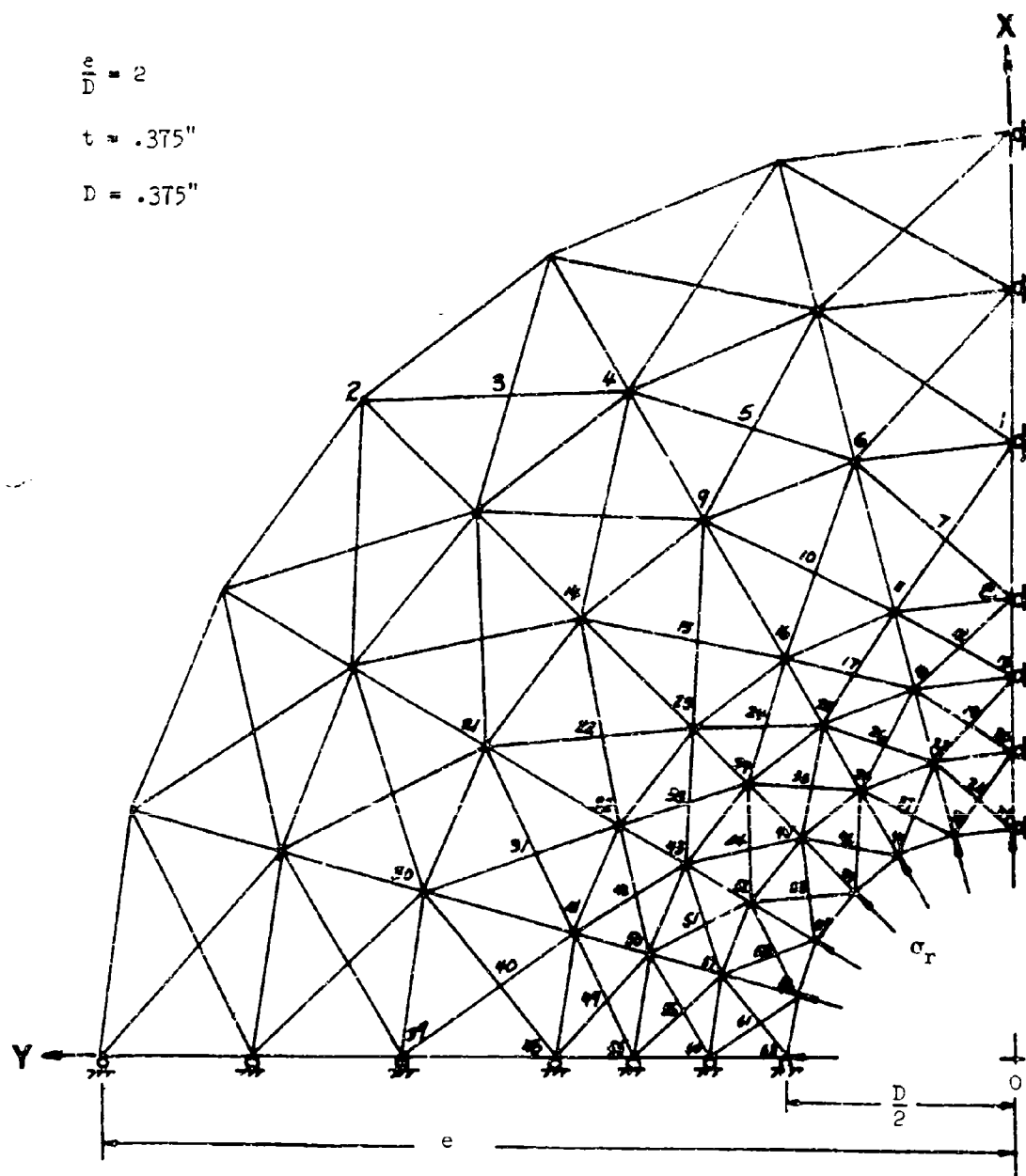


Fig. 42 Mathematical Model Used in the Analysis of Axi-symmetrically Loaded Plate.

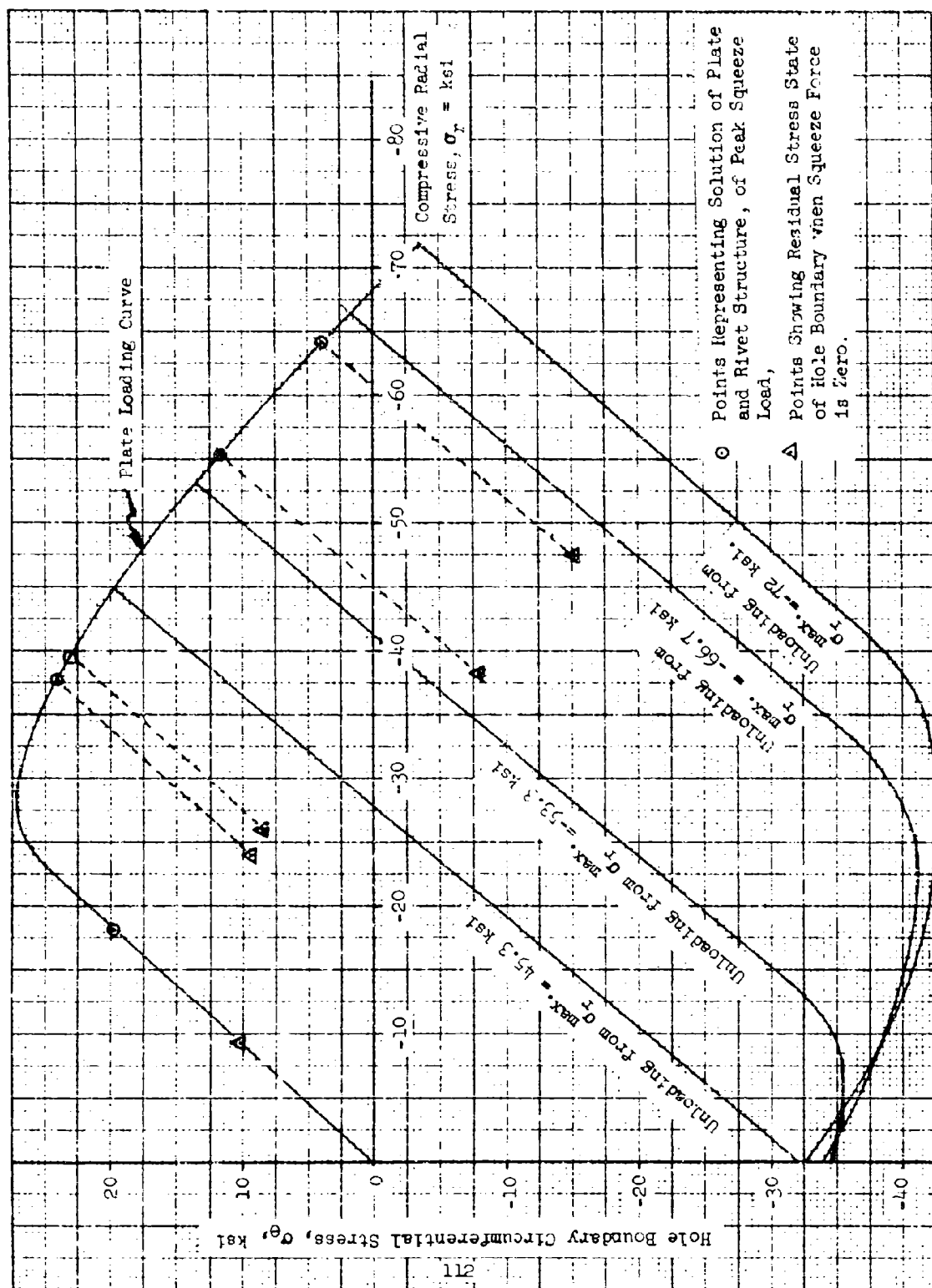


Fig. 43 Circumferential Stress vs Compressive Radial Stress at the Hole Boundary (2024-T351 Al. Plate)

2024-T351 AL. PLATE

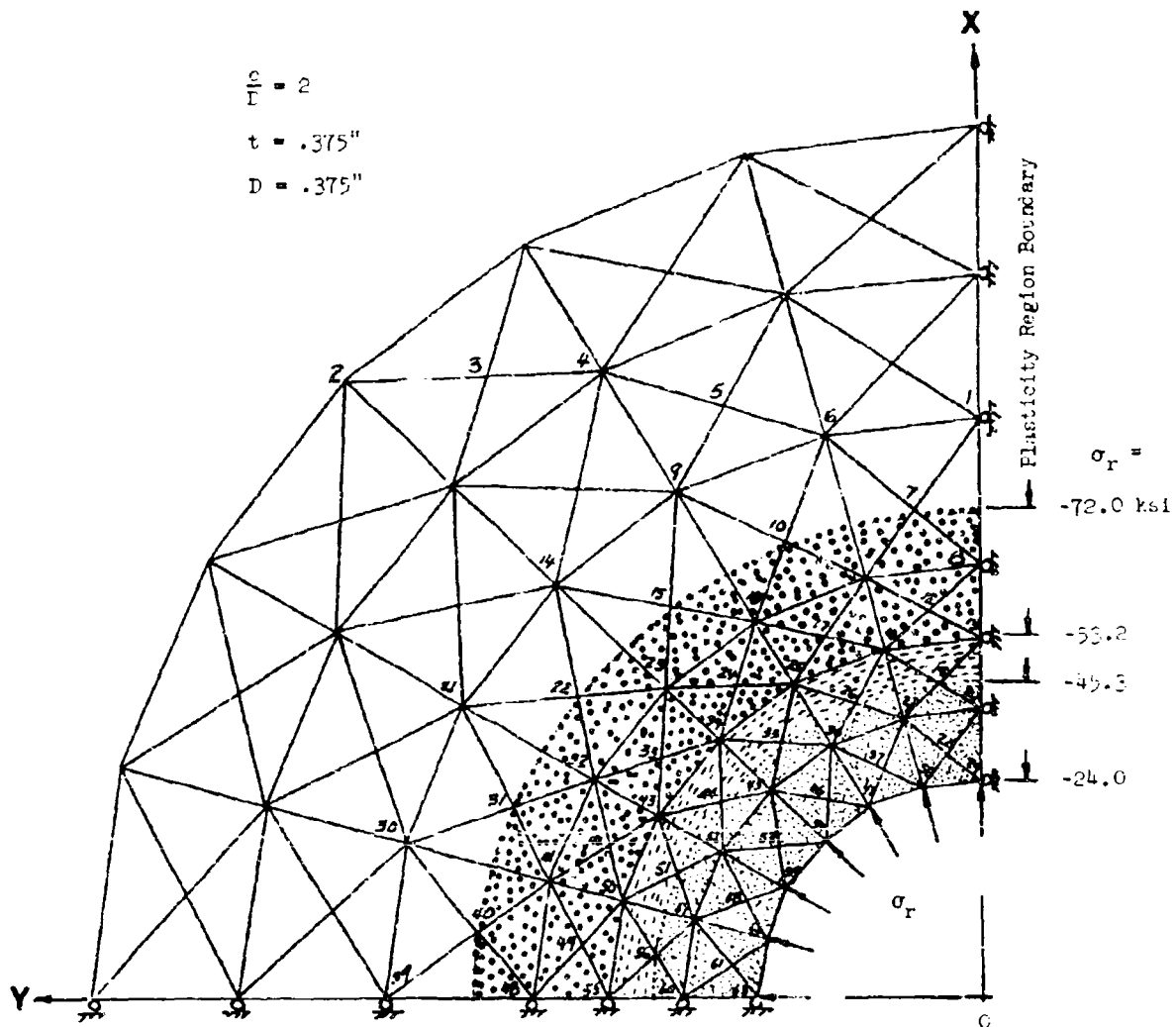


Fig. 44 Growth of the Plasticity Region in a 2024-T351 Aluminum Plate with Increasing Radial Stress

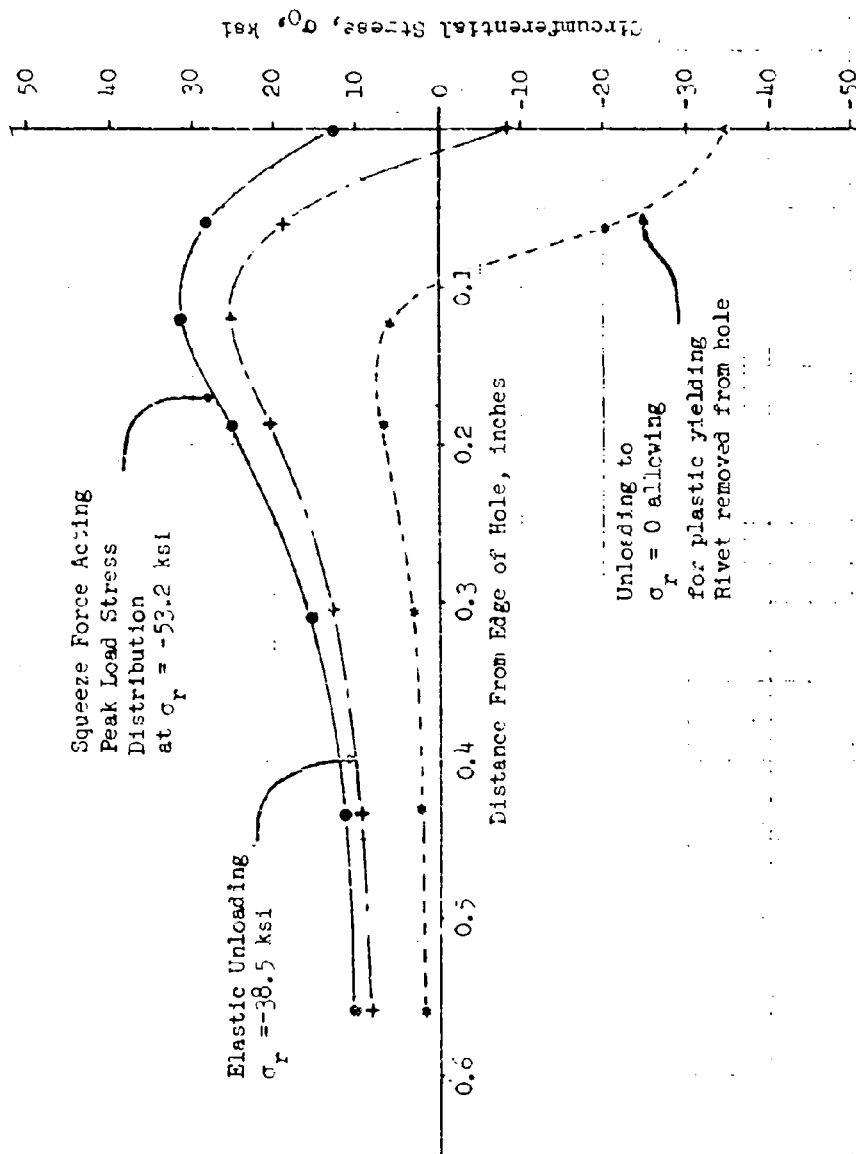
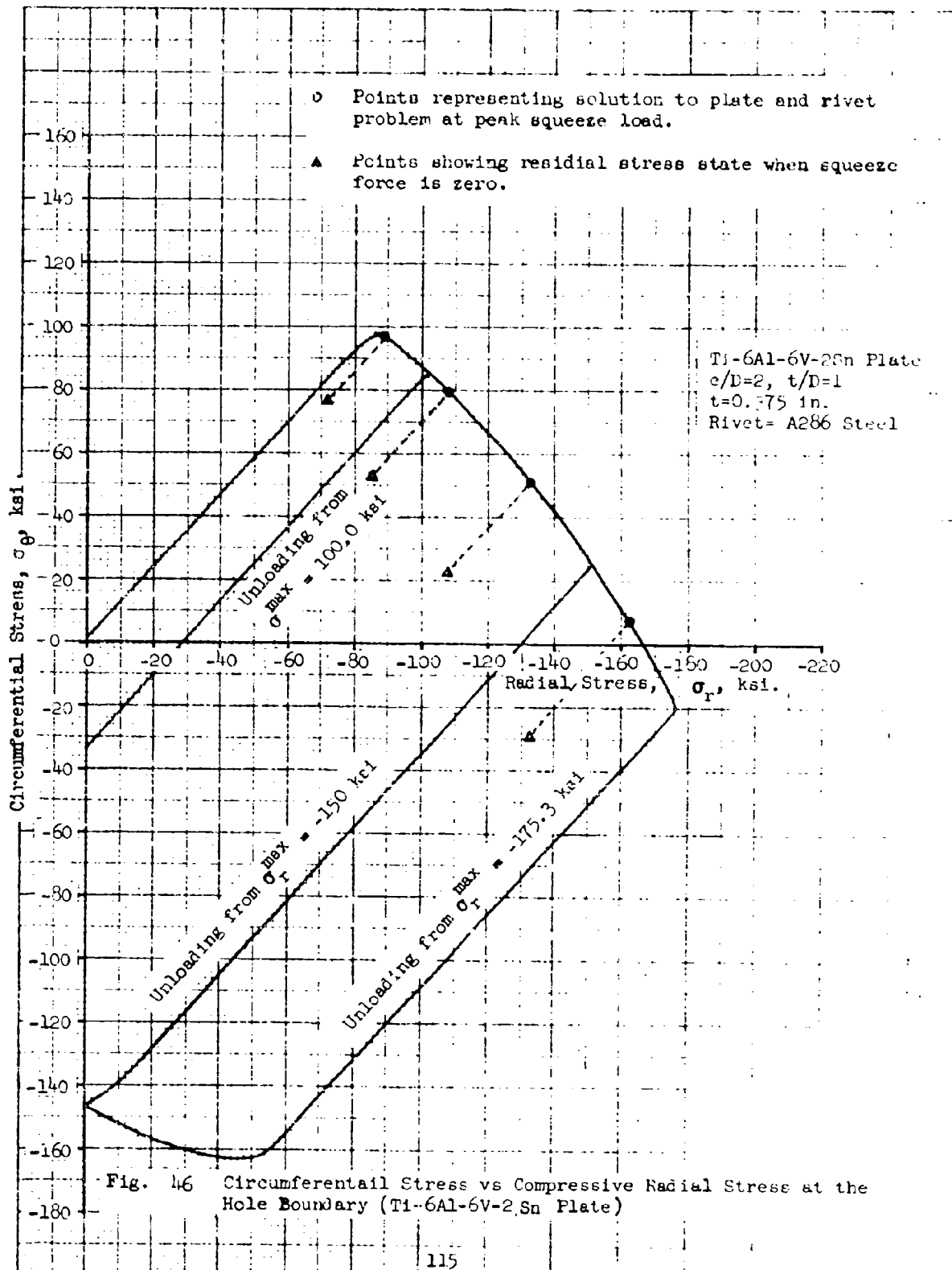


Fig. 45 Circumferential Stress Distribution, σ_θ , Across Aluminum Plate For Two Types of Unloading



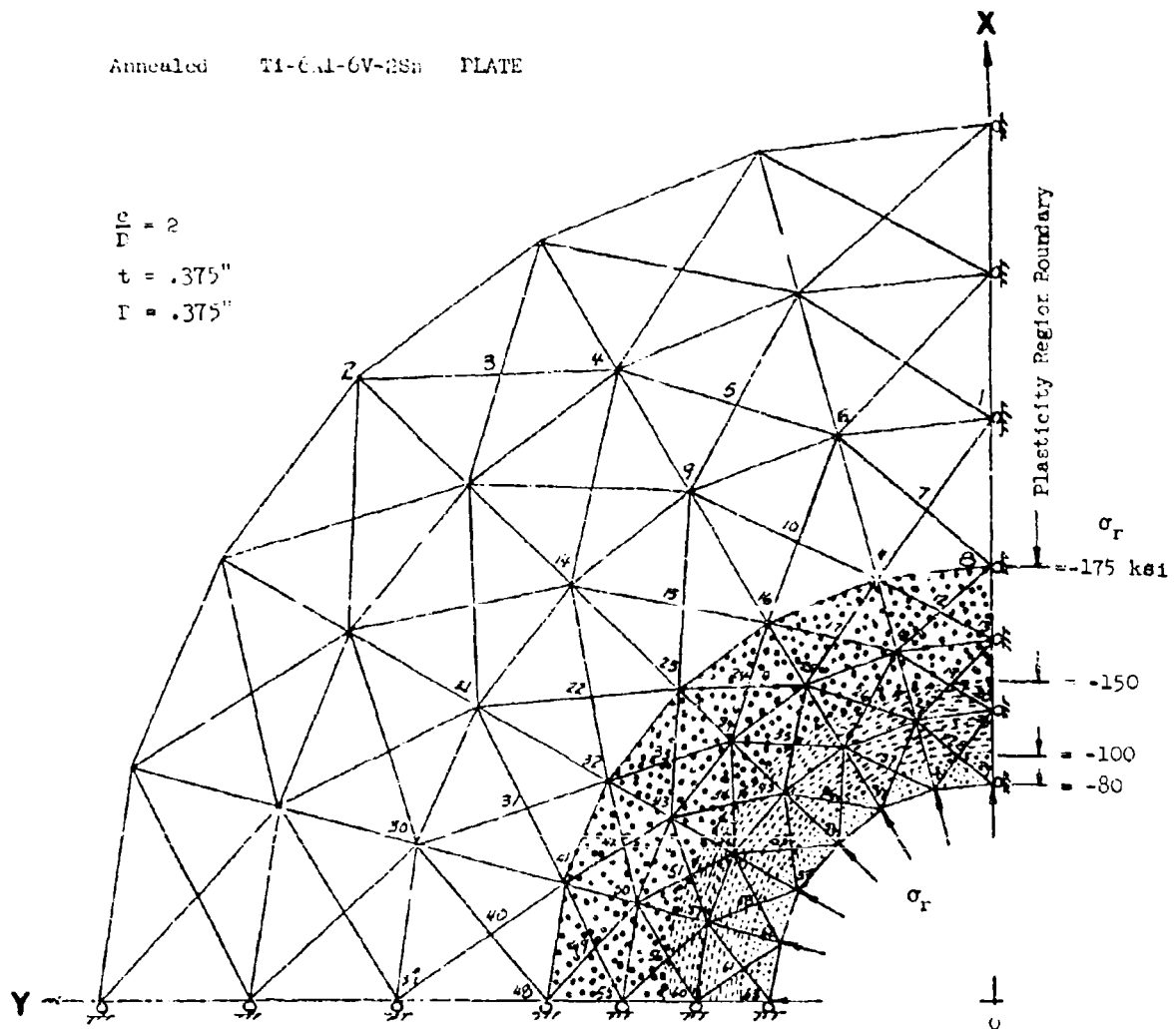
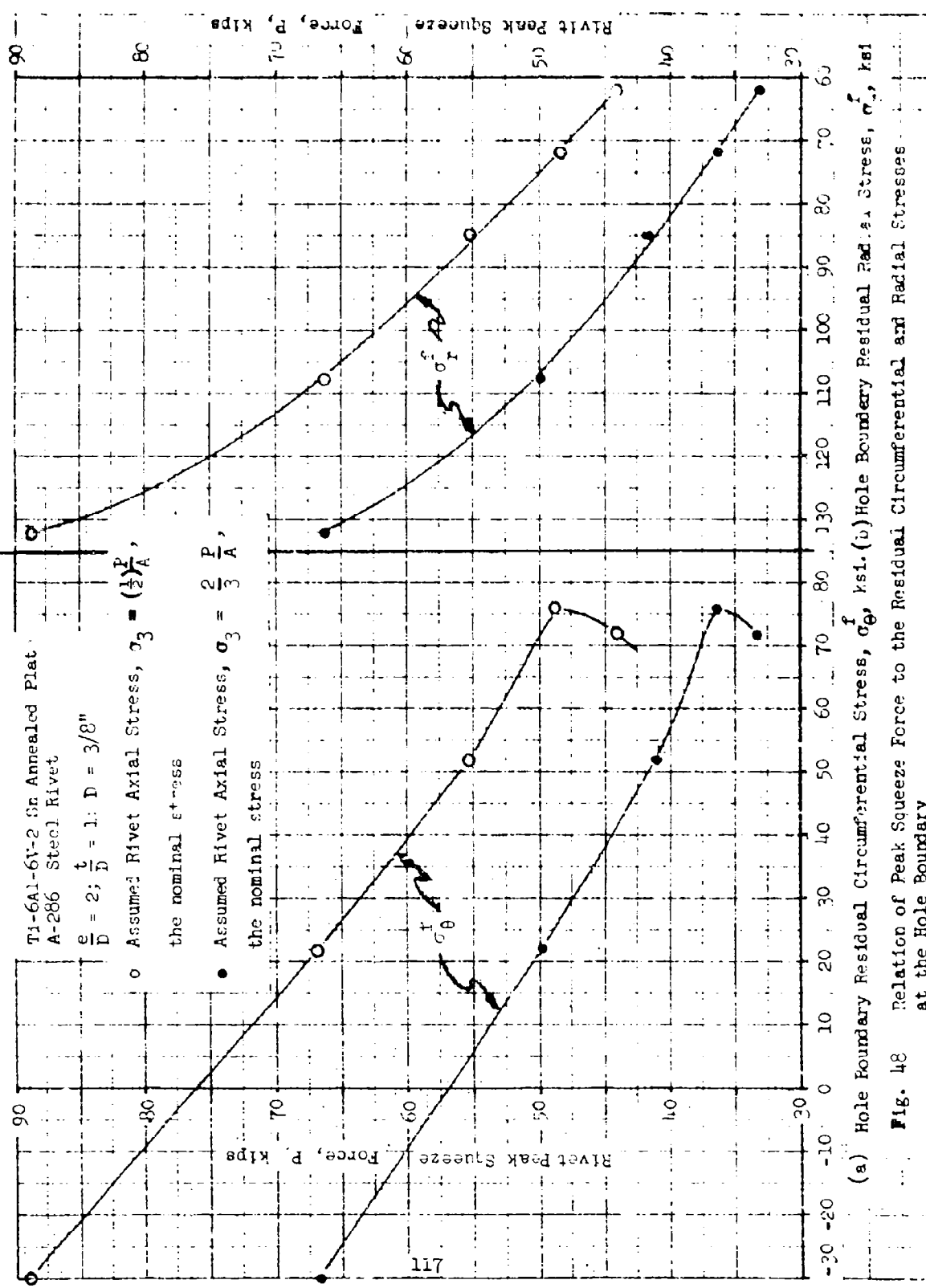


Fig. 47 Growth of the Plasticity Region in the Annealed Titanium Plate with Increasing Radial Stress



Ti-6Al-6V-2Sn Annealed Plate
A-286 Steel Pivot

$$e/D = 2; t/D = 1; D = 3/8 \text{ in.}$$

Assumed Rivet Axial Stress,

$$\sigma = \left(\frac{1}{2}\right)^2 \frac{1}{A}, \text{ the nominal stress.}$$

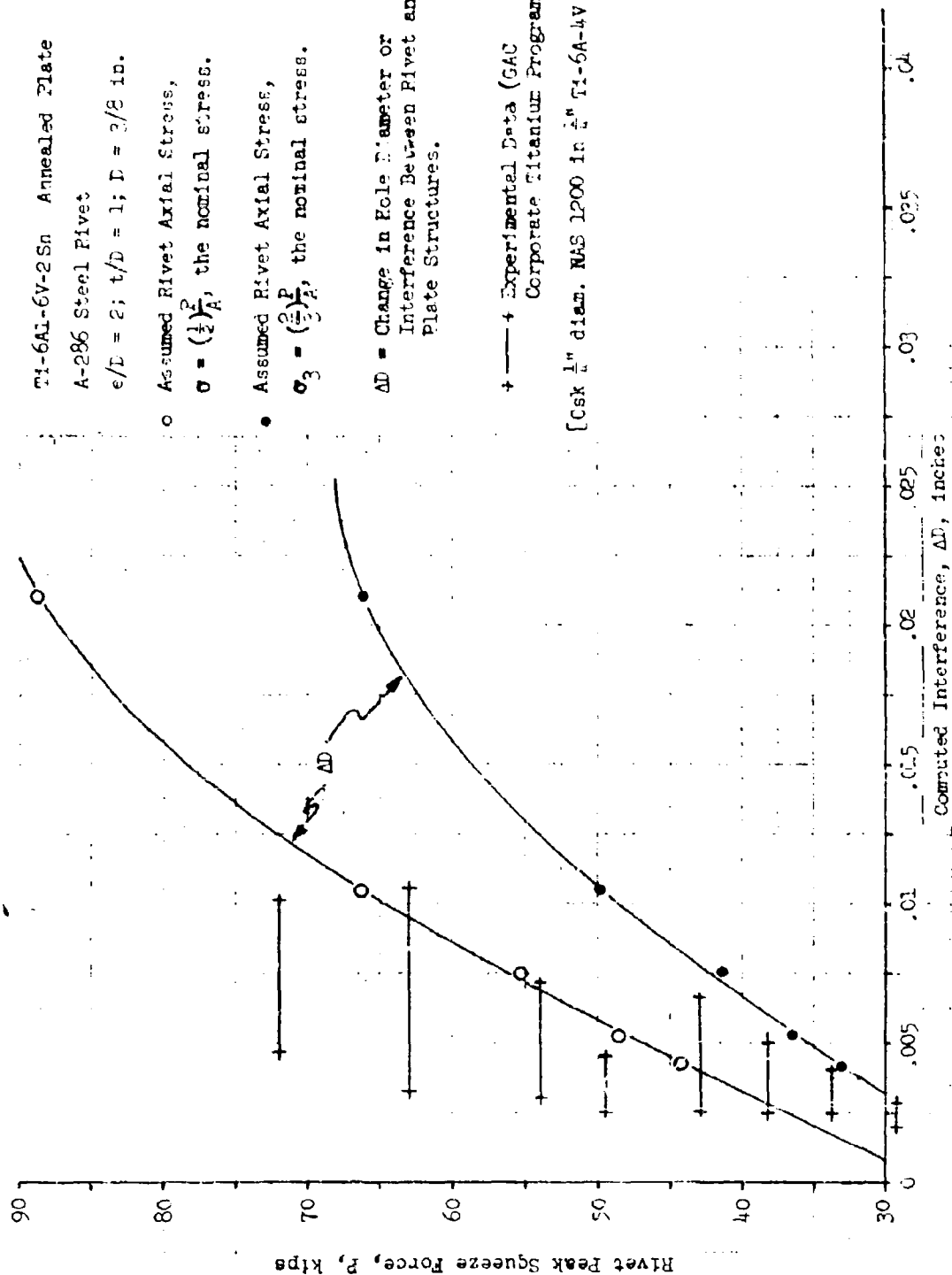
Assumed Rivet Axial Stress,

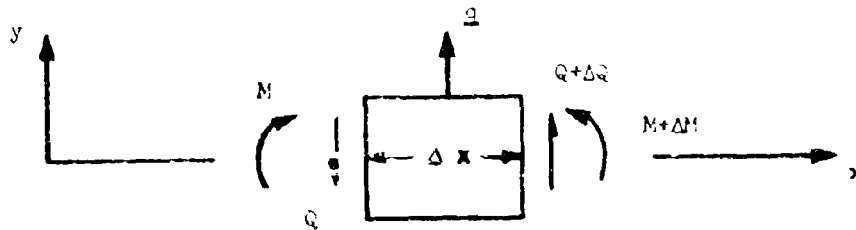
$$\sigma_3 = \left(\frac{\sigma}{3}\right)^2, \text{ the normal stress.}$$

ADD - Change in Hole Diameter or Interference Between Rivet and Plate Structures.

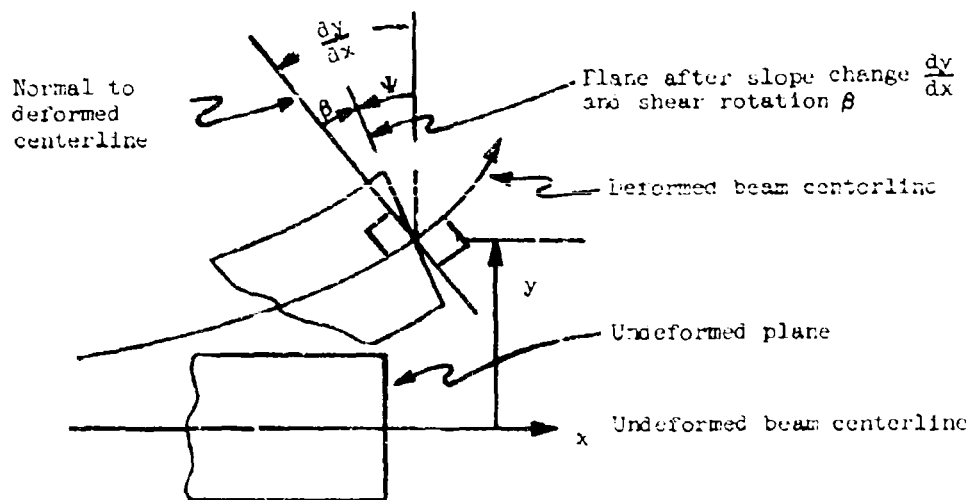
+ --- + Experimental Data (GAC
Corporate Titanium Program)

[Csk $\frac{1}{4}$ " diam. NAS 1200 in $\frac{1}{4}$ " T1-6A-4V Plate]



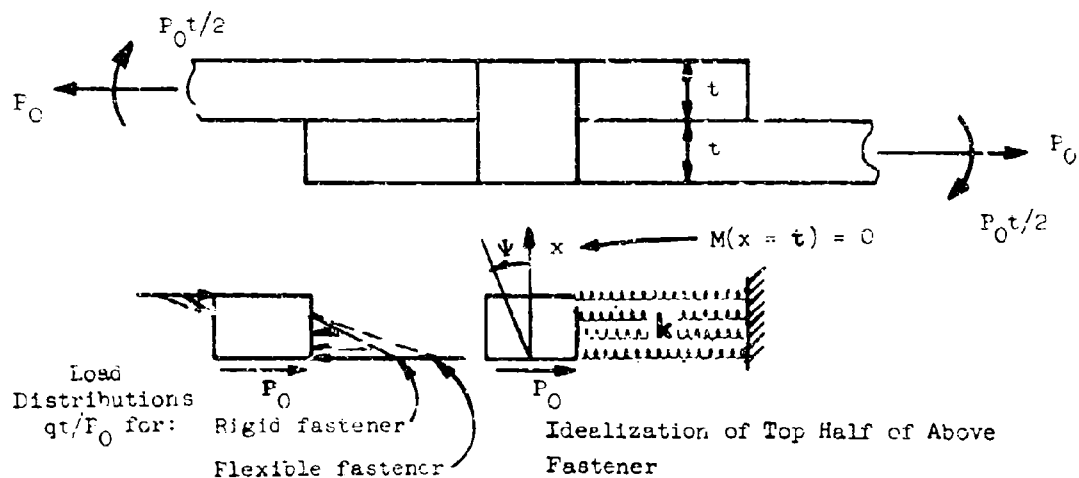


(a) Forces and Moments

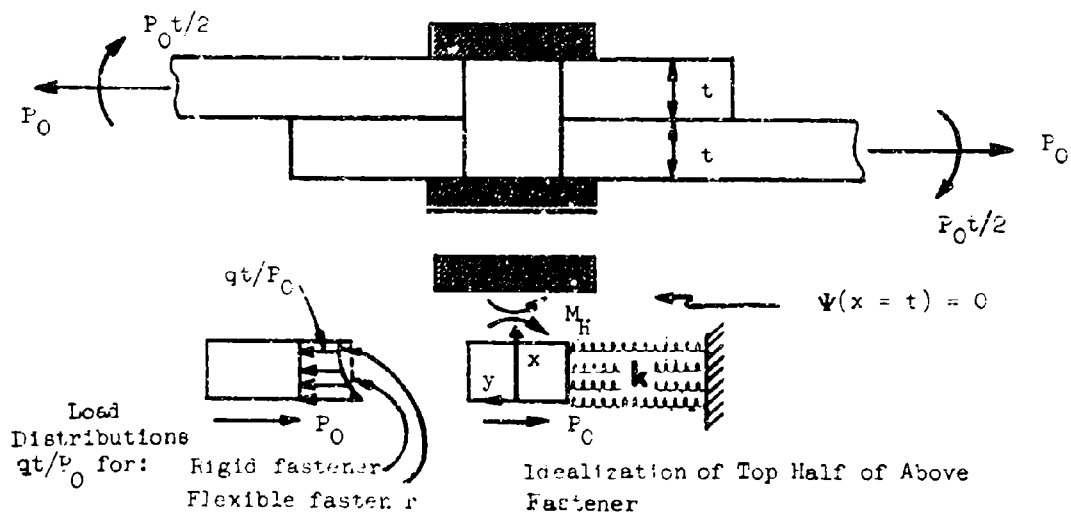


(b) Kinematics of Deformation

Fig. 50 Equilibrium and Deformation of a Typical Beam Element



(a) Fastener With Negligible Head Stiffness



(b) Clamped Head Condition

Figure 51 Fastener in Single Shear with Idealizations and Typical Solutions

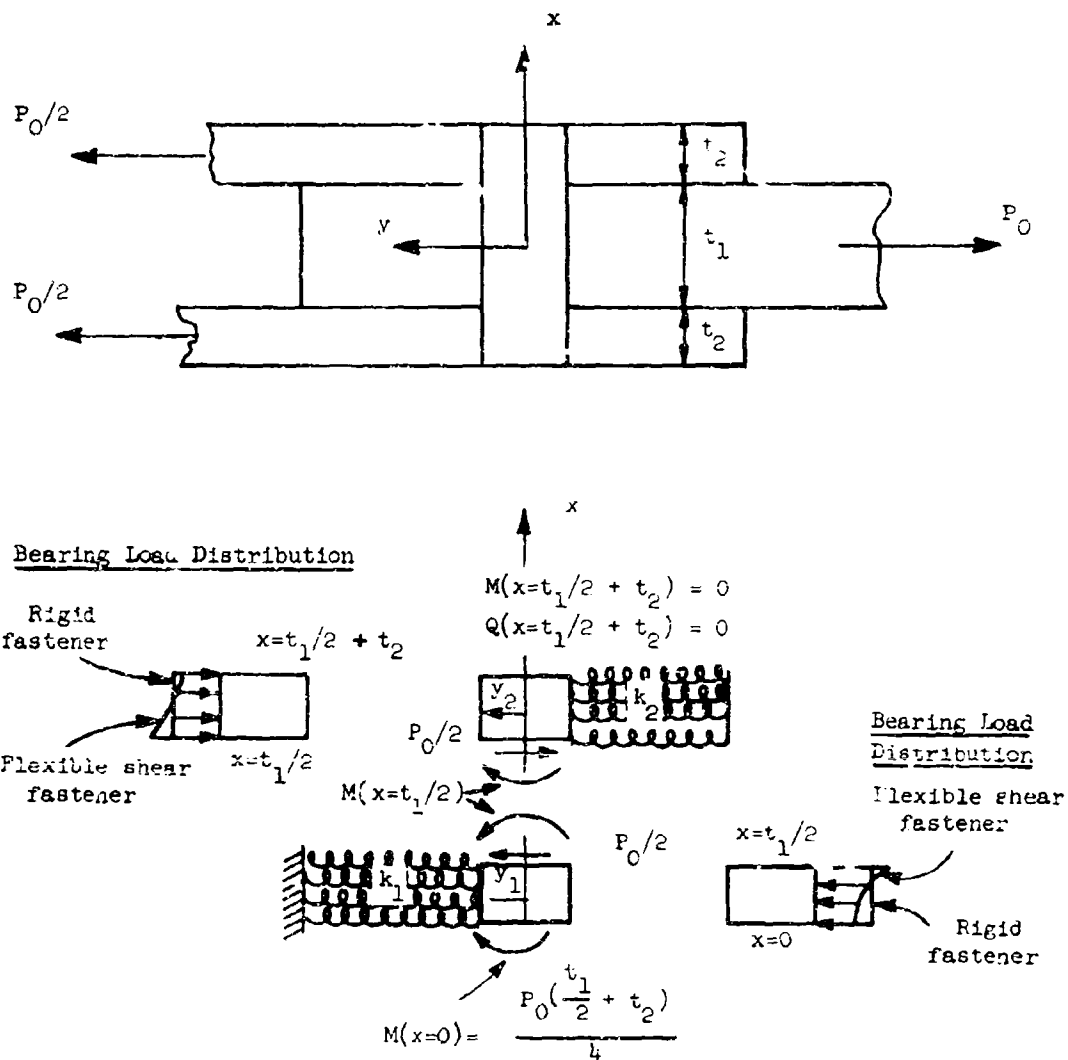
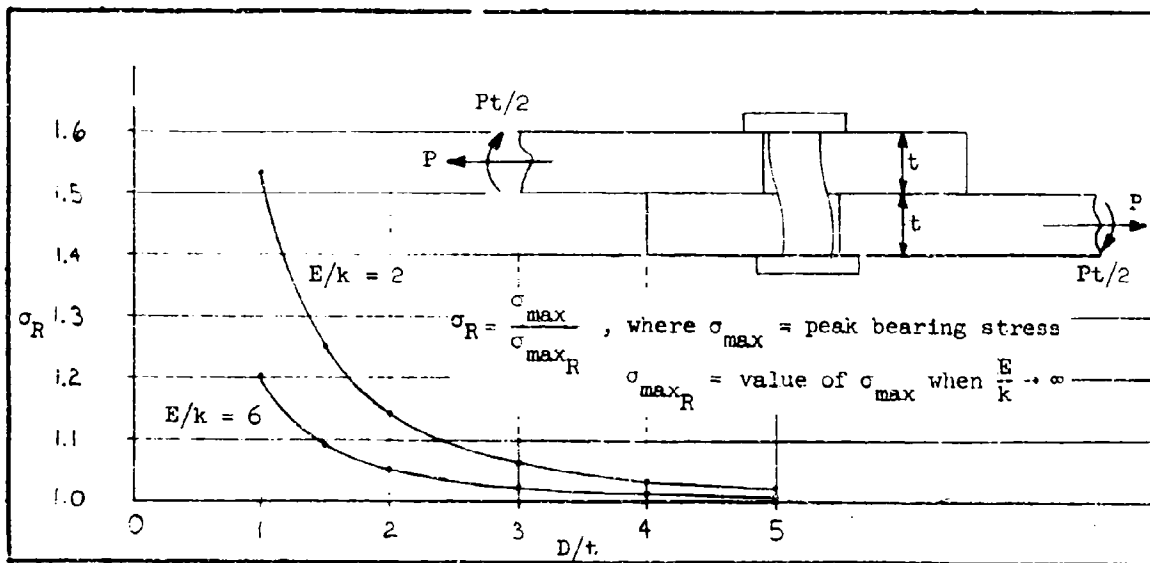
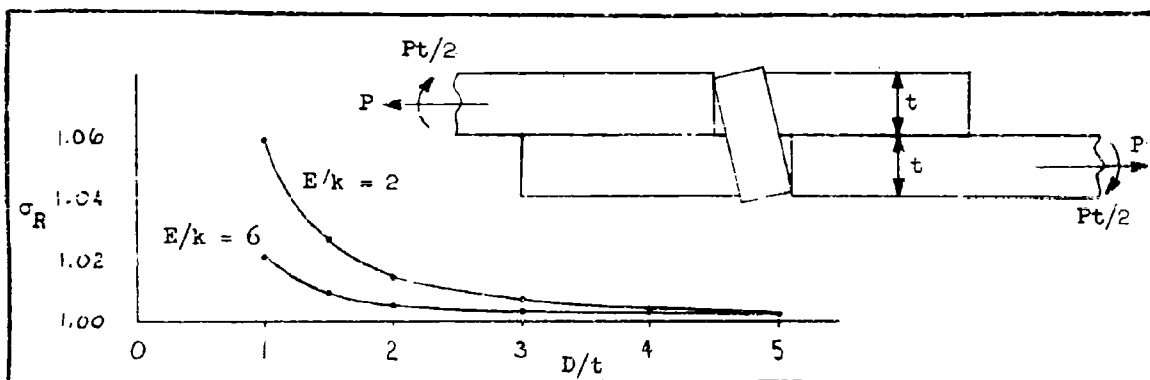


Figure 52 Fastener in Symmetrical Double Shear with Idealization and Typical Solution.



(a) Single Shear, Clamped Fastener Head



(b) Single Shear, Free Fastener Head.

Fig. 53 Effect of Fastener Stiffness on Peak Bearing Stress

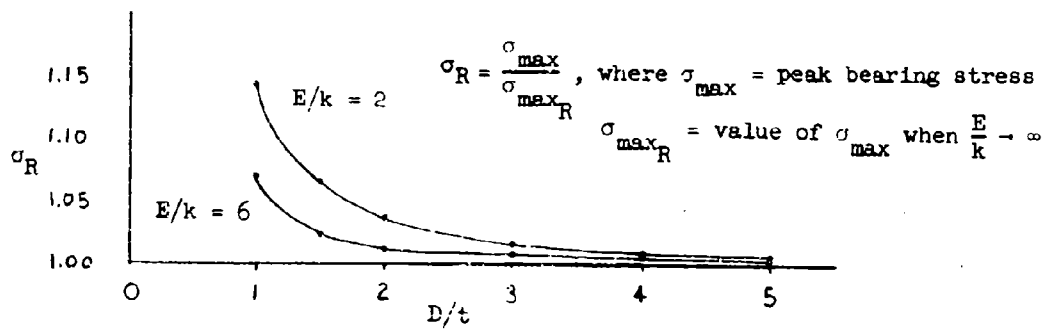
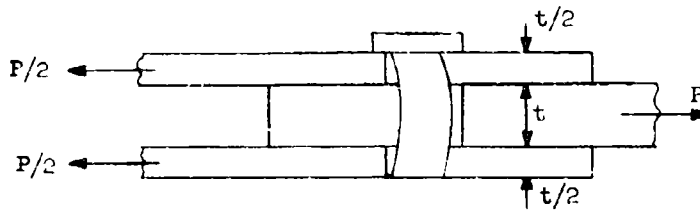


Fig. 54 Effect of Fastener Stiffness on Peak Bearing Stress
(Double Shear Case With Arbitrary Fastener Head End Conditions)

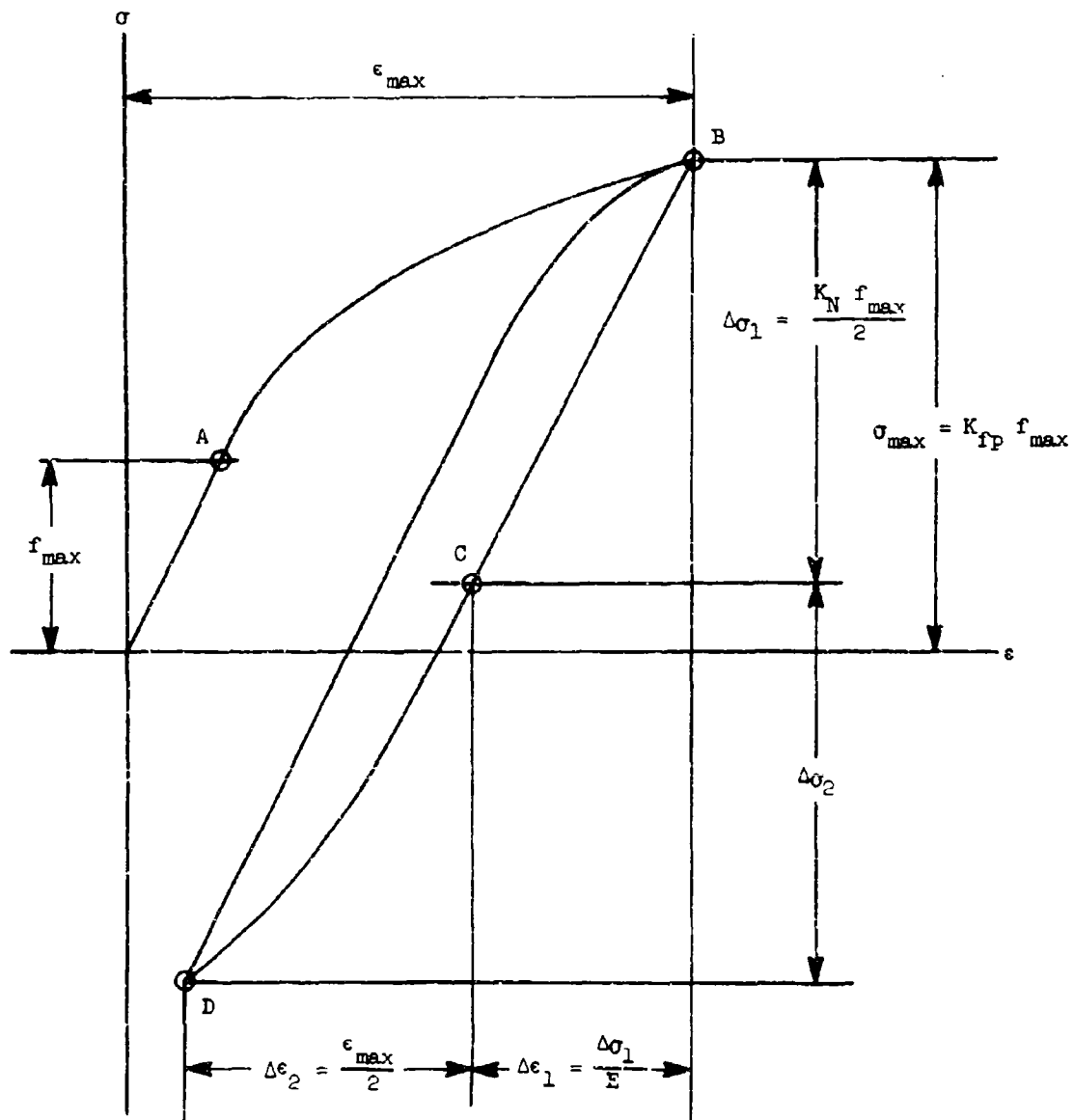


Fig 55 Stress-Strain Cycle at Edge of Notch

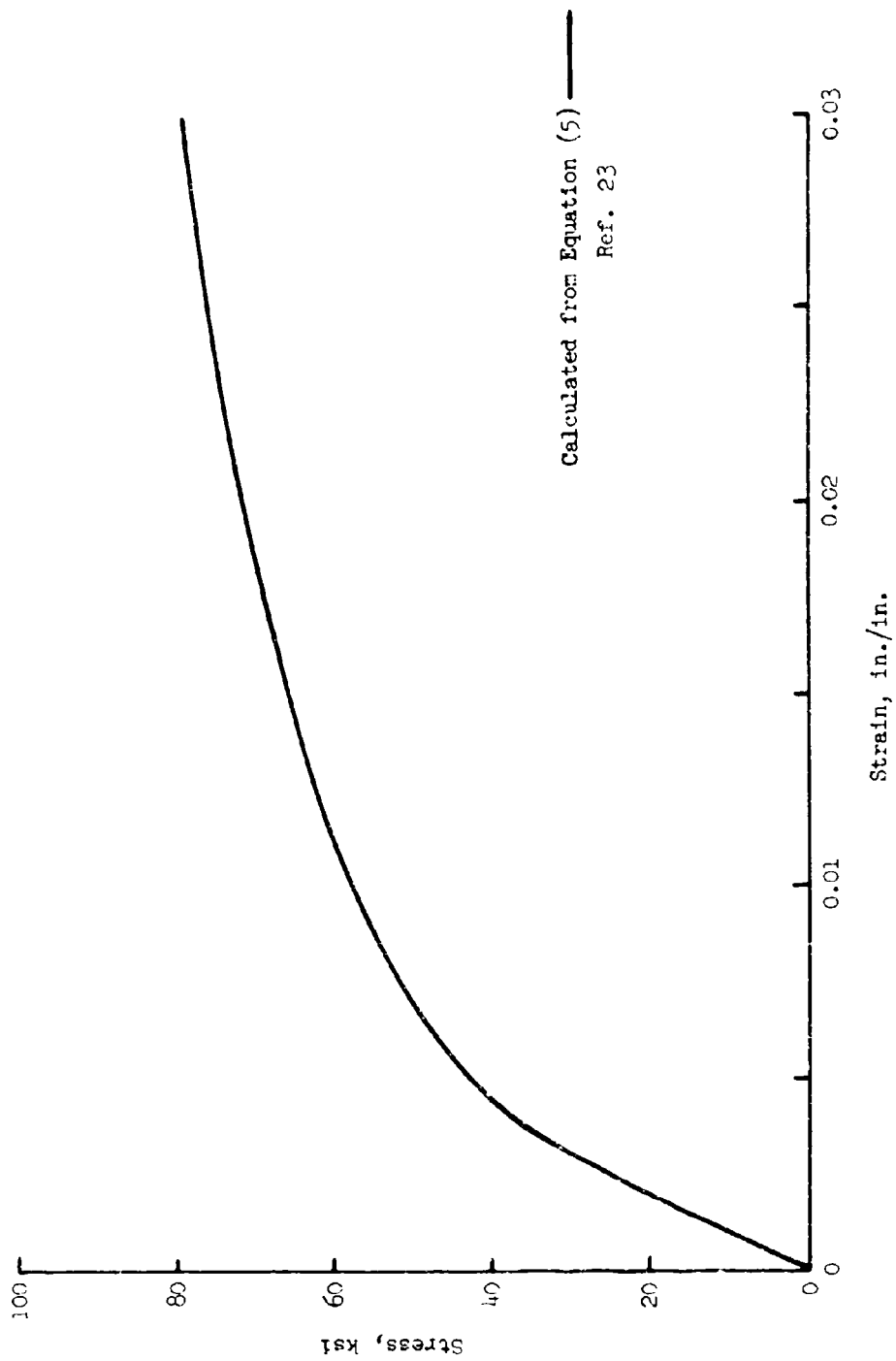


Fig. 56 Cyclic Stress - Strain Curves - 7075-T6 Aluminum Alloy

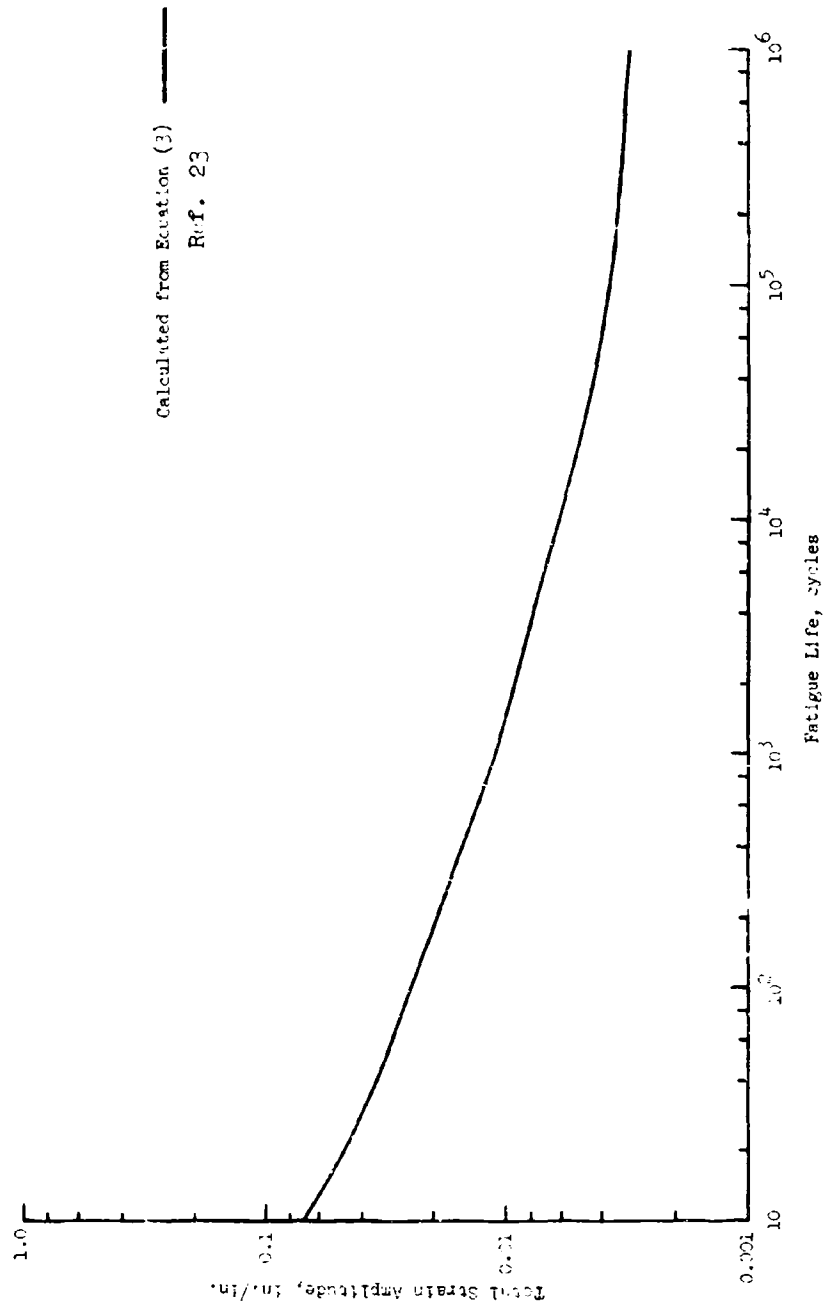


Fig. 57 Variation of Total Strain Amplitude with Fatigue Life for Completely Reversed Cyclic Loading - 7075-T6 Aluminum Alloy

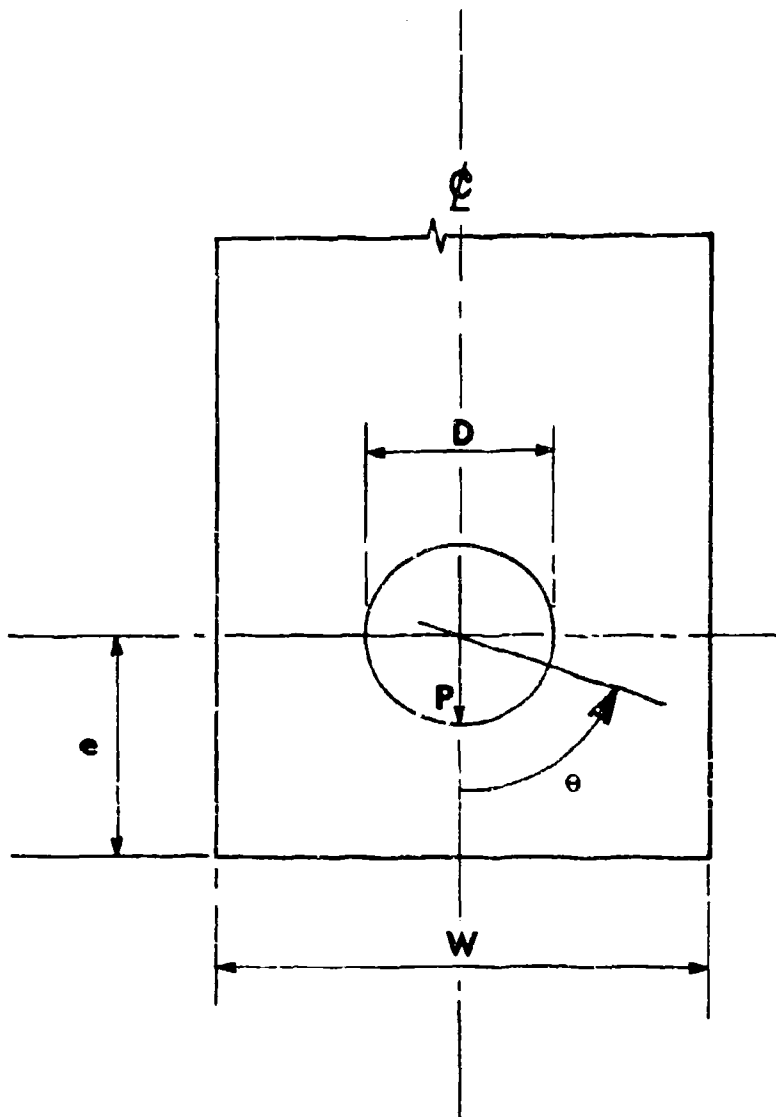


Fig. 58 Typical Element Geometry

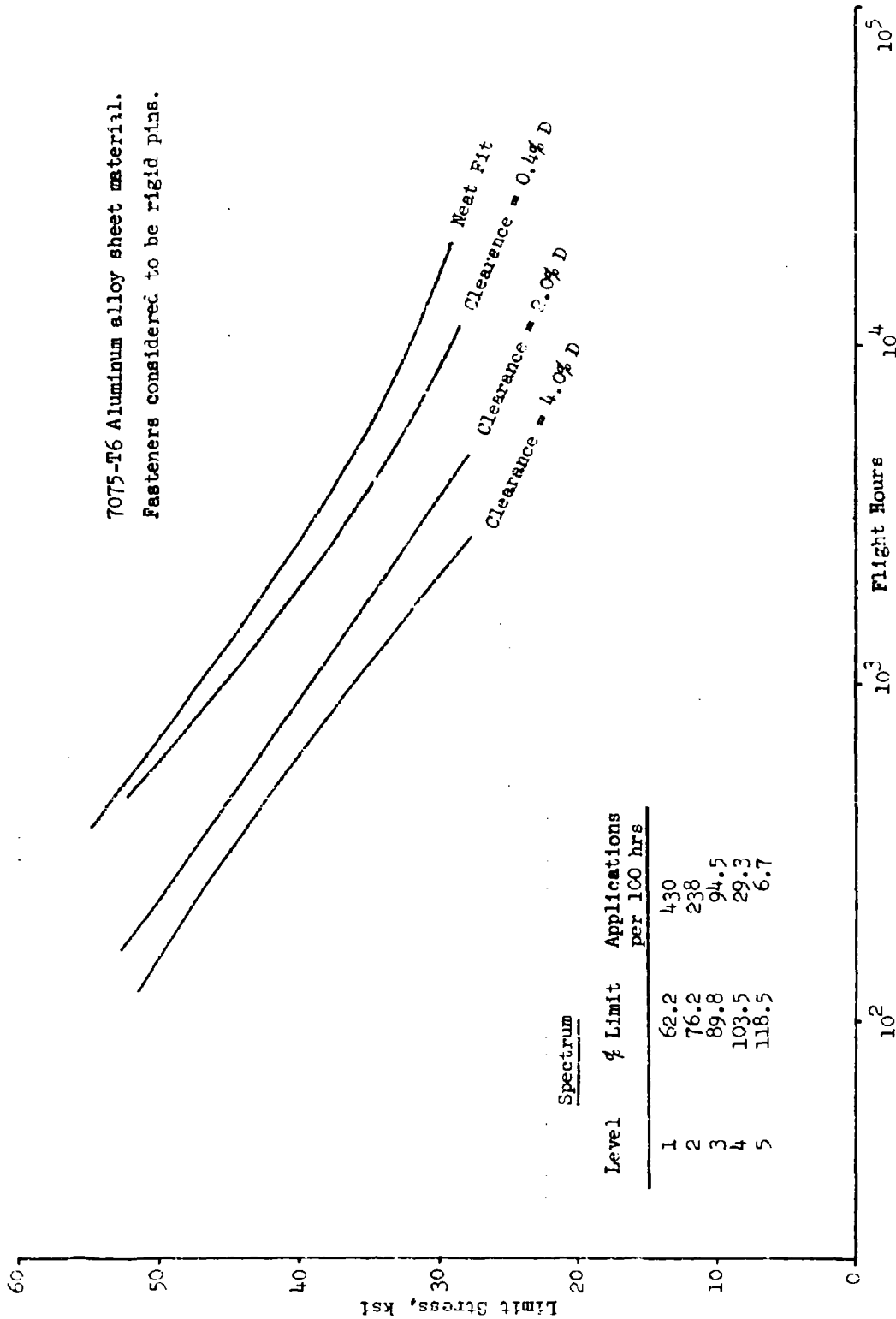


Fig. 59 Spectrum Fatigue Life Predictions For Aluminum Alloy Joint

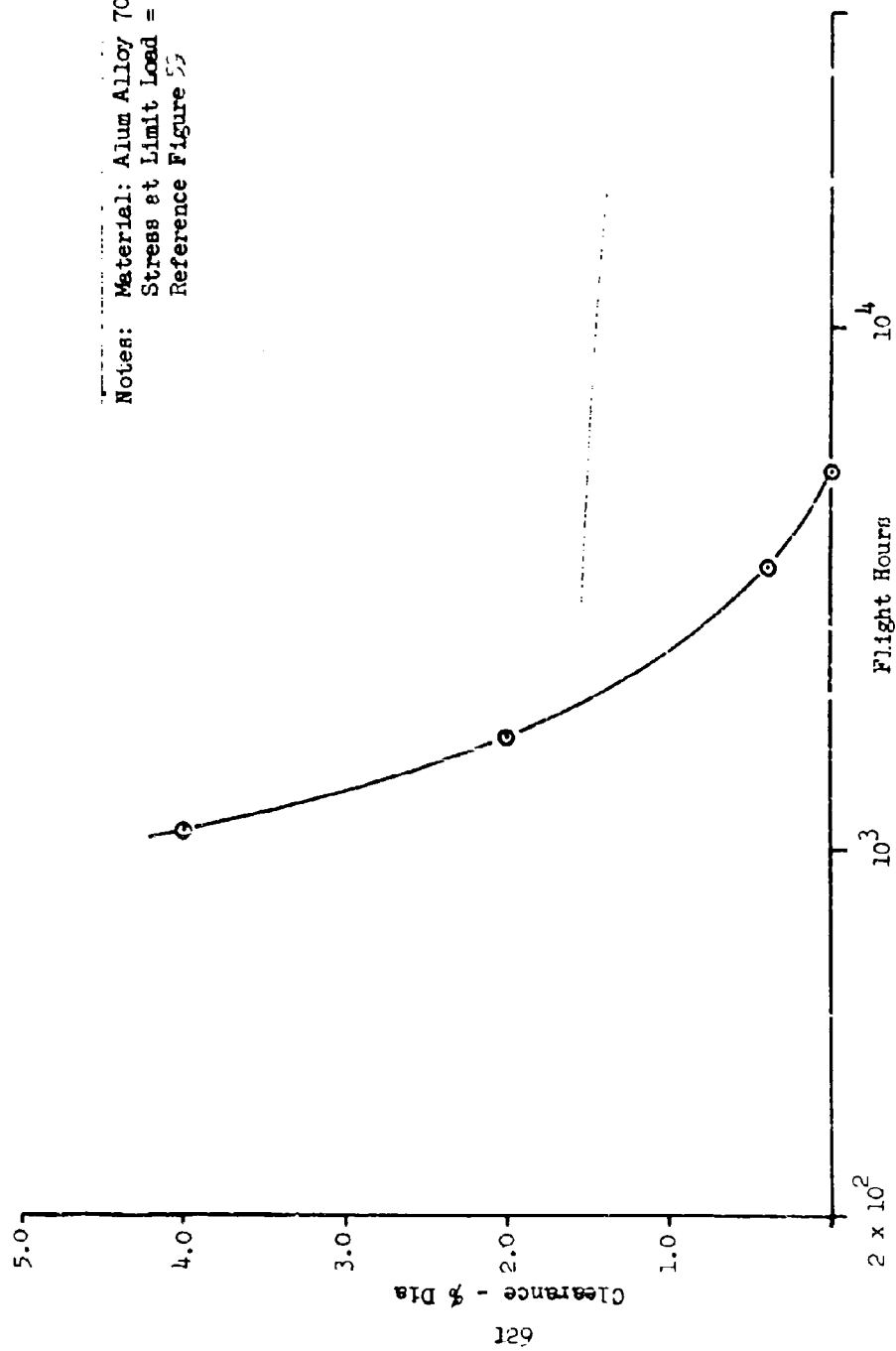


Fig. 60 Effect of Hole Clearance on Spectrum Fatigue Life

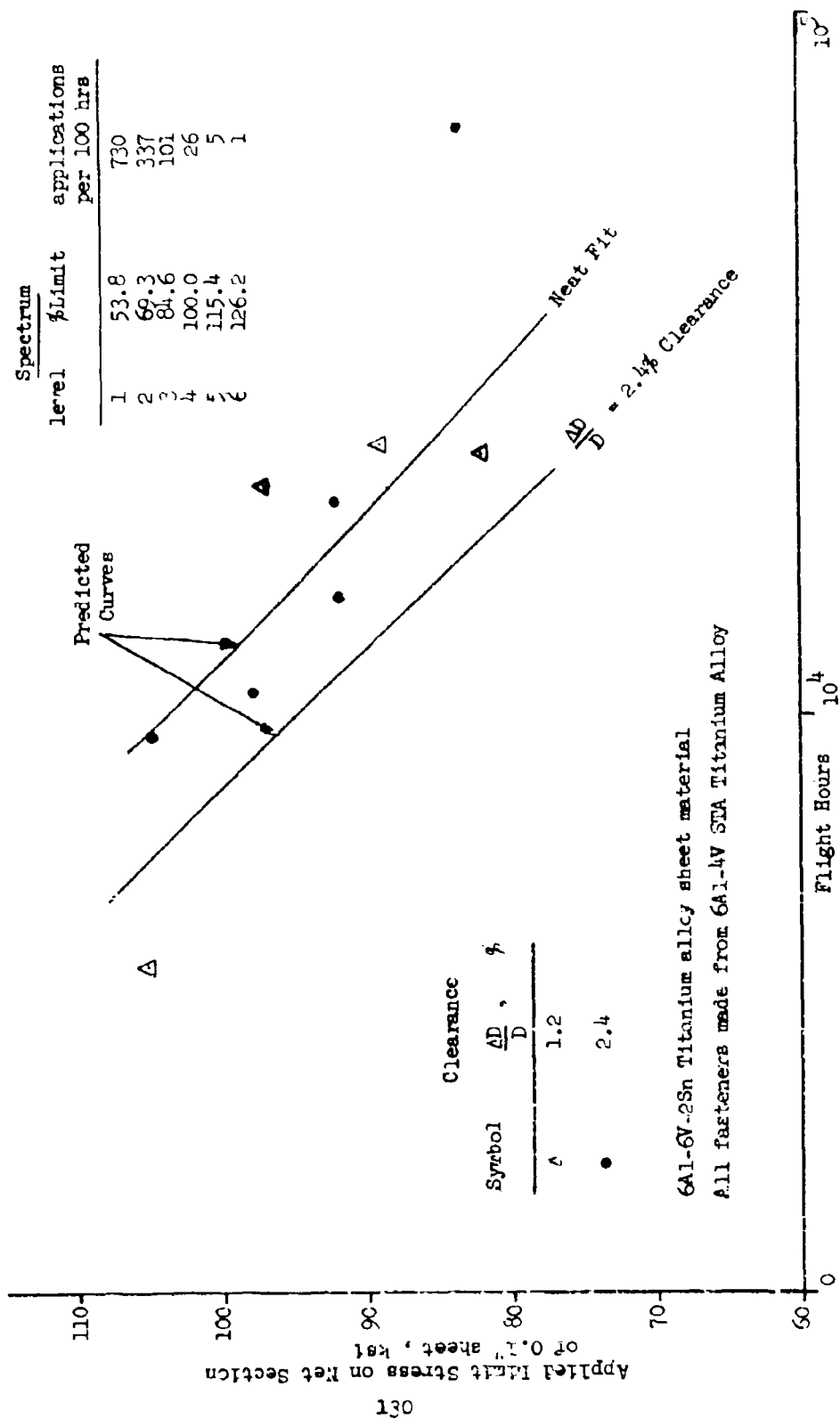


Fig. 61 Comparison of Spectrum Fatigue Life Predictions With Test Results For Titanium Alloy Joint

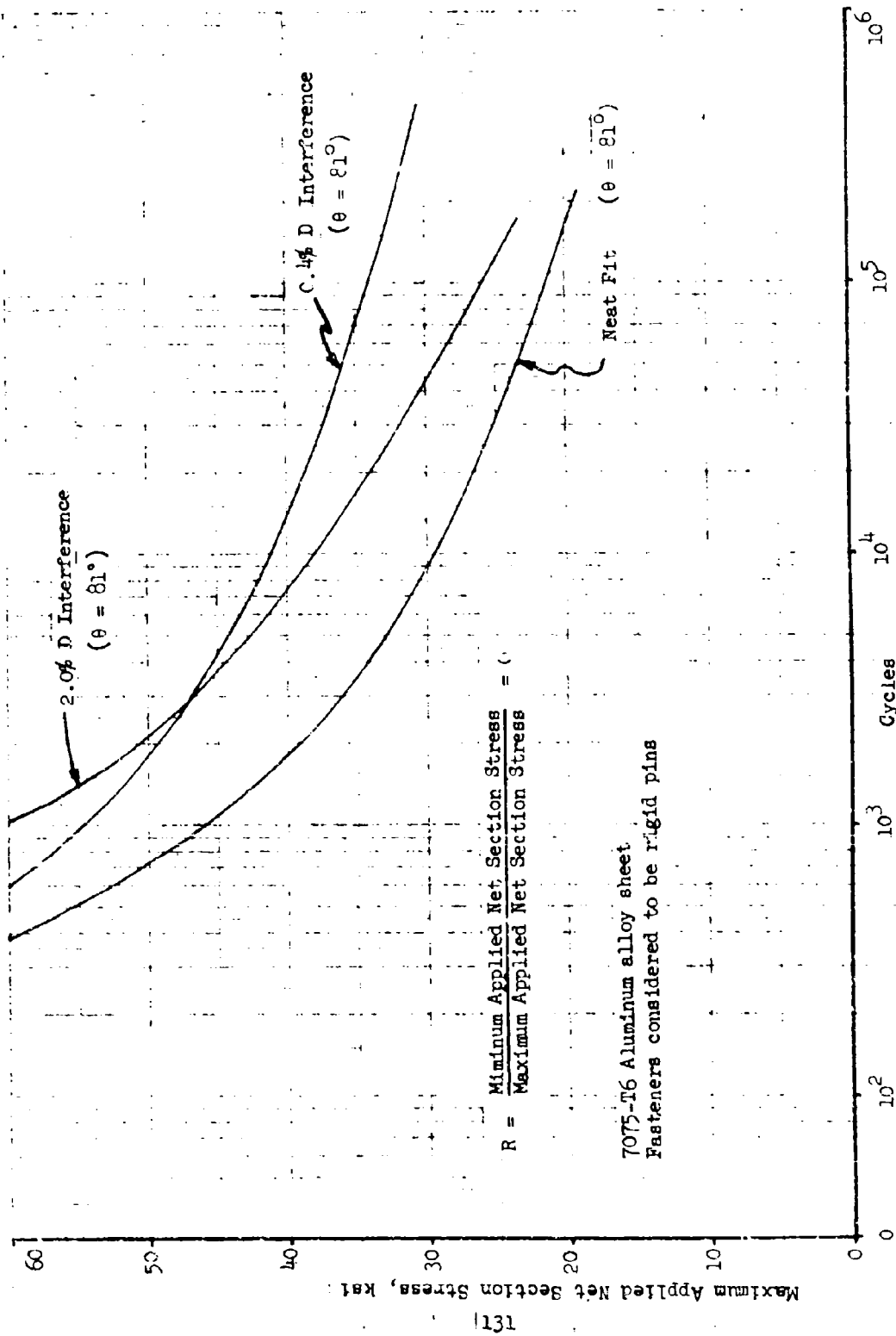


Fig. 62 Constant Amplitude Fatigue Life Predictions for Aluminum Alloy Joint

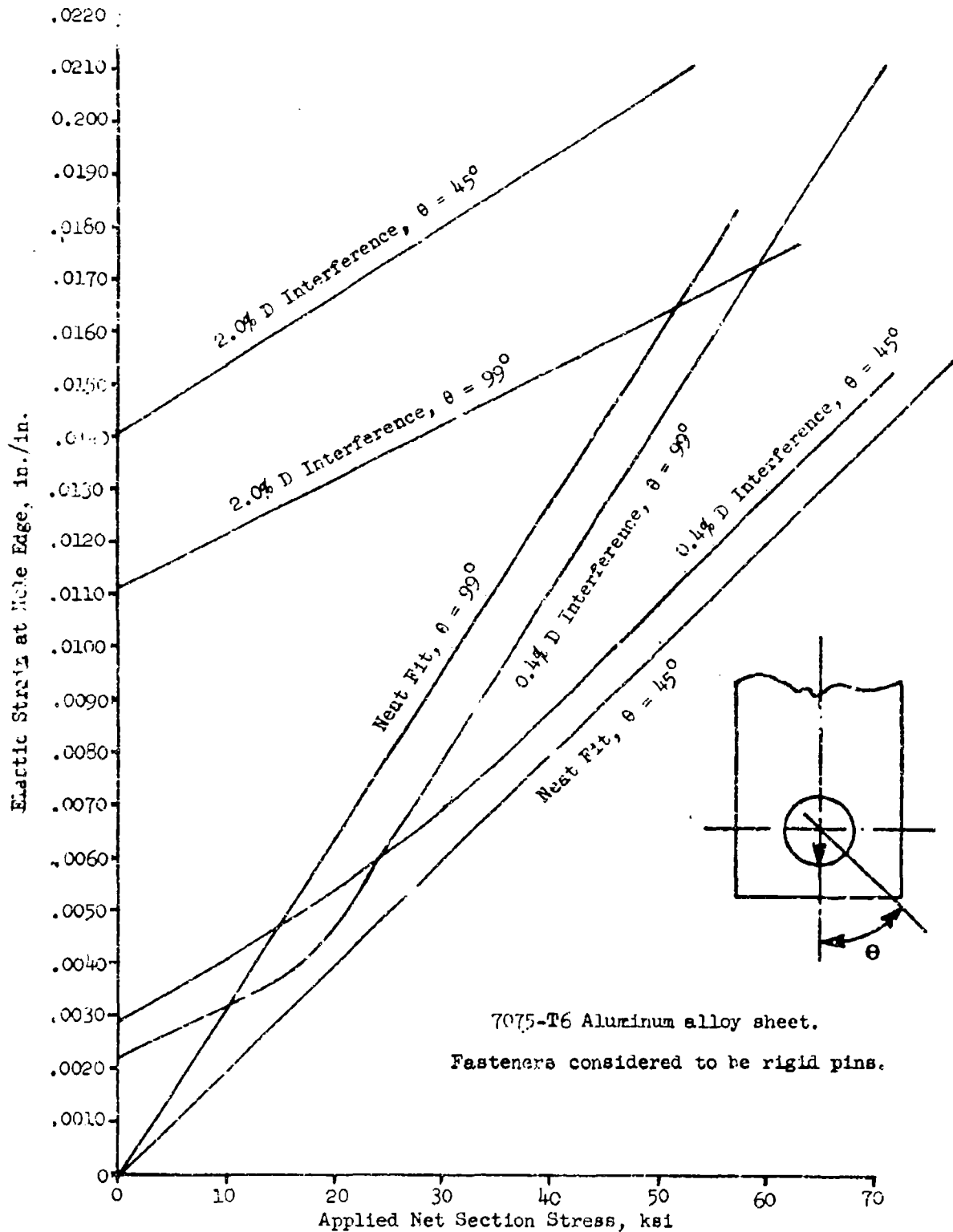
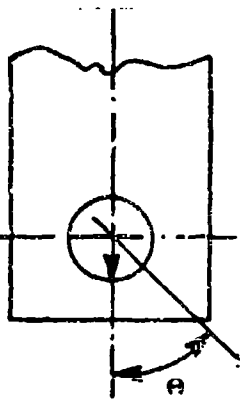
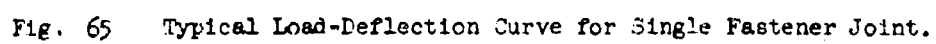


Fig. 63 Variation of Elastic Strain at the Hole Edge With Applied Net Section Stress for Interference and Neat Fits



1.33



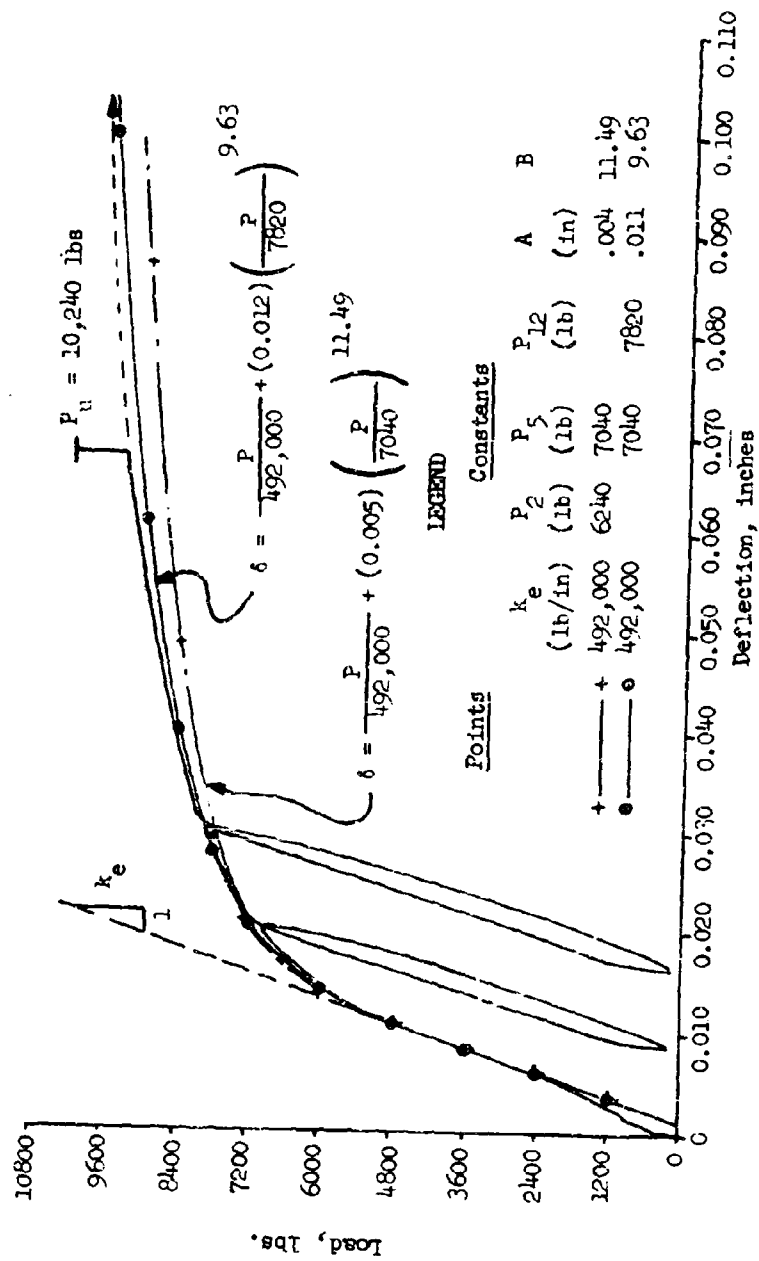


Fig. 66 Comparison of Experimental and Parametrically Derived Joint Load-Deflection Curves.

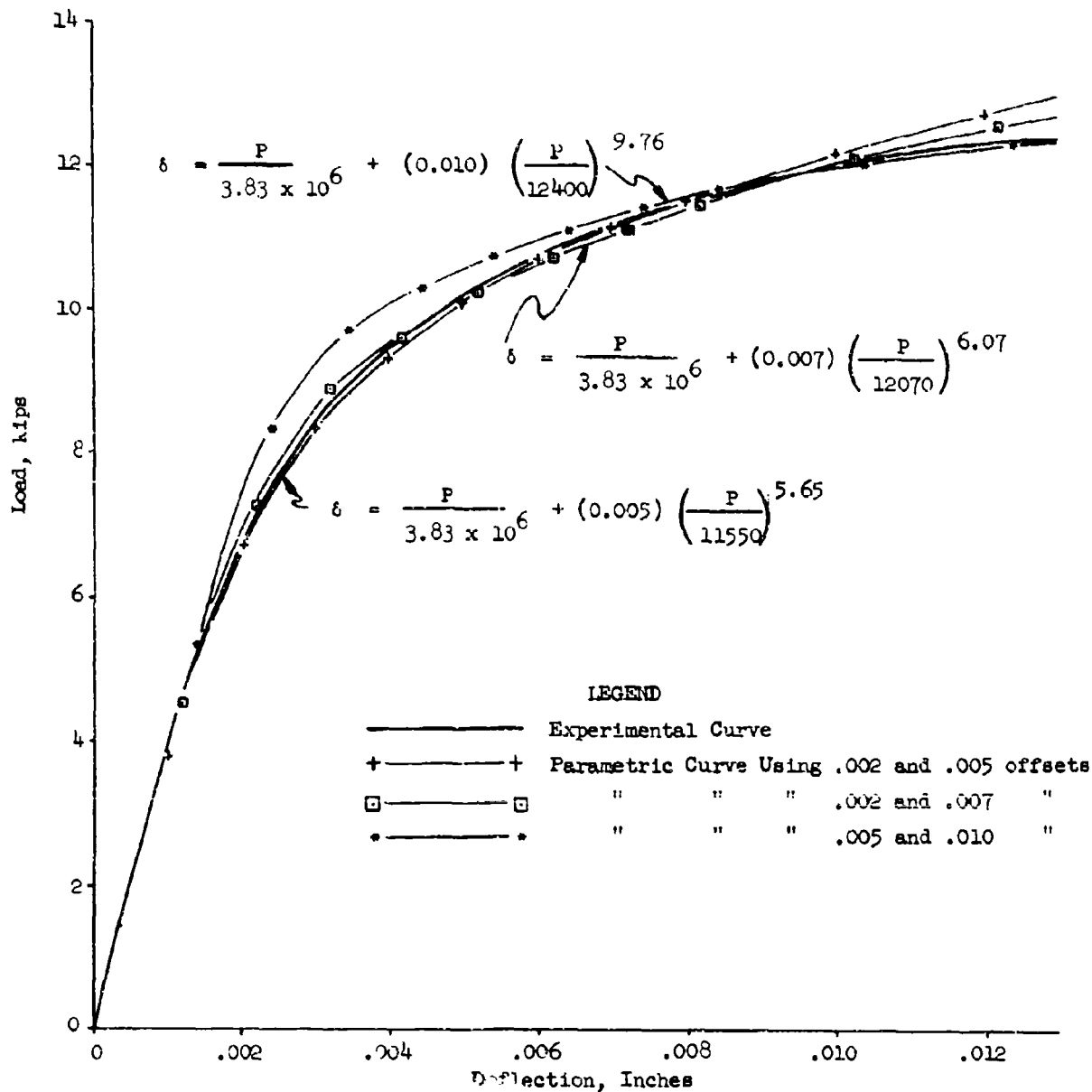


Fig. 67 Comparison of Three Parametric Curves With Experimentally Determined Load-Deflection Curve.

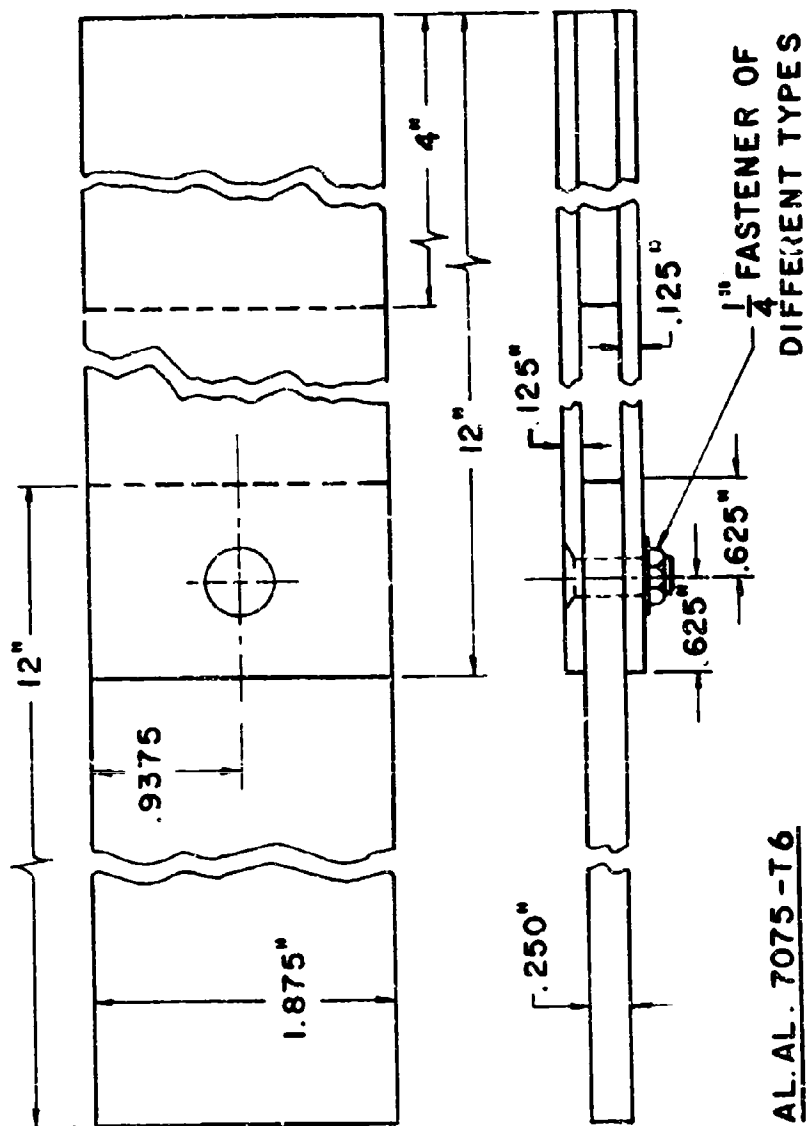


Fig. 68 - Test Specimen for Single Fastener in Double Shear

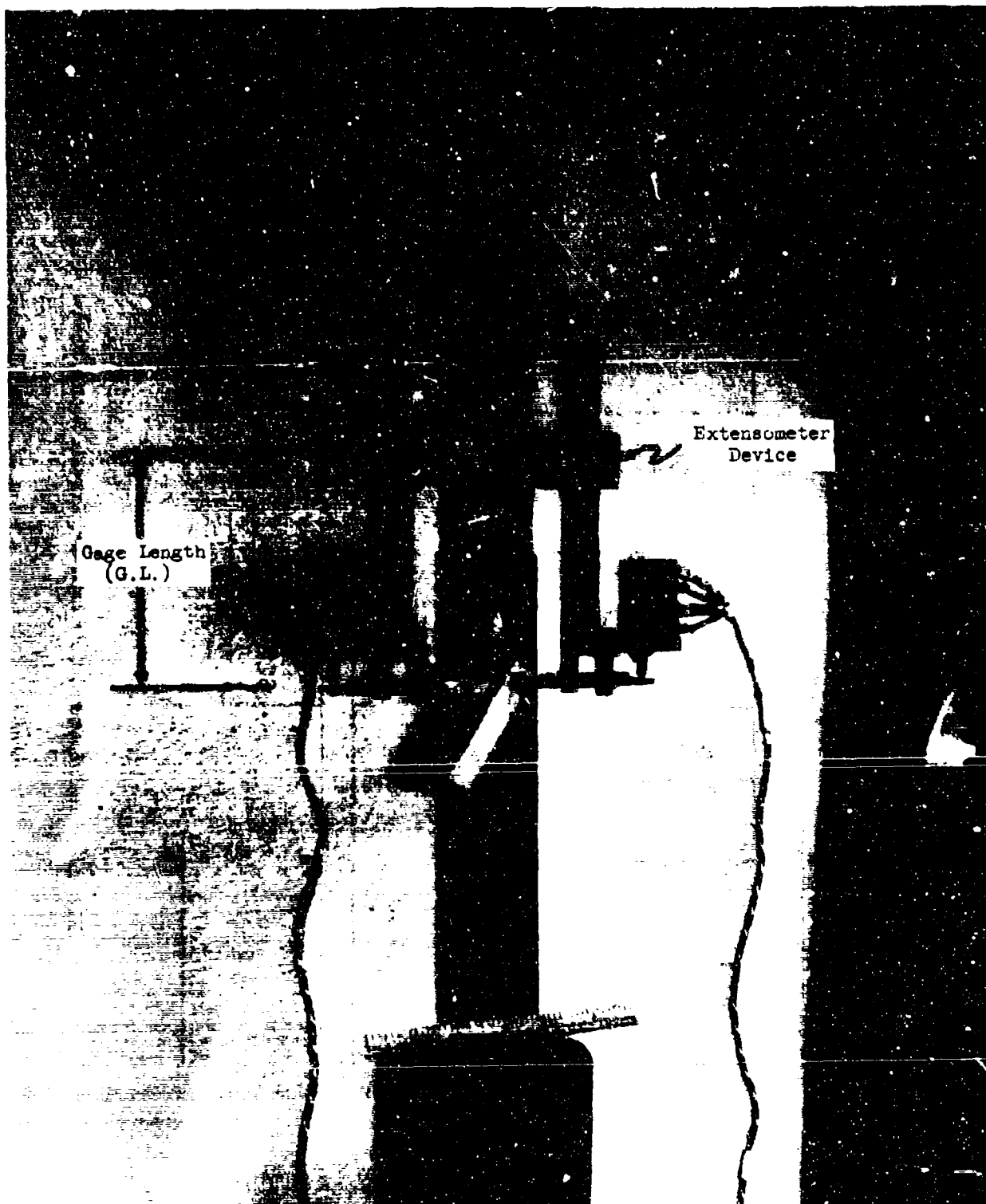


Fig. 69 Test Set Up for Single Fastener Joint in Double Shear



HL-10B

7/30/1969

Fig. 70 Typical Failed Single Fastener Joint Specimen Showing Ovalization of the Holes and Rupture of the Fastener.

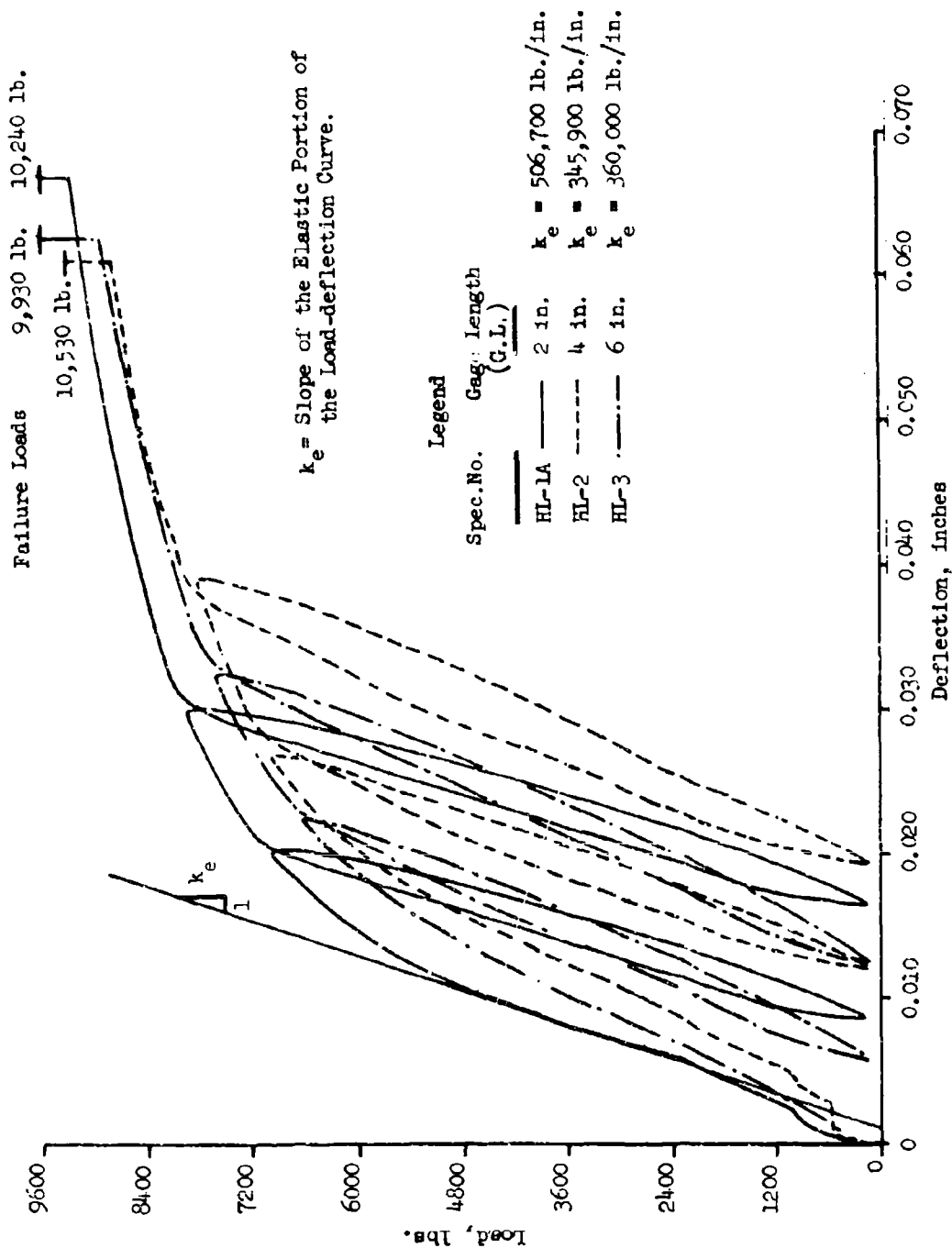


Fig. 71 Effect of Gage length (G.L.) on the Load-Deflection Curve of a Single Fastener Joint.

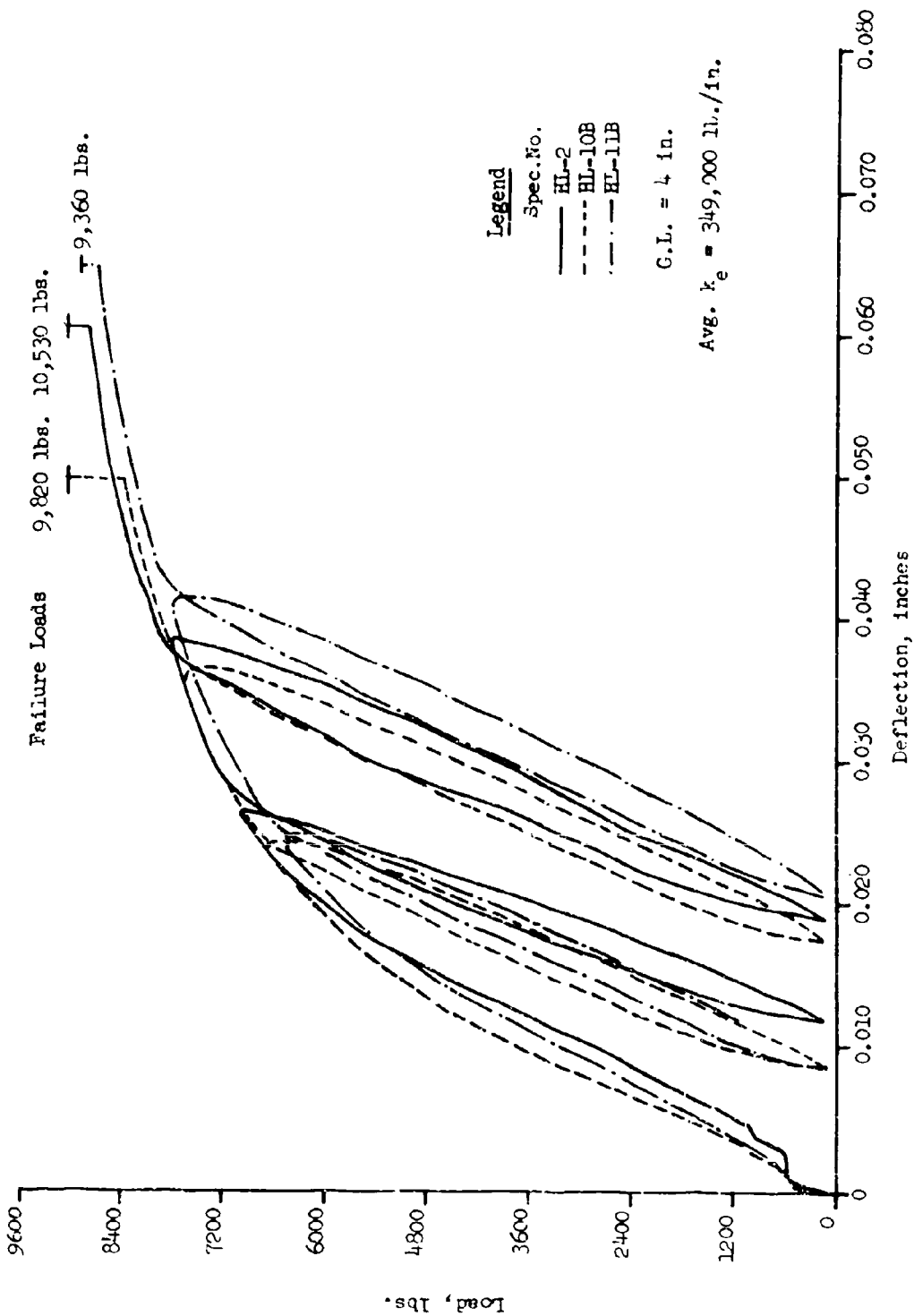


Fig. 72 Load-Deflection Curves of Single 1/4" H1-Lok Fastener in Double Shear With Plates With No Surface Treatment

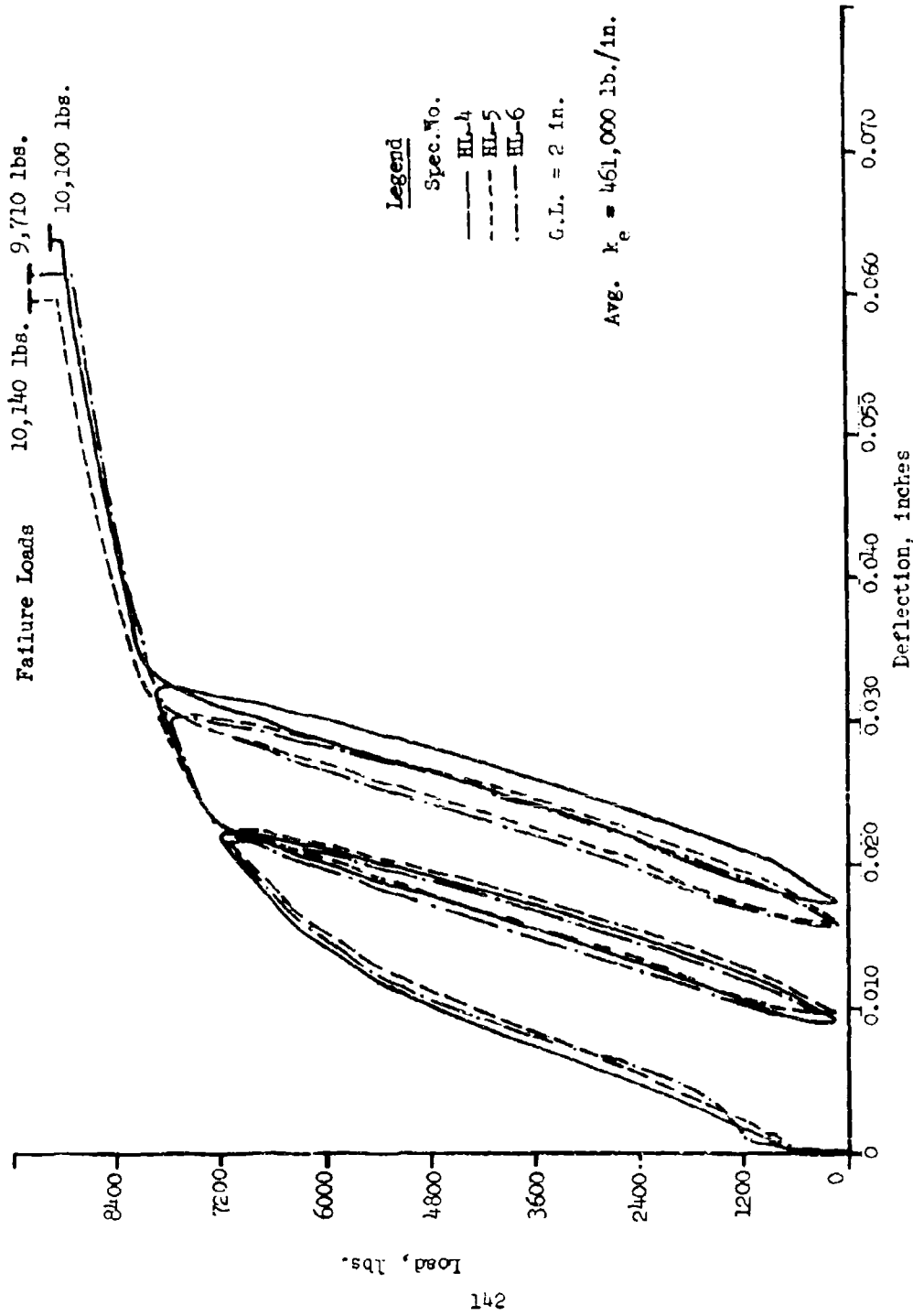


Fig. 73 Load-Deflection Curves of Single 1/4" H1-Lok Fastener in Double Shear with Alodine Finish on Fasteners of the Plates.

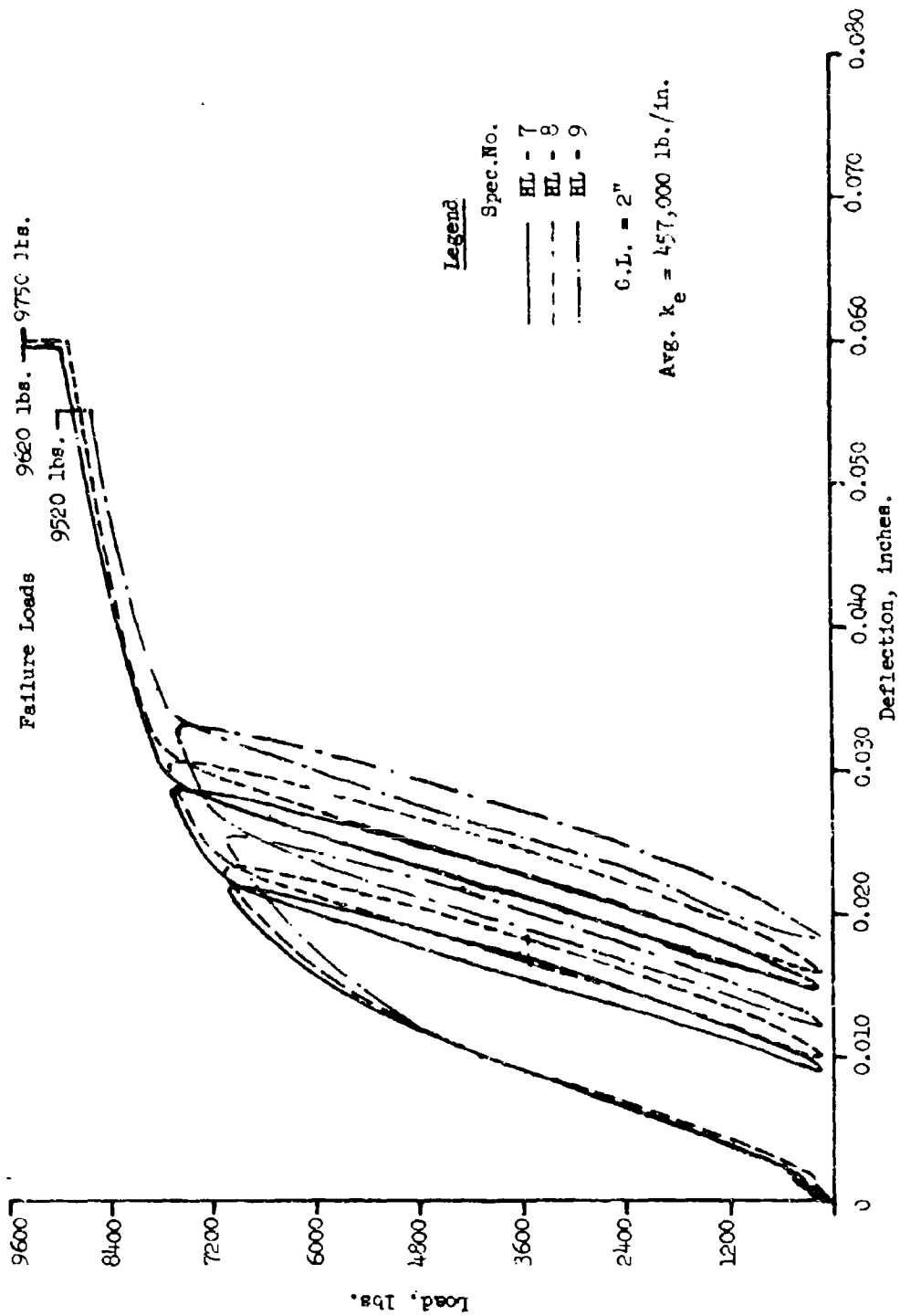


Fig. 74 Load Deflection Curves of Single 1/4" HI-Lok Fastener in Double Shear with Teflon Coated Facing Surfaces of the Plate.

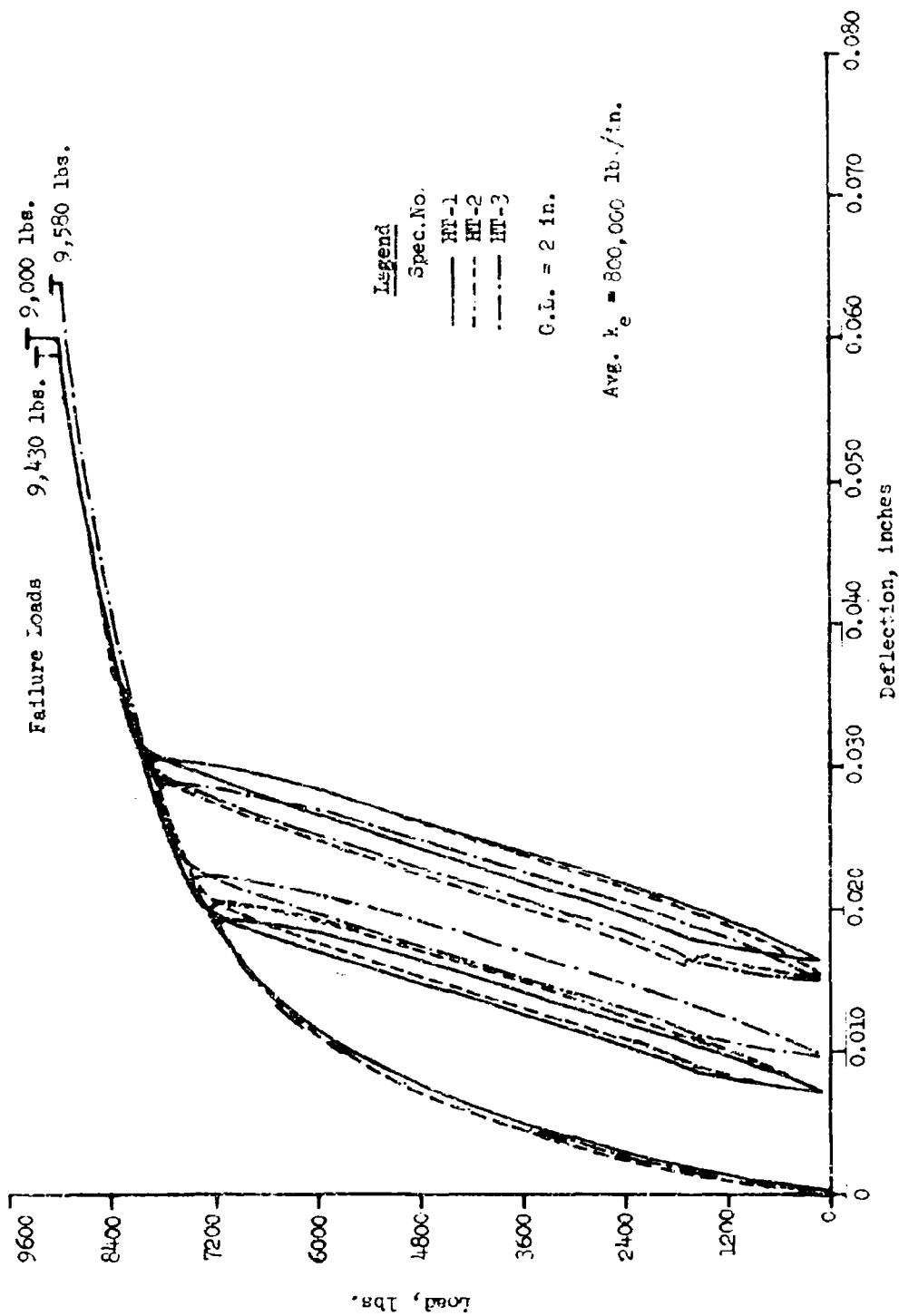


Fig. 75 Load-Deflection Curves of Single Hi-Lok-Hi-Tigue Fastener in Double Shear
in 7075-T6 Aluminum Plates with no Surface Treatment

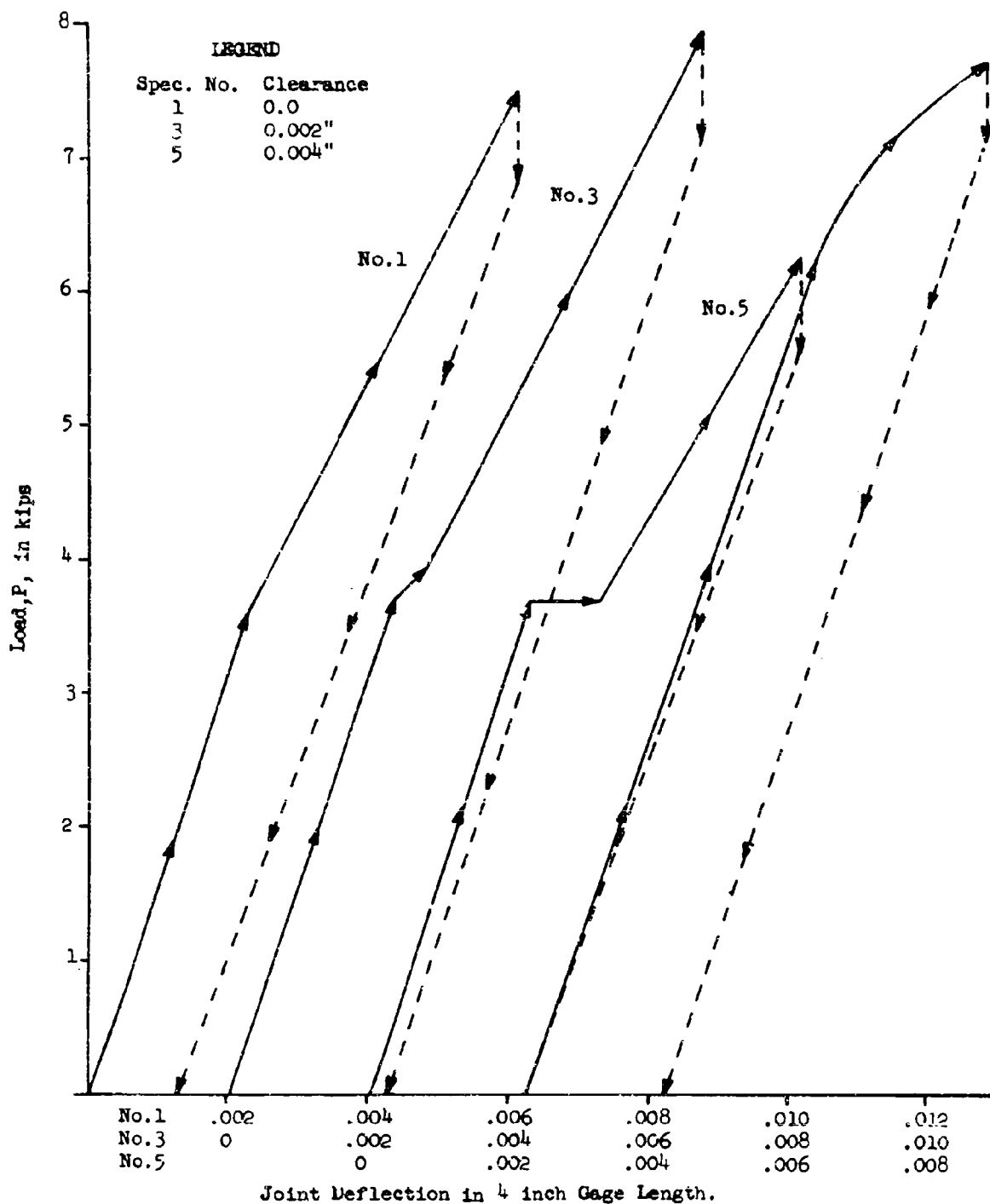


Fig. 77 Effect of Clamp up on the Static Load-Deflection Curves of Joint with Four Hi-Lok Fasteners and Three Different Amounts of Clearance

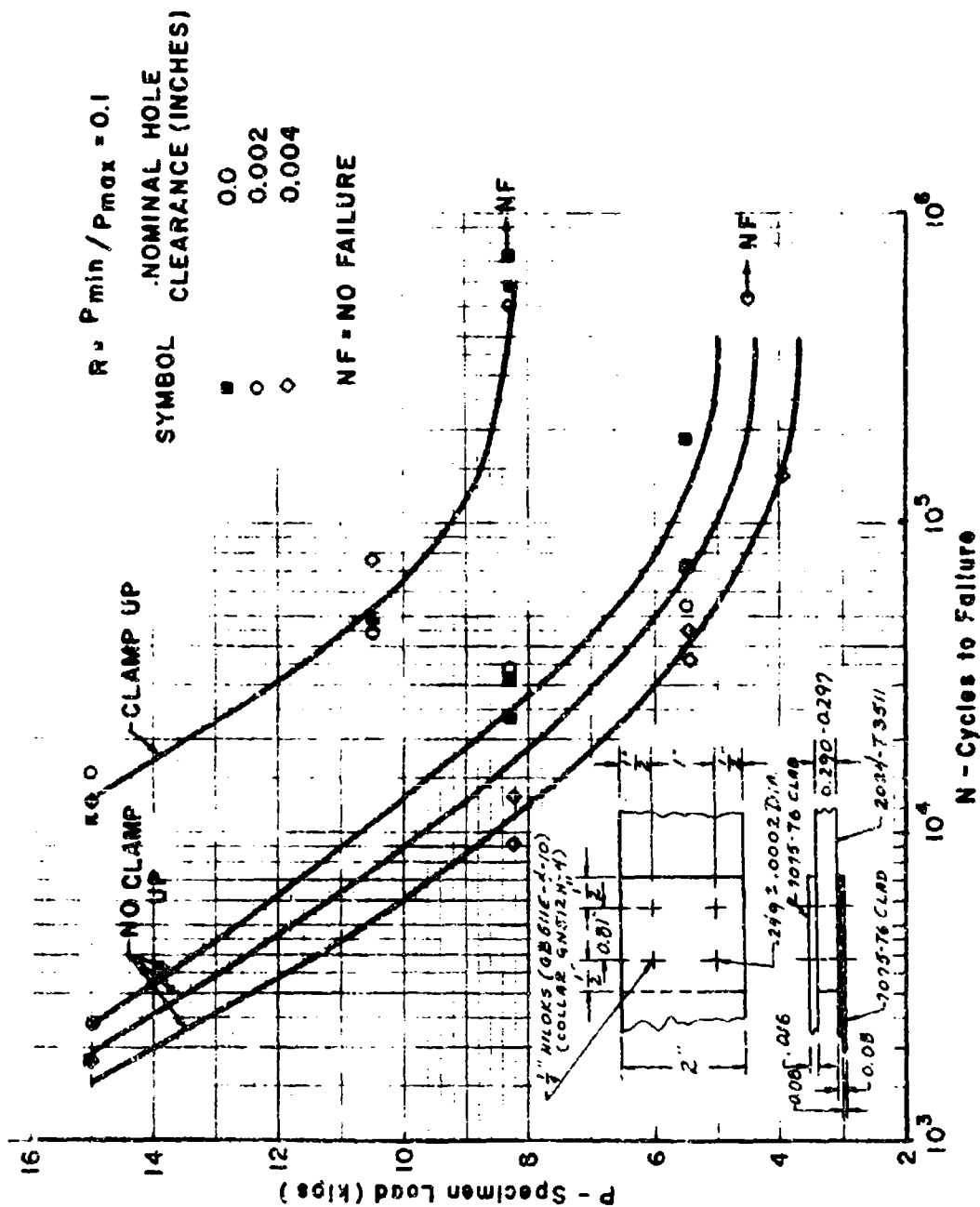


Fig. 78 Example of Fatigue Test Results Showing Effect of Interference Fit and Clamping Force

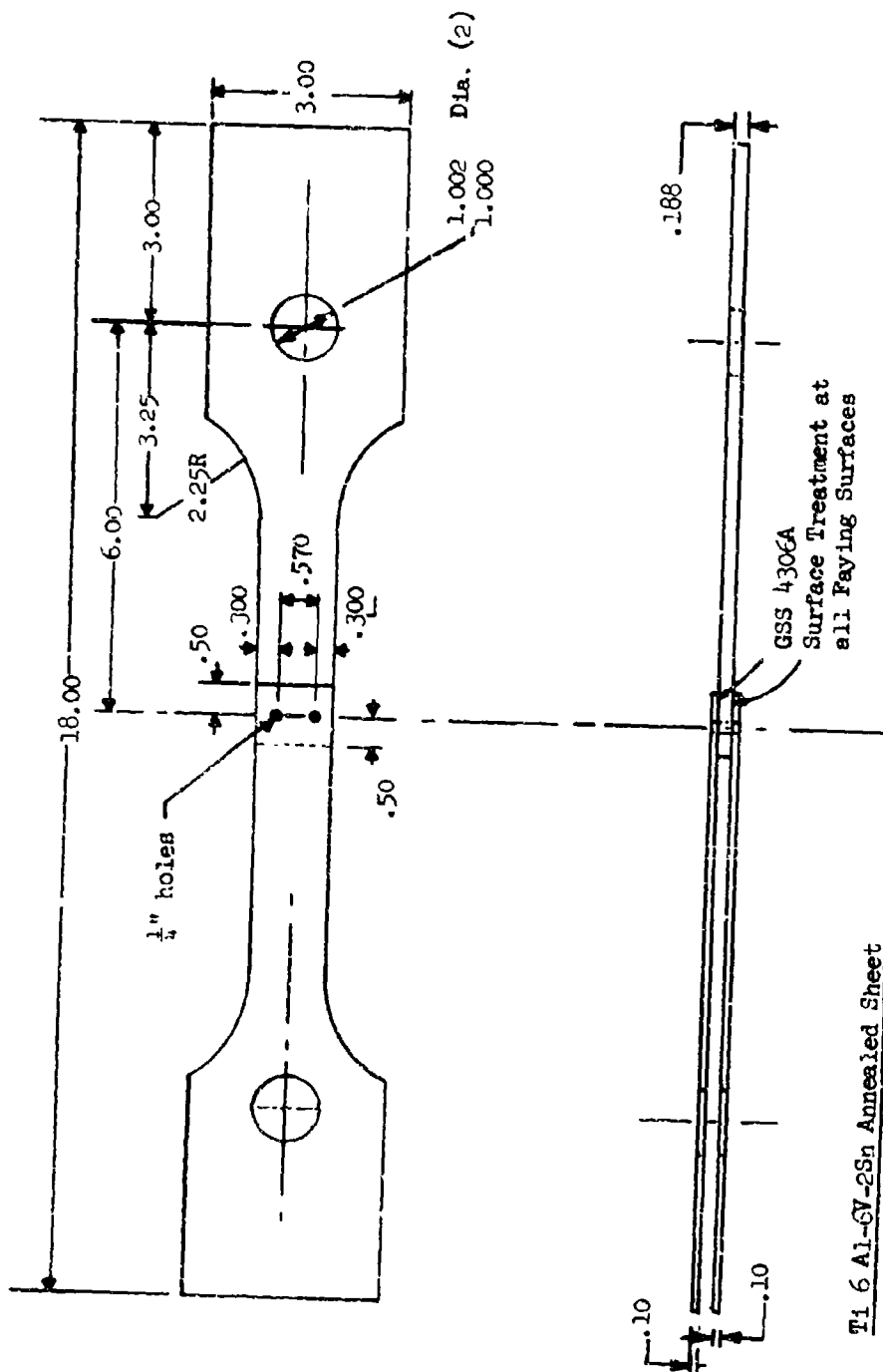


Fig. 79 - Fatigue Specimens Tested Under Spectrum Loading

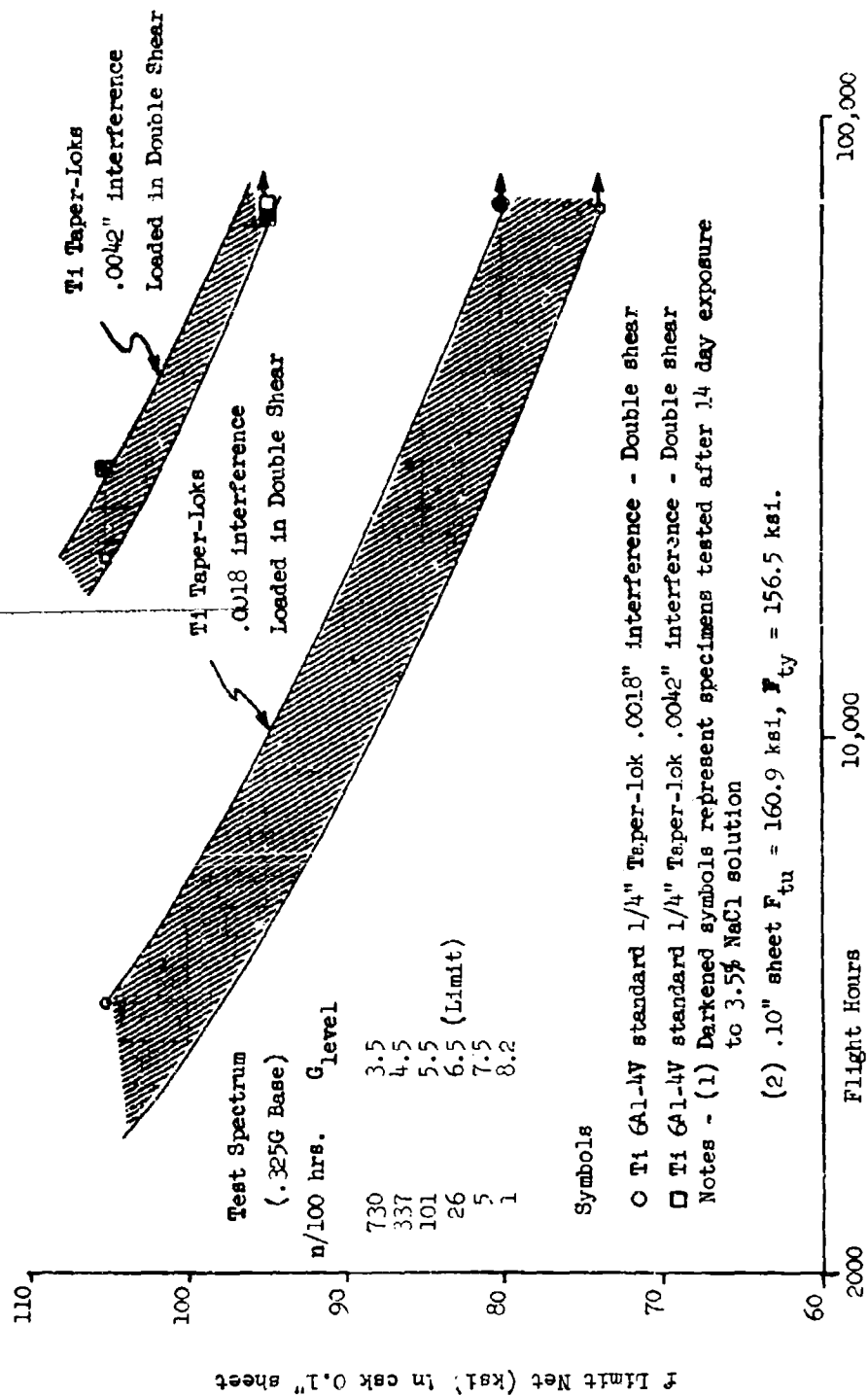


Fig. 80 Spectrum Fatigue Test Results of 1/4" Dia. Taper-Loks in Ti-6Al-6V-2Sn Annealed Sheet

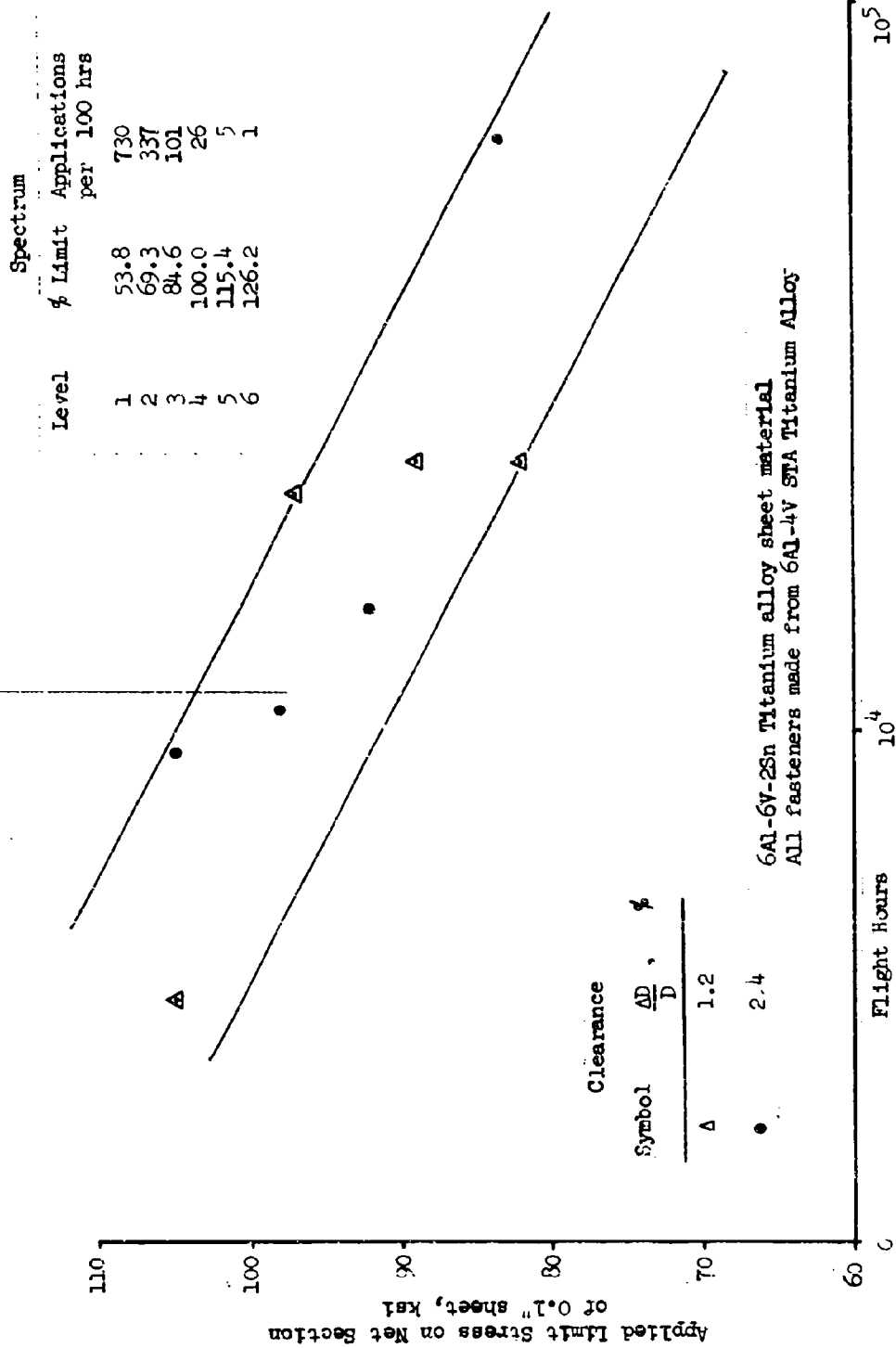


Fig. 81 Spectrum Fatigue Test Results for 1/4" D' in Ti-6Al-6V-2Sn Annealed Sheets

APPENDIX A INELASTIC STRESS AND DEFLECTION ANALYSIS OF PLANAR STRUCTURES

(A) Program Description

This is a brief description of the Grumman computer program used for plastic analysis of mechanically fastened joints. This program is a modified version of deck 45128 originally written to do elastic-plastic and creep analysis of isotropic and anisotropic planar structures. (Reference 3).

The program was originally written in Fortran II for the IBM 7094. It was modified and expanded to run on the IBM System/360 (Fortran IV). On this machine, available hardware facilities permitted expansion to sixty plastic "nodes"*, after allowance was made for some additional features. The version submitted to AFFDL to operate on IBM 7094 under IBSYS control (Fortran IV) is restricted in size to thirty four plastic nodes, due to core size limitations. Use of direct-access storage (disk or drum) for the two large data matrices permits a substantial increase in problem size, at the cost of a factor of approximately ten in running time. This has been done on the IBM/360, increasing the size to 175 nodes with the increase in machine time as described.

The program includes these features. Print output may be obtained at each cycle of calculation, or at load levels which are multiples of the load increments used, as desired. The stress-strain relationship may be defined as a table of ten points on a curve, with automatic interpolation. Alternatively, it may be defined by the Ramberg-Osgood equation, with a reference stress, exponent and Young's modulus as input. The program will accept decreasing steps in applied load. Unloading follows Hooke's law when the node has been stressed in the inelastic region. At any pre-determined load level, the program can be

* The term "node" as used herein refers to either element centroids for constant strain finite elements or actual nodal points for linearly varying strain elements.

forced to spill all the memory tables and clues into a binary save tape, then exit. This permits the programmer to inspect his results and decide whether to continue the analysis from that point, or to modify or abandon the job. A timing feature is built in the coding, but temporarily patched out of the AFFDL version, to allow this save tape to be generated upon reaching a time estimate supplied with the data.

Each link of the program contains the non-IBM subroutines needed for operation. Standard input-output subroutines etc., will be taken from the library tape. As a point of information, the program contains a main program and four overlays.

- (1) LKONE reads the first control card, and reads all other decimal input supplied.
- (2) LKTWO is used only on a restart job. It reads the modified step table, if provided, and the binary input tape.
- (3) LKFOUR is the processing link. It does all the calculation, print control, and writing of print output on tape 6.
- (4) LKFIVE writes a binary tape for restart.

Sequencing and details of the data cards follow. The symbols used in the program for various items of input data are listed on page 153 and are shown on the sample key-punching sheets, pages 160 and 161.

The data cards are used in the following sequence, immediately after the \$DATA card required by IBSYS:

- (1) General clue card (FORMAT 1) containing KLU4, KLU7, NA, NDE, GNU, and a title or caption.
- (2) Table of load steps desired. Up to ten cards defining ten steps may be used. The maximum level for each load step may be above or below the previous maximum level of load. The program verifies the algebraic sign of the increment, and corrects it if necessary. Each card

contains four variables TEMP1, TEMP2, TEMP3, TEMP4 in
FORMAT 2.

- (3) Data matrices. These may be provided in any sequence.
Each matrix has a header card in FORMAT 3, one or more
data cards in FORMAT 4, and a blank card to end it.
The last input matrix on the system input tape must be
followed by one added blank card (two total) to trip
the program into operation.

(B) Symbols and Format of the Data Cards

(1) General Clue Card - FORMAT 1

Cols.	Field	Symbol	
1	I1	KIU4	This gives the number of input matrices on the auxiliary input tape logical #9. If all matrices are on the monitor tape, leave this blank. If using binary restart tape on #8, use a digit from 5 to 9 Maximum number of decimal input matrices on the auxiliary input tape is 4.
2	I1	KIU7	0, or blank, prints output on cycles indicated by the step table; 1 prints on all cycles.
3-5	I3	NA	This sets the frame size for the problem to be handled. NA is three times the number of nodes. Maximum value is 102 (34 nodes).
6-8	I3	NDE	This indicates the number of load conditions for deflection calcula- tions.
9-14	F6.3	GNU	Poisson's ratio "nu"

(B) Symbols and Format of the Data Cards - (Continued)

(1) General Clue Card - FORMAT 1 (Continued)

Cols. Field Symbol

15-80	11A6	TA	Any 66 characters of alpha-numeric text to be printed as a heading for identification purposes.
-------	------	----	---

(2) Table of Steps (limited to ten entries) - FORMAT 2

Cols. Field

1-6	6X	Not used.
-----	----	-----------

7-10	A4	LOAD indicates a load step, TAPE indicates write memory on a save-tape on logical #11 then exit. ESTM indicates a time estimate in minutes to automatically terminate run. ESTM does <u>not</u> count as one of the ten load steps. FENS or blank indicates end of table (this may be the 11th card in the table).
------	----	--

11-20	E10.6	Upper limit of step in pounds or estimated time in minutes.
-------	-------	---

21-30	E10.1	Interval or increment for calculation.
-------	-------	--

31-40	E10.1	Interval or increment for print output. Prints are generated on tape 6 for the current cycle when the current load level is an integral multiple of the print interval. If the print interval is left blank, no prints are generated for cycles in this step. If print interval is
-------	-------	--

(B) Symbols and Format of the Data Cards - (Continued)

(2) Table of Steps (limited to ten entries) - FORMAT 2 (Continued)

Cols.	Field	
31-40	E10.1	very small compared to the current level, then numeric problems sometimes occur in the print control subroutine (OUTPUT), and it may be necessary to re-run with the every-cycle print control "1" for KLU7 punched in the first control card.
41-80		Ignored.

(3) Data Matrices - Header Card - FORMAT 3

Cols.	Field	
1-6	6X	Not used.
7-10	4X	Not used. We use the letters MTRX for compatibility with the GISMO Matrix System, which reads and writes matrices in this format.
11	1X	Not used.
12-17	A6	This is the identification name for the input matrices and must correspond exactly with one of the following names:
or	bbbSIM	{ Matrix of stresses for applied loads maximum size 102x1
	SIMbbb	
or	bbbSIJ	{ Matrix of stresses for member strains maximum size 102x102
	SIJbbb	
	bTSIGN	Table of stress values 11x1 }
	bTEPSN	Table of strain values 11x1 }

(B) Symbols and Format of the Data Cards - (Continued)

(3) Data Matrices - Header Card - FORMAT 3 - (Continued)

Cols. Field

These two matrices define the stress-strain curve as a series of chords. The data is entered in this format, merely to conform to the format of the SIM-SIJ matrices which were generated using the GISMO Matrix System. Note that the first value in both TSIGN and TEPSN must be zero to avoid upsetting the interpolation procedure.

bbbbDIM } Matrix of deflection for applied loads
or DIMbbb } maximum size 102x1

bbbdIJ } Matrix of deflection for unit induced
or DIJbbb } strains - maximum size 102x102

RAMOSC three parameters for Ramberg-Osgood equation - reference stress, exponent and Young's modulus.

18	1X	Not used.
19-21	I3	Number of rows in this matrix.
22-24	I3	Number of columns in this matrix.
25-30		Not used.

(B) Symbols and Format of the Data Cards - (Continued)

(4) Data Matrices - Data Cards - FORMAT 4

Cols.	Field	
1	IX	Not used.
2-4	I3	Row index for the first element.
5-7	I3	Column index for the first element.
8-23	E16.8	The first element on this card.
24	IX	Not used.
25-27	I3	Row index for the second element.
28-30	I3	Column index for the second element.
31-46	E16.8	The second element on this card.
47	IX	Not used.
48-50	I3	Row index for the third element.
51-53	I3	Column index for the third element.
54-69	E16.8	The third element on this card.
70-80		Not used.

The last card of a matrix must be completely blank (tested in Col. 2-4). The last matrix on the Monitor input tape 5 must be followed by one added blank card (two total) to trip the program into operation.

The input matrices on the Monitor tape may be in any sequence as long as each matrix starts with a header card, has all its data cards next, and ends with a blank card.

(C) Ramberg-Osgood Equation

If a run encounters the name RAMOSG in the data matrices, an internal clue (KIU3) is set and the run will use the three input parameters with the Ramberg-Osgood equation for stress:-

(C) Ramberg-Osgood Equation - (Continued)

strain relationships. Note that the second value in TSIGN (TSIGN(2)) must be provided to indicate the effective stress for start of plasticity. Without this value, plasticity is assumed to start at zero stress level.

(D) Stress-Strain Curve

If a run has not found the Ramberg-Osgood parameters, it will calculate Young's Modulus from the first two points in Tables TSIGN and TEPSN (Table of SIGma sub N and Table of EPSilon sub N). TSIGN (2) is also used to define the start of plasticity.

(E) Restart Procedure

If the run being set up is a restart, the input deck can be in several forms. The first control card must have a digit from 5 to 9 in KIU4 so that the program will read tape 8; KIU7 is read from this card. The other factors are carried from the previous run.

The table of steps may be read in again (modified) if the previous run is stopped due to the time estimate, or it may be retained and continued from the previous run. However, if the previous run is stored on tape by using a TAPE card in the step table, then a new step table must be read in.

In either case, the last data card on a restart job must be FINS or blank in columns 7-10. This means a restart data deck will have a minimum of two cards (clue card and a blank), or a maximum of 12 cards (clue card, 10 step cards and a FINS or blank).

(F) Time Estimates

For time estimates, allow 3.5 minutes to compile the Fortran, 1.5 minutes to read the input tape, 2.0 seconds per cycle printed, and 100 to 110 cycles of calculation per minute.

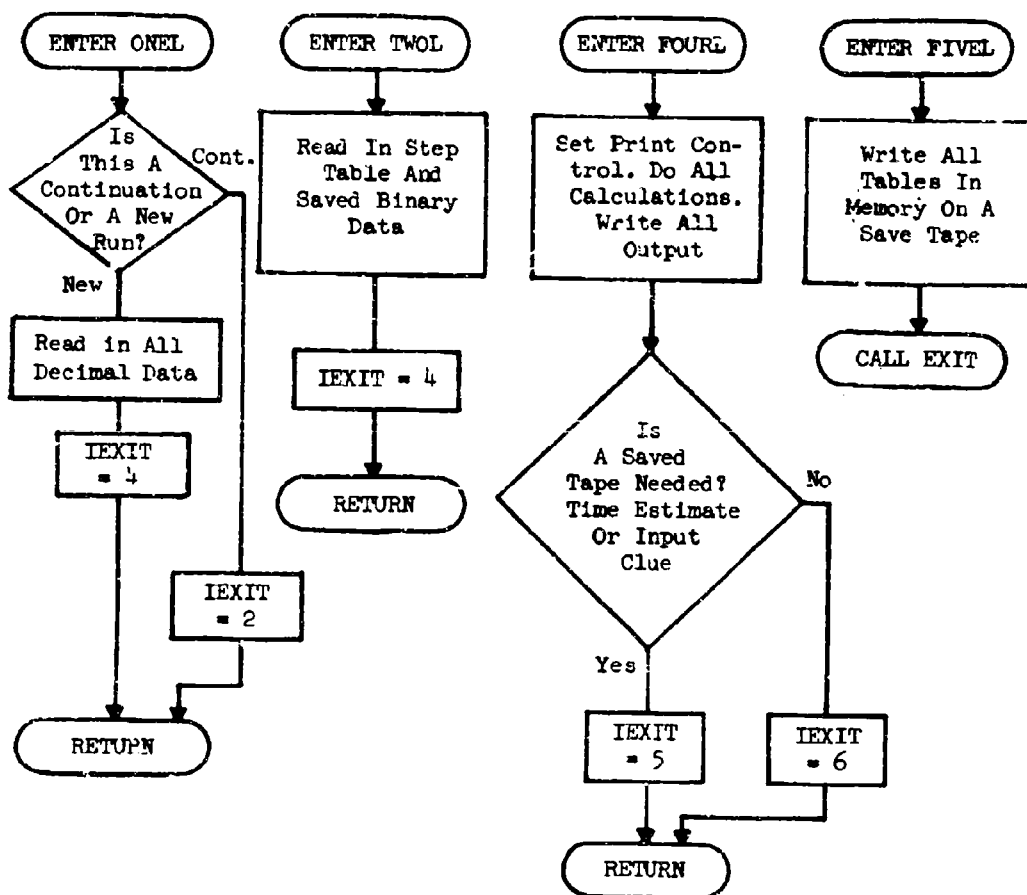
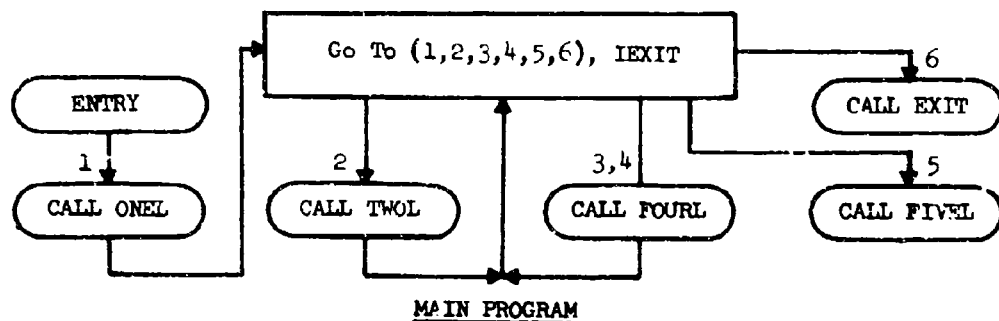
(G) Special Input Formats and Options

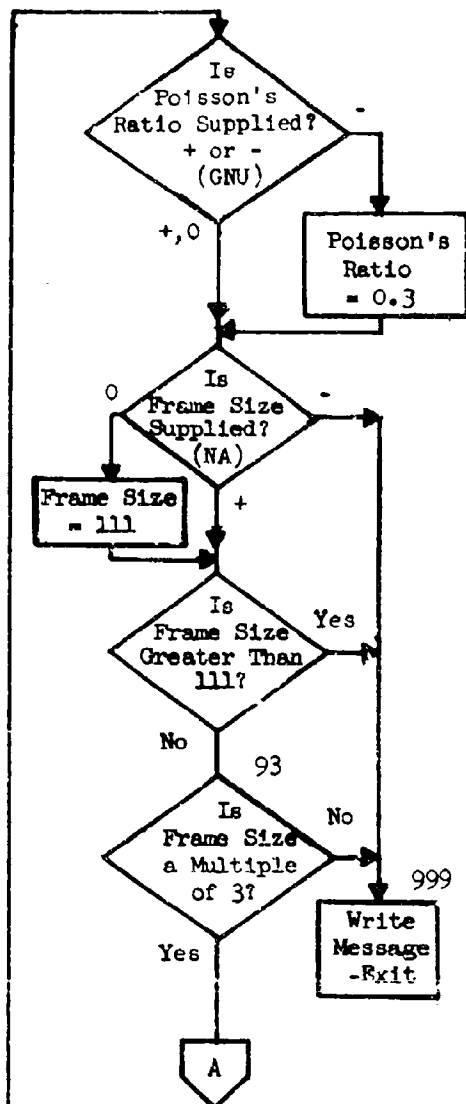
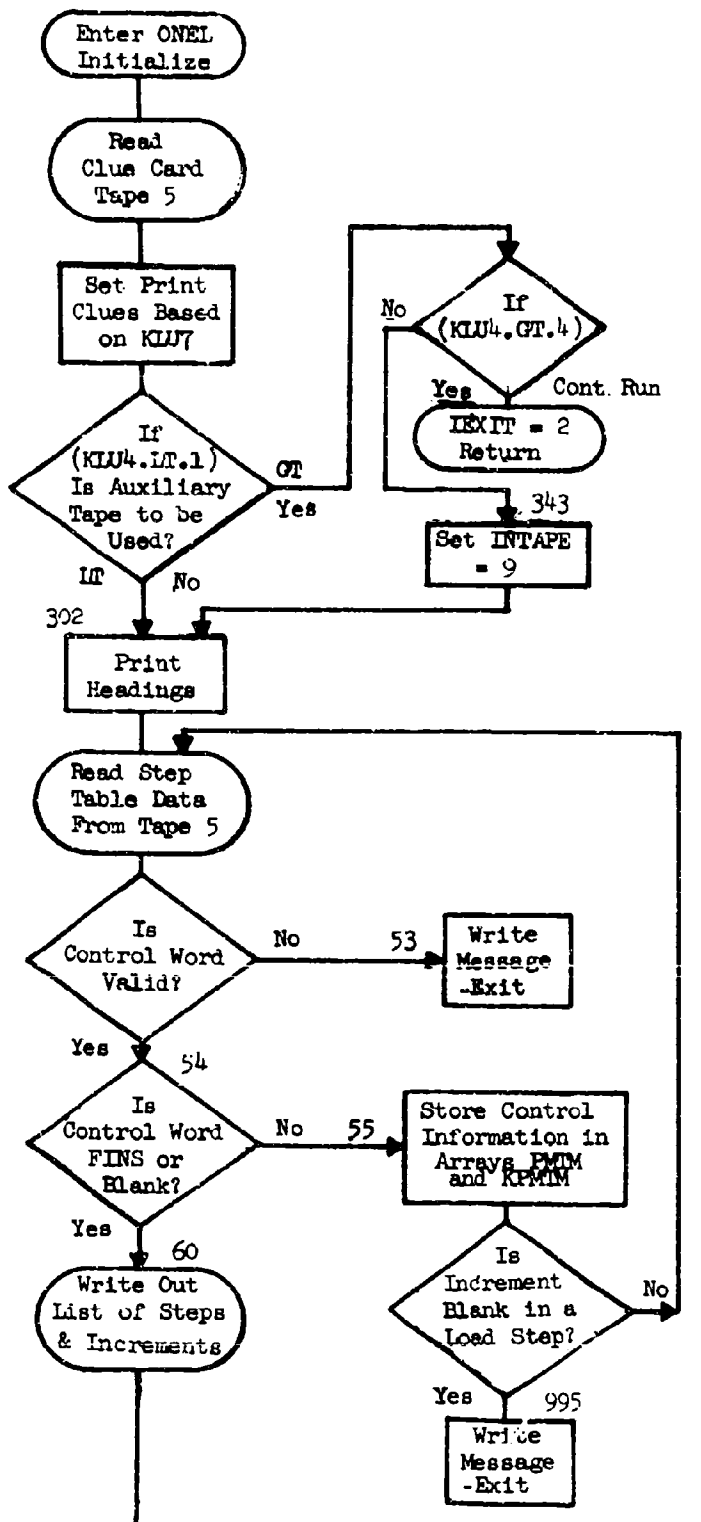
The plastic analysis program was set up to use common matrix card formats for the auxiliary tape as well as the system input tape, due to the simplicity of the logic required to switch between the units. Original formats were those used in the GISMD matrix package. The formats used on the auxiliary tape were modified (as an option at "compile time") to accept formats from a later matrix package (COMAP card images) on the auxiliary tape. The option has been extended to include FORMAT matrix package card images on the auxiliary tape only, still using the original card formats on the system input tape. The option is selected when compiling the program. It is not set by clues at execution time. The card formats are illustrated on the sample key punch sheets pgs. 160 and 161.

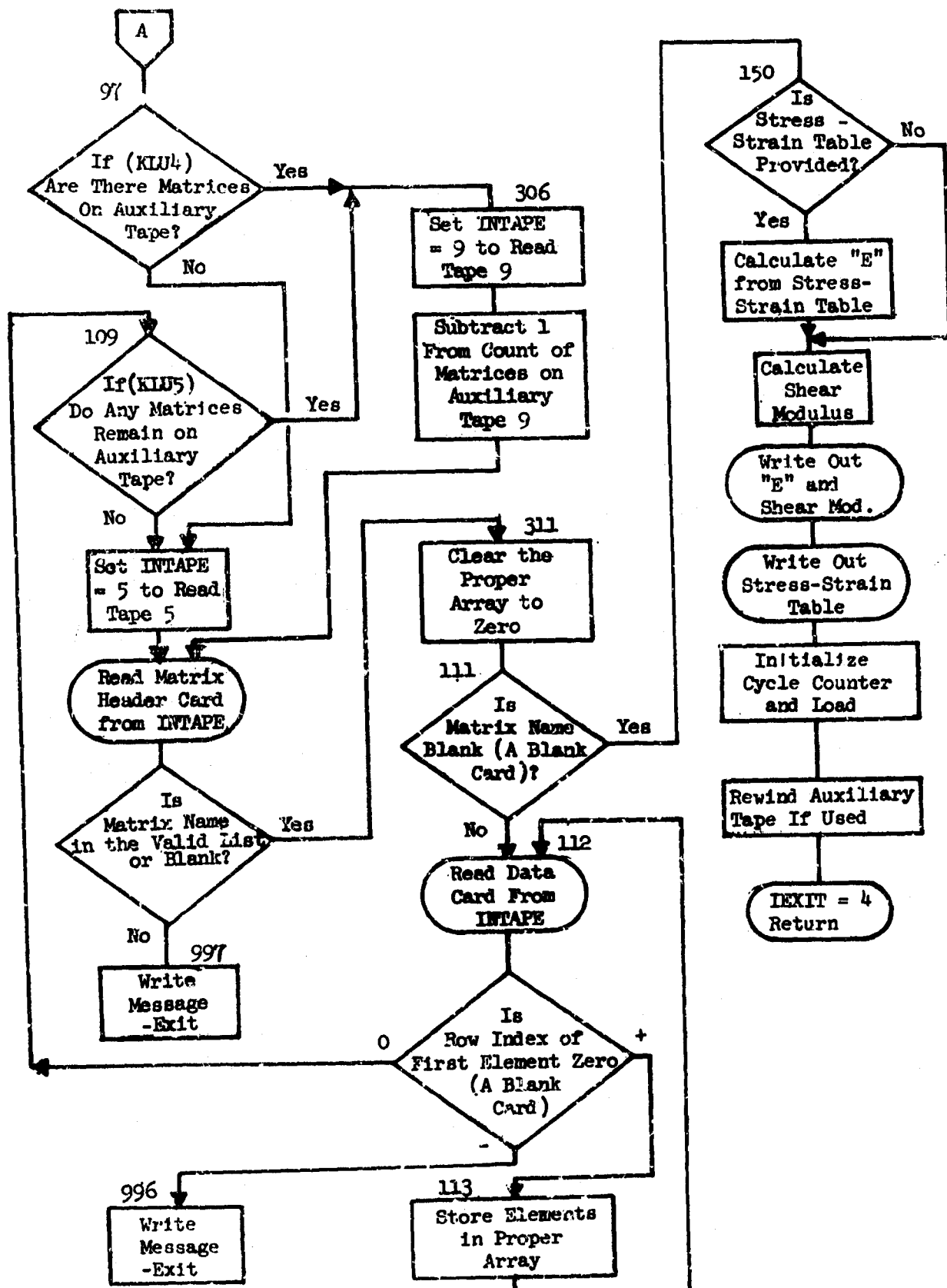
Internal clue NFC is set to 0 to use the original card formats (#3 and #4) for the auxiliary tape as well as on the system input tape. If NFC = 40, the program will expect COMAP card formats (#43 and #44) only on the auxiliary tape. If NFC = 42, the program will expect FORMAT cards (#45 and #46) only on the auxiliary tape. Note that all these clues, formats and modified lists are provided in the program, with identifying comment cards. It is only necessary to reproduce the desired card for NFC dropping out the C from column 1 (and adding C to the conflicting card) and the program logic will be modified.

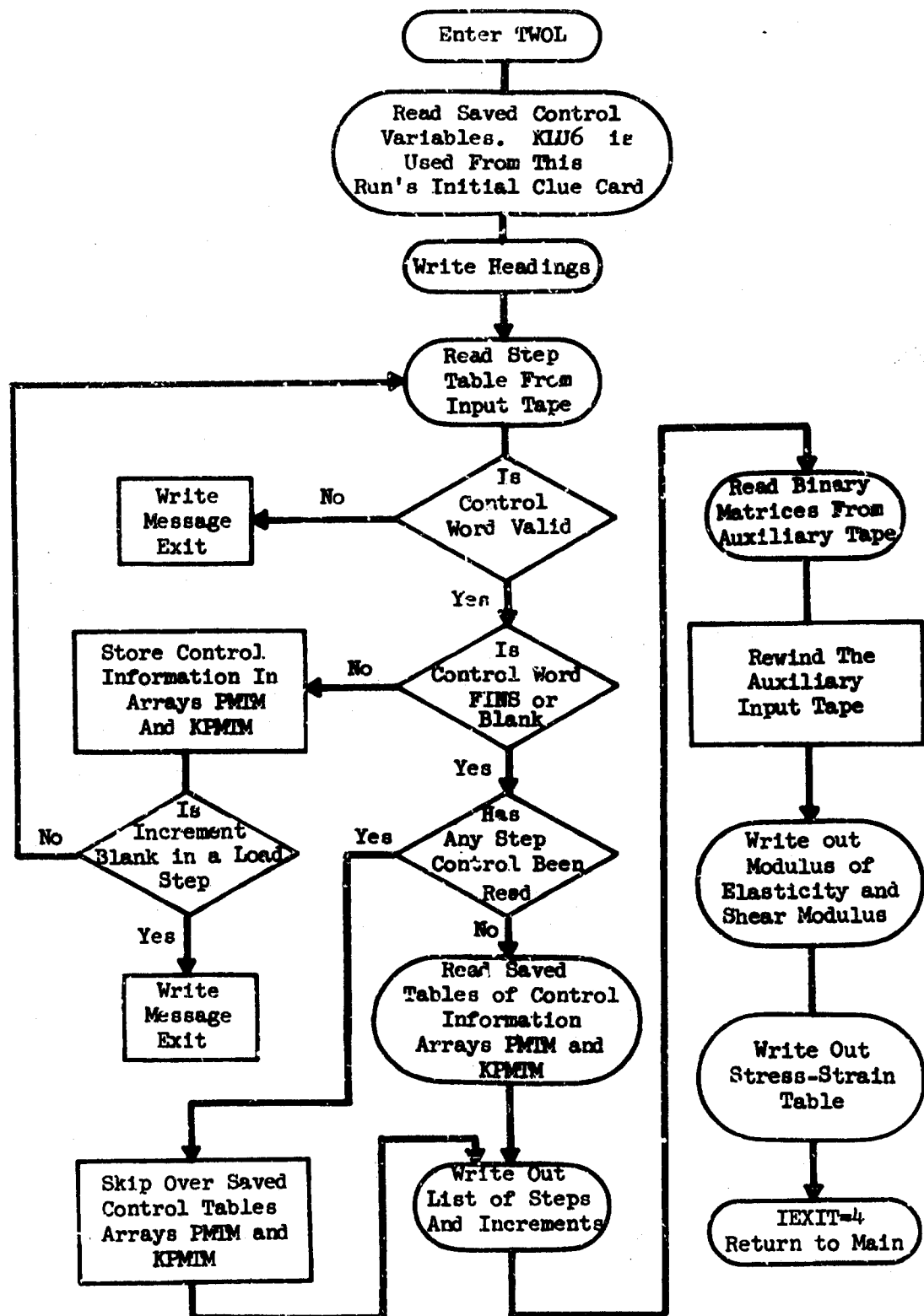
APPENDIX B INELASTIC STRUCTURAL ANALYSIS PROGRAM - FLOW CHARTS

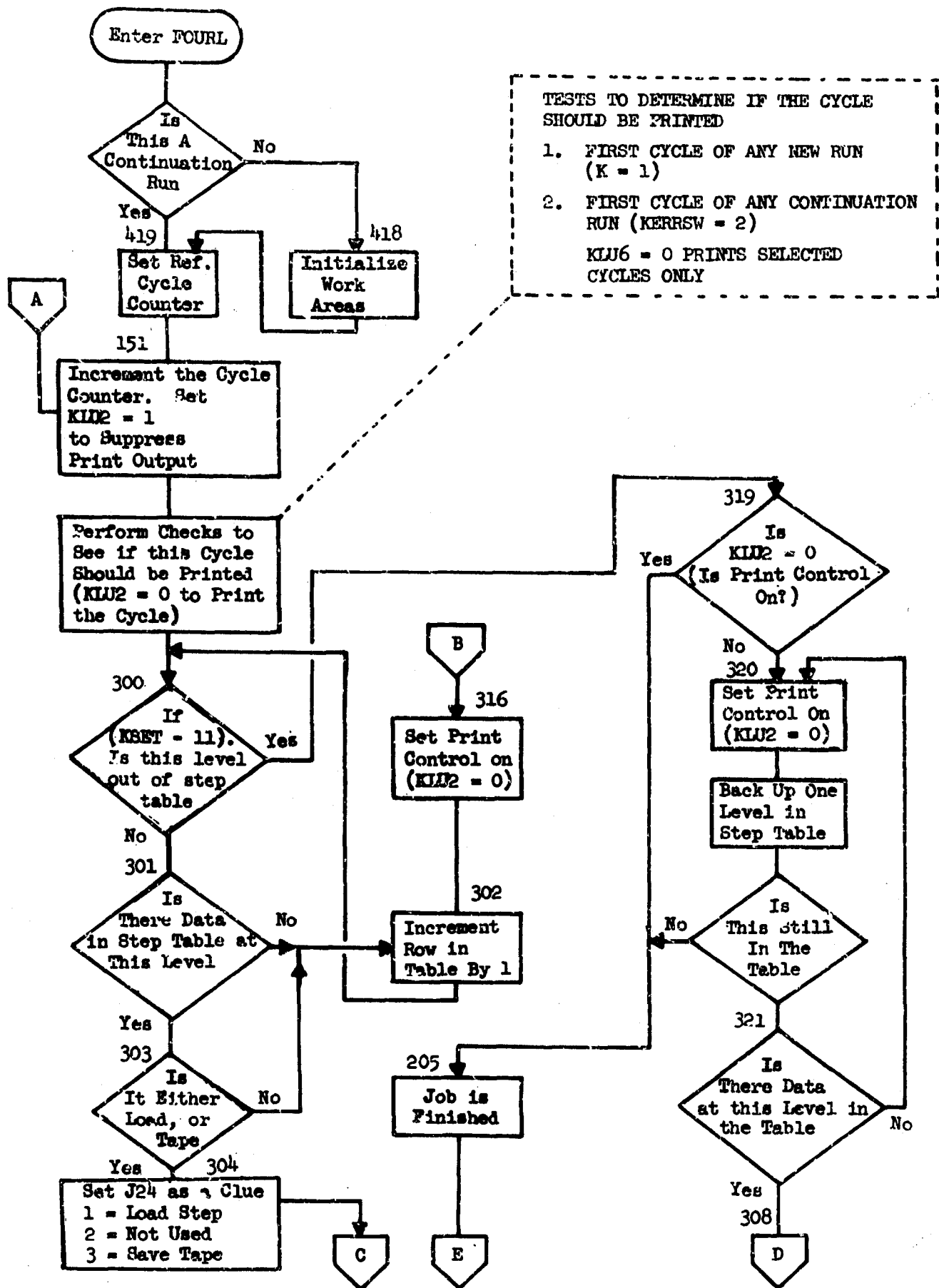
The Main Program is Simply an Overlay Caller. The Clue (IEXIT) to Direct the Main Program is Set in Each Overlay, as Determined by the Data and the Execution of the Problem.

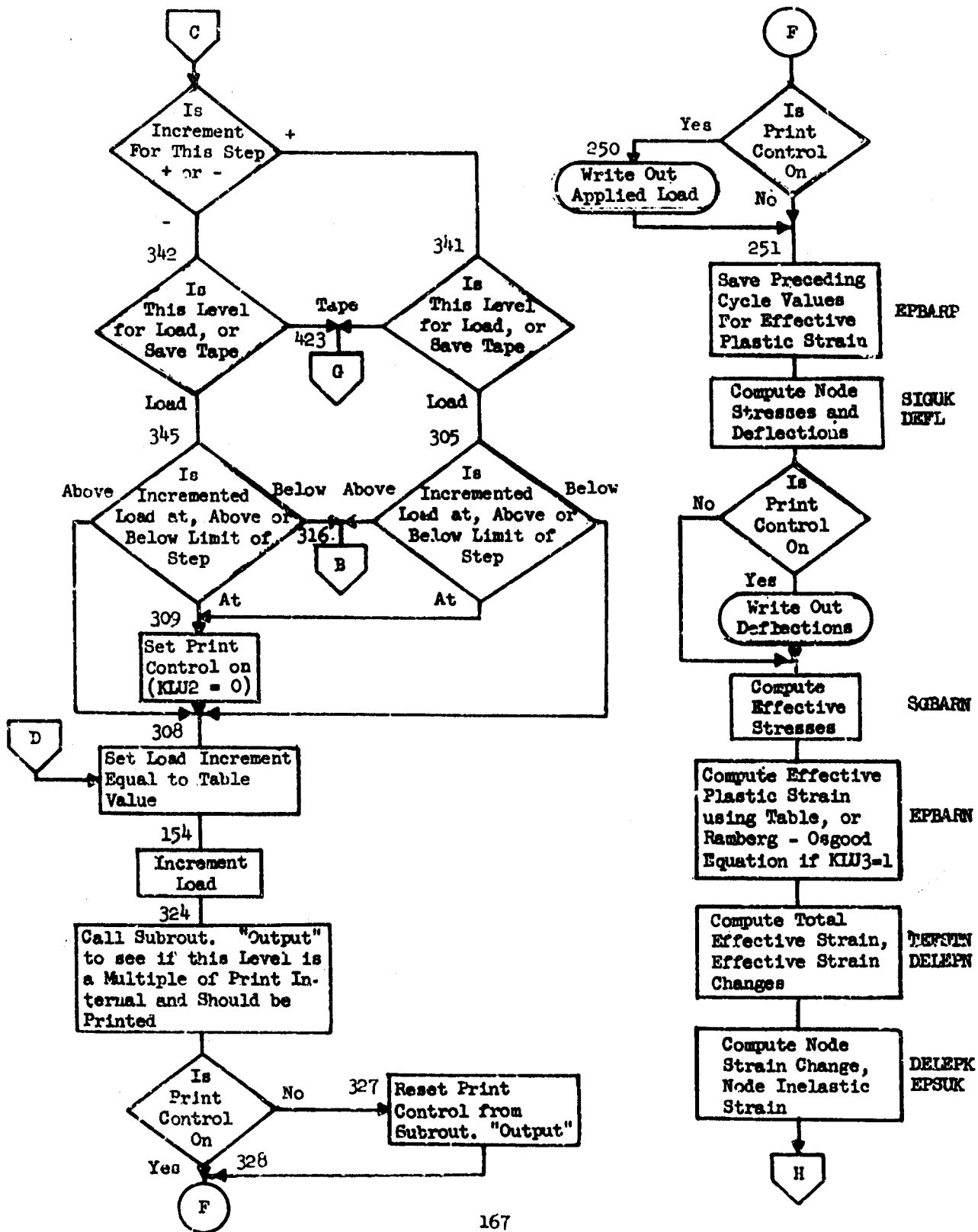


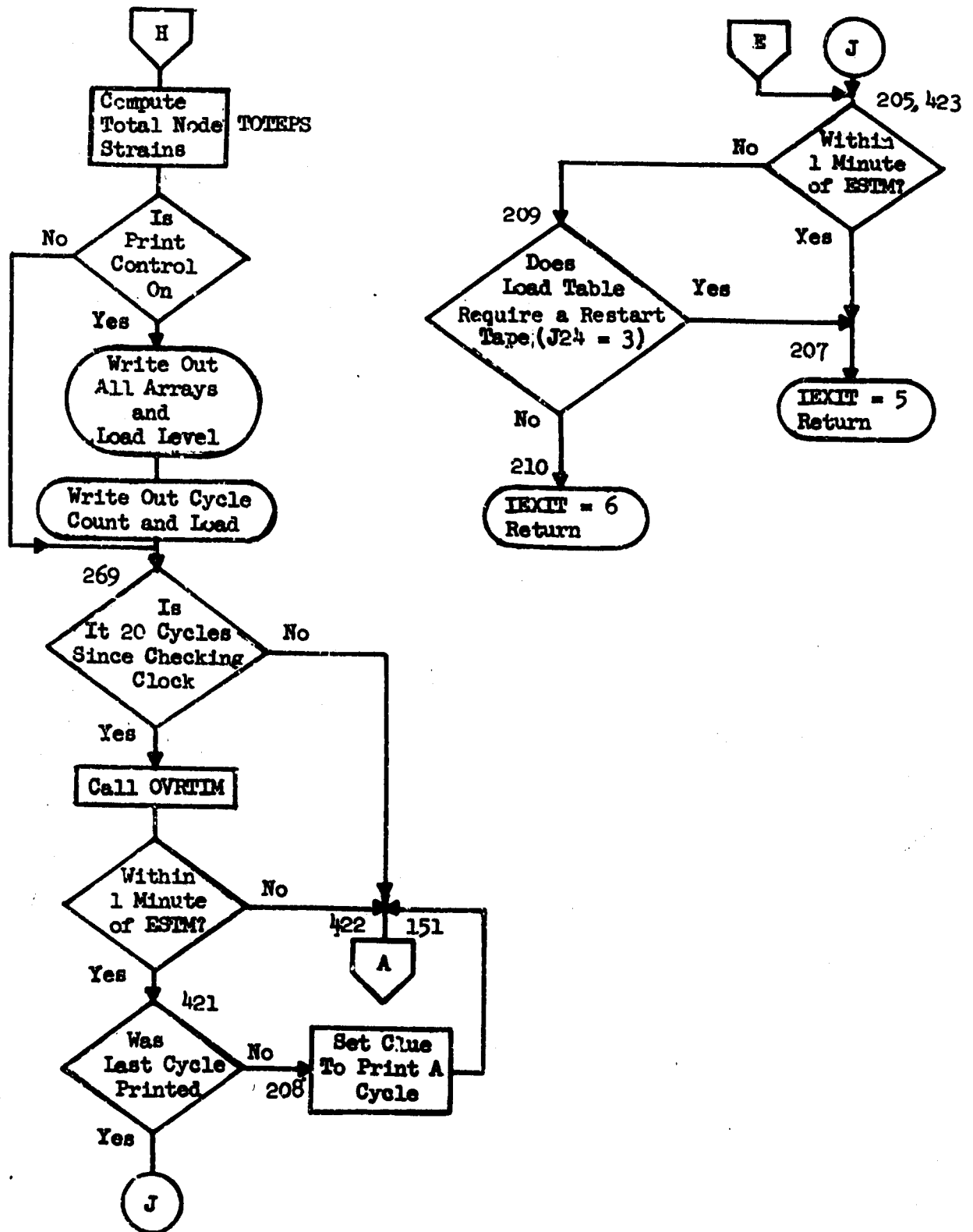


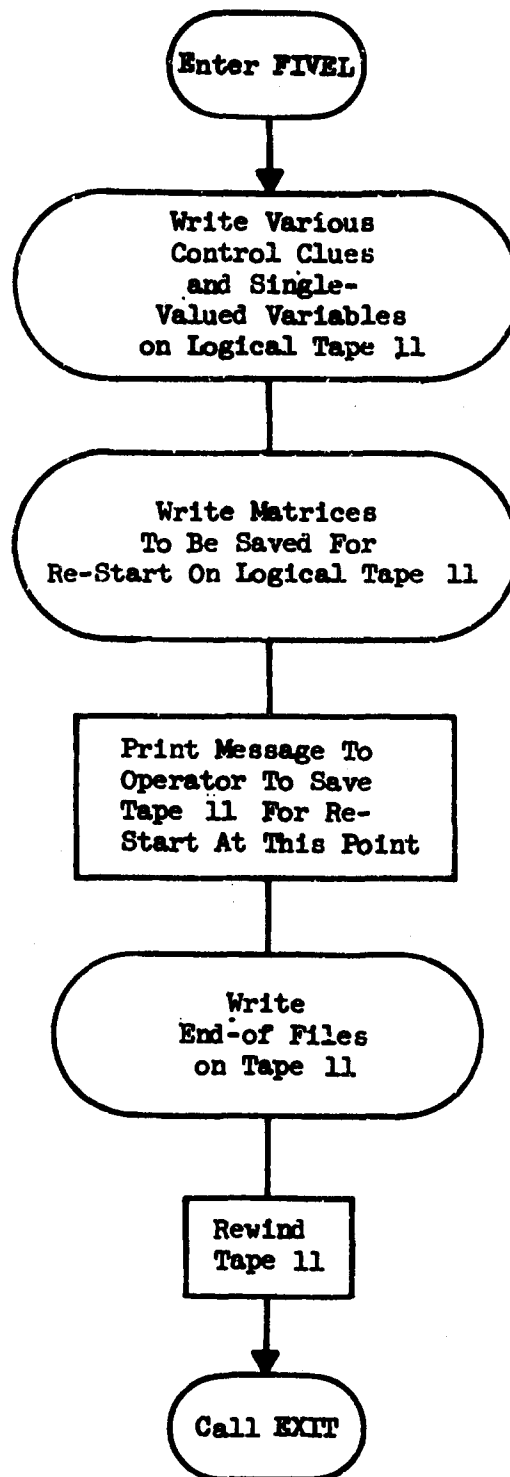












NOT REPRODUCIBLE

APPENDIX C FORTRAN LISTING OF INELASTIC STRUCTURAL ANALYSIS PROGRAM

```

$GROUP      CPNCT=01,BUFCT=001,'UNIT04','UNIT08','UNIT10'
$GROUP      CPNCT=01,BUFCT=001,'UNIT01','UNIT02','UNIT11'
$*          TAPE 08 IS USED FOR A BINARY INPUT TAPE (SAVED FROM PREVIOUS RUN)
$IDMAP TAPE08
          ENTRY      .UN08.
          .UN08. PZE   UNIT08
          UNIT08 FILE  .L03,MOUNT,INPUT,BLK=256,BIN
          END
$*          TAPE 09 IS USED FOR A BCD MATRIX INPUT TAPE (AUXILIARY TAPE)
$IDMAP TAPE09
          ENTRY      .UN09.
          .UN09. PZE   UNIT09
          UNIT09 FILE  .CK1,MOUNT,INPUT,BLK=14,BCD
          END
$*          TAPE 11 IS USED FOR A BINARY SAVE TAPE (FOR CONTINUATION RUN)
$IDMAP TAPE11
          ENTRY      .UN11.
          .UN11. PZE   UNIT11
          UNIT11 FILE  .L04,READY,OUTPUT,BLK=256,BIN
          END
```

NOT REPRODUCIBLE

APPENDIX C FORTRAN LISTING OF ELASTIC STRUCTURAL ANALYSIS PROGRAM

```
IGROUP      CPNCT=01,DPNCT=001,'UNIT04','UNIT05','UNIT10'  
$GROUP      DPNCT=01,DPNCT=001,'UNIT01','UNIT02','UNIT11'  
$*          IS USED FOR A BINARY INPUT TAPE (SAVED FROM PREVIOUS RUN)  
$IDMAP TAPE08  
ENTRY      .UN08.  
.UN08. PZE   UNIT08  
UNIT08 FILE .L03,OUNT,INPUT,BLK=256,BIN  
END  
$*          IS USED FOR A BCD MATRIX INPUT TAPE (AUXILIARY TAPE)  
$IDMAP TAPE09  
ENTRY      .UN09.  
.UN09. PZE   UNIT09  
UNIT09 FILE .CK1,OUNT,INPUT,BLK=14,BCD  
END  
$*          IS USED FOR A BINARY SAVE TAPE (FOR CONTINUATION RUN)  
$IDMAP TAPE11  
ENTRY      .UN11.  
.UN11. PZE   UNIT11  
UNIT11 FILE .L04,READY,OUTPUT,BLK=256,LIN  
END
```

Best Available Copy

GRUMMAN AEROSPACE CORP. DECK NO. 45128 MAIN PROGRAM
MATRIX ANALYSIS FOR ISOTROPIC INELASTIC STRUCTURES
IN THE PLASTIC REGIME

THIS PROGRAM WRITTEN FOR AIR FORCE CONTRACT F33615-69-C-1263

TABLE OF SYMBOLS USED

*** ARRAYS ***

DEFL - MATRIX OF NODE DEFLECTIONS
DELEPK - MATRIX OF NODE STRAIN CHANGES (DELTA EPSILON)
FOR THE X AND Y DIRECTIONS, AND THE SHEAR STRAIN CHANGE
DELEPN - MATRIX OF EFFECTIVE INELASTIC STRAIN CHANGES
DIJ - MATRIX OF DEFLECTIONS FOR MEMBER STRAINS
DIM - MATRIX OF DEFLECTIONS FOR APPLIED LOADS
EPBARN - MATRIX OF EFFECTIVE INELASTIC STRAINS (EPSILON BAR SUB N)
FOR K-TH CYCLE
EPBARP - MATRIX OF EFFECTIVE INELASTIC STRAINS (EPSILON BAR SUB N)
FOR K-1 CYCLE (THE PRECEDING CYCLE)
EPSUK - MATRIX OF NODE PLASTIC STRAINS (EPSILON SUB U) FOR THE
X, Y AND SHEAR STRAINS, FOR THE K-TH CYCLE
KPMTH - TABLE OF CONTROL CLUES (LOAD STEP, WRITE SAVE
TAPE) READ-IN CONTROL INFORMATION USED WITH PMTH
PMTH - TABLE OF LOAD INCREMENTS, PRINT CONTROL INCREMENTS AND
UPPER LOAD LEVEL PER STEP (CONTROL INFORMATION)
SGBARN - MATRIX OF EFFECTIVE NODE STRESSES FOR LAST CYCLE THAT
SHOWED AN INCREASE AT A PARTICULAR NODE
SGBARN - MATRIX OF EFFECTIVE NODE STRESSES FOR CURRENT CYCLE
SGBARP - MATRIX OF EFFECTIVE NODE STRESSES FOR PREVIOUS CYCLE
SIGUK - MATRIX OF NODE STRESSES (SIGMA SUB U) FOR K-TH CYCLE
SIJ - MATRIX OF STRESSES FOR MEMBER STRAINS
SIN - MATRIX OF STRESSES FOR APPLIED LOADS
TEFSTN - MATRIX OF TOTAL EFFECTIVE STRAINS
TEPSN - TABLE OF STRAIN VALUES DEFINING THE STRESS-STRAIN CURVE
TOYENS - MATRIX OF TOTAL NODE STRAINS
TSIGN - TABLE OF STRESS LEVELS DEFINING THE STRESS-STRAIN CURVE

*** VARIABLES AND CLUES ***

E - MODULUS OF ELASTICITY
EXP - INPUT EXPONENT FOR RAMBERG-OSGOOD EQUATION
GNU - POISSONS RATIO
INTAPE - THE DATA INPUT TAPE (MAY BE AN AUXILIARY TAPE OR
THE SYSTEM INPUT TAPE - VARIES DURING INPUT PHASE)
K - THE CYCLE COUNTER
KERRSW - CLUE USED FOR TEMPORARY INDICATOR BETWEEN LINKS
- ALSO AS AN ERROR INDICATOR FOR LINK 1 ONLY
KL03 - CLUE INDICATING USE OF RAMBERG-OSGOOD EQUATION
KL04 - CLUE INDICATING TOTAL COUNT OF MATRICES ON AUXILIARY TAPE
KL05 - CLUE INDICATING MATRICES STILL NOT READ FROM AUXIL. TAPE
KL06 - CLUE FOR PRINT CONTROL (WHICH CYCLES)
KL07 - INPUT CLUE FOR PRINT CONTROL (1 TO PRINT ALL CYCLES)
KS07 - CLUE FOR THE LOAD LEVEL CURRENTLY IN USE
NA - 3 TIMES THE NODE COUNT (INPUT)
NC - NUMBER OF NODES

```

C      NAUX  - THE AUXILIARY MATRIX DATA INPUT TAPE
C      NIN   - THE SYSTEM INPUT TAPE
C      NOUT  - THE SYSTEM PRINT OUTPUT TAPE
C      PM    - CURRENT LOAD LEVEL
C      SHMOD - SHEAR MODULUS
C      S7    - INPUT REFERENCE STRESS FOR RAMBERG-OSGOOD EQUATION
C
C      OUTPUT - SUBROUTINE TO CONTROL PRINTING AT VARYING LOAD LEVELS
C
C      ELASTIC UNLOADING (FOLLOWS HOOKES LAW WHEN UNLOADING)
C
      DIMENSION TAlF12(11),TAlF23(11),TAlF31(11),TAlF44(11)
      DIMENSION AL1212(34),AL1223(34),AL1231(34),ALFA44(34)
      DIMENSION PMTM(10,3),KPMTN(10),SIM(102),SIJ(102,102),TSIGN(11)
      DIMENSION TEPSN(11),SIGUK(102),EPSUK(102),SGBARN(34),SGBARP(34)
      DIMENSION SGBARN(34),EPBARN(34),EPBARP(34),DELEPN(34)
      DIMENSION DIM(102),DIJ(102,102),DEFL(102)
      COMMON /COMA/ KLU3,KLU4,KLU5,KLU6,NA,NC,NDE,INTAPE,NIN,NOUT,NAUX
2     , K      , KERRSW , KSET , RUNEST , E      , EXP      , S7
3     , GNU    , SHMOD , PM
4     , PMTM   , KPMTN , SIM   , DIM    , DEFL   , TSIGN   , TEPSN
5     , TAlF12 , TAlF23 , TAlF31 , TAlF44 , AL1212 , AL1223 , AL1231
6     , ALFA44 , SIGUK , EPSUK , SGBARN , SGBARP , SGBARN , EPBARN
7     , EPBARP , DELEPN , SIJ   , DIJ
1    CALL ONEI( IEXIT)
      GO TO 10
2    CALL TWOL( IEXIT)
      GO TO 10
3    CONTINUE
4    CALL FOURL( IEXIT)
      GO TO 10
5    CALL FIVEL( IEXIT)
10   CONTINUE
      WRITE(6,11) IEXIT
      GO TO (1,2,3,4,5,6) , IEXIT
6    CONTINUE
      CALL EXIT
11  FORMAT(10H1 IEXIT = ,I2)
      STOP
      END

```

Note: Subroutine OVRTIM, listed on Page 190, is put in the root segment of the program, after the main program, so it will be accessible to all its callers.

```

SUBROUTINE ONEL(IEXIT)
C GRUMMAN AEROSPACE CORP. DECK NO. 45128 LINK 1
DIMENSION TALF12(11),TALF23(11),TALF31(11),TALF44(11)
DIMENSION AL1212(34),AL1223(34),AL1231(34),ALFA44(34)
DIMENSION PMTM(10,3),KPMTH(10),SIM(102),SIJ(102,102),TSIGN(11)
DIMENSION TEPSN(11),SIGUK(102),EPSUK(102),SGBARN(34),SGBARP(34)
DIMENSION SGBARN(34),EPBARN(34),EPBARP(34),DELEPN(34)
DIMENSION DIM(102),DIJ(102,102),DEFL(102)
DIMENSION BL(6),ID(12),NR(3),ND(3),EL(3),TA(11)
DIMENSION PSTEP(3)
DIMENSION NFA(1),NFB(1)
COMMON /COMA/ KLU3,KLU4,KLU5,KLU6,NA,NC,NDE,INTAPE,NIN,NOUT,NAUX
2 . K , KERRSW , KSET , RUNEST , E , EXP , S7
3 . GNU , SHRMOD , PM
4 . PMTH , KPMTH , SIM , DIM , DEFL , TSIGN , TEPSN
5 . TALF12 , TALF23 , TALF31 , TALF44 , AL1212 , AL1223 , AL1231
6 . ALFA44 , SIGUK , EPSUK , SGBARN , SGBARP , SGBARN , EPBARN
7 . EPBARP , DELEPN , SIJ , DIJ
DATA BL/36HLOAD VOID TAPE FINS ESTN /
DATA ID/ 72H SIM SIJ TSIGN TEPSN DIM DIJ RAMOSG SIM
1SIJ DIM DIJ /
KERRSW = 1
INTAPE = NIN
NFA(1) = 3
NFB(1) = 4

C
C * * SELECT CLUE NFC AS APPROPRIATE WHEN COMPILING * *
C NFC = 0 FOR GISMO CARD FORMATS ON AUXIL. TAPE (SIM,SIJ,DIM,DIJ)
C NFC = 0
C NFC = 40 FOR COMAP FORMAT CARDS WHEN DEBUGGING (SIM,SIJ,DIM,DIJ)
C NFC = 40
C NFC = 42 FOR FORMAT PACKAGE DECIMAL MATRIX CARD FORMATS ON
C AUXILIARY TAPE 9 ONLY (SIM,SIJ,DIM,DIJ) FOR AFFDL
C NFC = 42

C
C KLU3 = 1 WILL INDICATE USE OF THE RAMBERG-OSGOOD EQUATION
C FOR STRAINS, NOT A TABLE INTERPOLATION
KLU3 = 0
KLU5 = 0
KLU6 = 0
PSTEP(1) = 0.0
PSTEP(2) = 0.0
PSTEP(3) = 0.0
RUNEST = 120.
K = C
K000FX = 3
C TIMING ROUTINE DISABLED
C IT = ICHRON(K000FX)
DO 51 J22=1,10
51 KPMTH(J22) = 0
LROW = 0
READ (NIN,1) KLU4,KLU7,NA,NDE,GNU,(TA(I),I=1,11)

WRITE (NOUT,21) (TA(I),I=1,11)
C KLU7 (INPUT) = 0 TO PRINT ON SELECTED CYCLES

```

```

C      KLU7 (INPUT) = 1 TO PRINT ON ALL CYCLES
      IF(KLU7)384,384,382
382 KLU6 = 1
C      KLU6 (OUTPUT) = 1 TO PRINT EACH CYCLE. 0 TO PRINT SELECTED CYCLES
384 CONTINUE
C      KLU4 = 5 TO 9 FOR A CONTINUATION RUN
C      = 1 TO 4 FOR INPUT ON AUXILIARY MATRIX TAPE NAUX (9)
C      DECIMAL INPUT CARDS MUST ALWAYS BE IN INPUT STREAM
C      DECIMAL INPUT TAPE MOUNTS ON LOGICAL UNIT NAUX (9)
      IF(KLU4.LT.1) GO TO 302
      IF(KLU4.GT.4) GO TO 344
      GO TO 343
344 CONTINUE
C      SAVED TAPE FROM PREVIOUS RUN OF INELASTIC PLATE
C      THIS RUN IS A CONTINUATION
      IEXIT = 2
      RETURN
343 CONTINUE
C      SEPARATE MATRIX INPUT TAPE - NOT IN INPUT STREAM
C      THIS RUN IS A NEW ONE
      INTAPE = NAUX
      KLU5 = KLU4
302 CONTINUE
C      NEW RUN - ALL THE INPUT IS DECIMAL IN THE INPUT STREAM
      WRITE (NOUT,9)
52 READ (NIN,2) TEMP1,TEMP2,TEMP3,TEMP4
      LROW = LROW + 1
      DO 53 J22=1,6
      IF(TEMP1.EQ.BL(J22)) GO TO 54
53 CONTINUE
C      RAD CONTROL CARD
      WRITE (NOUT,12) LROW
      WRITE (NOUT,2) TEMP1,TEMP2,TEMP3,TEMP4
      GO TO 998
54 GO TO (55,66,55,60,60,65),J22
55 IF(LROW-10)56,56,53
C      KPMTM( N ) = 1 FOR A LOAD STEP
C      KPMTM( N ) = 2 NOT USED
C      KPMTM( N ) = 3 TO DUMP MEMORY INTO A SAVE TAPE
C      PMTM (N,1) = UPPER LIMIT OF STEP
C      PMTM (N,2) = INTERVAL (INCREMENT) FOR CALCULATION
C      PMTM (N,3) = INTERVAL (INCREMENT) FOR PRINT OUTPUT
56 KPMTM(LROW) = J22
      PMTM(LROW,1) = TEMP2
      TEMPS = TEMP2-PSTEP(J22)
      PMTM(LROW,2) = SIGN(TEMP3,TEMPS)
      PSTEP(J22) = TEMP2
      IF(PMTM(LROW,2))57,63,57
63 GO TO(995,995,57),J22
57 CONTINUE
      PMTM(LROW,3) = TEMP4
      GO TO 52
65 RUNEST = TEMP2 - 1.0
68 LROW = LROW - 1
      GO TO 52
60 CONTINUE

```

```

DO 361 J23 = 1,10
IF(KPMTM(J23))361,361,362
362 IF(KPMTM(J23)-4)363,361,361
363 J24 = KPMTM(J23)
GO TO (364,364,368),J24
364 WRITE (NOUT,22) PMTM(J23,2),PMTM(J23,1)
IF(PMTM(J23,3))361,361,365
365 WRITE (NOUT,23) PMTM(J23,3)
GO TO 361
368 WRITE (NOUT,27)
361 CONTINUE
IF(GNU)86,87,87
86 GNU = .3
87 CONTINUE
IF(NA)99,91,92
91 NA = 102
92 IF(NA-102) 93,93,999
93 CONTINUE
96 IF(NA-3*(NA/3))959,57,999
97 NC = NA/3
DO 303 I21 = 1,11
TSIGN(I21) = 0.0
303 TEPST(I21) = 0.0
IF(KLU4) 308,308,306
306 CONTINUE
INTAPE = NAUX
KLUS = KLUS - 1
C STATEMENT 108 READS INPUT STREAM OR AUXILIARY TAPE NAUX (9)
108 CONTINUE
IF(INTAPE.EQ.NIN) GO TO 1108
IF(NFC-40) 1108,2108,3108
C GISMC FORMAT CARD READING LIST
1108 READ (INTAPE, 3) NAME,NROWS,NCOLS
GO TO 1001
C COMAP FORMAT CARD READING LIST
2108 READ (INTAPE,43) NAME,NROWS,NCOLS
GO TO 1001
C AFFDL FORMAT CARD READING LIST
3108 READ (INTAPE,45) NROWS,NCOLS,NAME
1001 CONTINUE
DO 110 I21 = 1,12
IF(NAME-ID(I21))110,311,110
110 CONTINUE
C BAD INPUT - MATRIX NAME NOT ACCEPTABLE
GO TO 997
311 GO TO (321,322,323,324,325,326,472,111,321,322,325,326),I21
321 DO 321 I1 = 1,NA
331 SIM(I1) = 0.0
WRITE (NOUT,20) NAME,NROWS,NCOLS,INTAPE
IF(NROWS.NE.NA) KERRSW=2
IF(NCOLS.NE.1) GO TO 990
GO TO 111
322 DO 322 I1 = 1,NA
DO 322 I2 = 1,NA
332 SIJ(I1,I2) = 0.0
WRITE (NOUT,20) NAME,NROWS,NCOLS,INTAPE

```

```

        IF(NROWS.EQ.NCOLS) GO TO 433
        WRITE (NOUT,33) NAME,NROWS,NCOLS
        KERRSW = 2
        GO TO 468
433 CONTINUE
468 CONTINUE
        GO TO 111
323 DO 323 I1 = 1,11
333 TSIGN(I1) = 0.0
        GO TO 111
324 DO 324 I1 = 1,11
334 TEP SN(I1) = 0.0
        GO TO 111
325 DO 325 I1 = 1,NRCWS
335 DIM(I1) = 0.0
        WRITE (NOUT,20) NAME,NROWS,NCOLS,INTAPE
        IF(NCOLS.NE.1) GO TO 590
        IF(NROWS.EQ.NDE) GO TO 449
        KERRSW = 2
        WRITE (NOUT,40) NROWS,NAME,NDE
449 CONTINUE
        GO TO 111
326 DO 326 I1 = 1,NRCWS
        DO 326 I2 = 1,NCCLS
336 DIJ(I1,I2) = 0.0
        WRITE (NOUT,20) NAME,NROWS,NCOLS,INTAPE
        IF(NROWS.EQ.NDE) GO TO 461
        KERRSW = 2
        WRITE (NOUT,40) NROWS,NAME,NDE
461 CONTINUE
        GO TO 111
472 S7 = 0.0
        EXP = 0.0
        E = 0.0
        KLU3 = 1
        WRITE (NOUT,37)
        GO TO 111
111 IF(I21.EQ.8) GO TO 150
112 CONTINUE
        IF(INTAPE.EQ.NIN) GO TO 1112
        IF(NFC-40) 1112,2112,3112
C      GISMC FORMAT CARD READING LIST
1112 READ (INTAPE, 4) (NR(I22),ND(I22),EL(I22),I22=1,3)
        GO TO 1002
C      COMAP FORMAT CARD READING LIST
2112 READ (INTAPE,44) (NR(I22),ND(I22),EL(I22),I22=1,3)
        GO TO 1002
C      AFFDL FORMAT CARD READING LIST
3112 READ (INTAPE,46) (NR(I22),ND(I22),EL(I22),I22=1,3)
1002 CONTINUE
        IF(NR(1))996,109,113
109 IF(KLU5)308,308,306
308 INTAPE = NIN
        GO TO 108
113 GO TO (I21,I22,I25,I26,201,202,474,150,I21,I22,201,202),I21
C      READ IN ARRAY SIGMA-IM

```



```

121 MROW = NR(1)
    WRITE (NOUT,4) (NR(I22),ND(I22),EL(I22),I22=1,3)
    SIM (MROW) = EL(1)
    IF(EL(2))127,128,127
127 MROW = NR(2)
    SIM (MROW) = EL(2)
128 IF(EL(3))129,112,129
129 MROW = NR(3)
    SIM (MROW) = EL(3)
    GO TO 112
C    READ IN ARRAY SIGMA-IJ
122 MROW = NR(1)
    MCOL = ND(1)
    SIJ (MROW,MCOL) = EL(1)
    IF(EL(2))130,131,130
130 MROW = NR(2)
    MCOL = ND(2)
    SIJ (MROW,MCOL) = EL(2)
131 IF(EL(3))132,112,132
132 MROW = NR(3)
    MCOL = ND(3)
    SIJ (MROW,MCOL) = EL(3)
    GO TO 112
C    READ IN ARRAY TSIGN (TABLE OF SIGMA BAR N)
125 MROW = NR(1)
    TSIGN(MROW) = EL(1)
    IF(EL(2))139,140,139
139 MROW = NR(2)
    TSIGN(MROW) = EL(2)
140 IF(EL(3))141,112,141
141 MROW = NR(3)
    TSIGN(MROW) = EL(3)
    GO TO 112
C    READ IN ARRAY TEPN (TABLE OF EPSILON BAR N)
126 MROW = NR(1)
    TEPN(MROW) = EL(1)
    IF(EL(2))142,143,142
142 MROW = NR(2)
    TEPN(MROW) = EL(2)
143 IF(EL(3))144,112,144
144 MROW = NR(3)
    TEPN(MROW) = EL(3)
    GO TO 112
C    READ IN ARRAY DIM
201 MROW = NR(1)
    WRITE (NOUT,4) (NR(I22),ND(I22),EL(I22),I22=1,3)
    DIM(MROW) = EL(1)
    IF(EL(2))221,222,221
221 MROW = NR(2)
    DIM(MROW) = EL(2)
222 IF(EL(3))223,112,223
223 MROW = NR(3)
    DIM(MROW) = EL(3)
    GO TO 112
C    READ IN ARRAY DIJ
202 MROW = NR(1)

```

```

        MCOL = ND(1)
        DIJ(MROW,MCOL) = EL(1)
        IF(EL(2))224,225,224
224  MROW = NR(2)
        MCOL = ND(2)
        DIJ(MROW,MCOL) = EL(2)
225  IF(EL(3))226,112,226
226  MROW = NR(3)
        MCOL = ND(3)
        DIJ(MROW,MCOL) = EL(3)
        GO TO 112
C      READ IN CONSTANTS FOR RAMBERG-OSGOOD EQUATION
474  S7 = EL(1)
        EXP = EL(2)
        E = EL(3)
        WRITE (NOUT,38) S7,EXP,E
        GO TO 112
150  CONTINUE
C      END OF LOOP THAT READS DECIMAL MATRICES
        IF(KLU4.GE.1) REWIND NAUX
165  DO 166 I = 1,11
        TALF12(I) = 0.5
        TALF23(I) = 0.5
        TALF31(I) = 0.5
166  TALF44(I) = 1.0
C *** IF(KLU3.EQ.1) TSIGN(2) = START OF INELASTICITY *****
C      TSIGN(2) MUST BE SUPPLIED OR THE PROGRAM ASSUMES PLASTICITY
C      STARTS AT ZERO EFFECTIVE STRESS
        IF(KLU3.EQ.1) GO TO 81
        E = TSIGN(2)/TEPSN(2)
        IE = IFIX(E/100.)
        E = FLOAT(100*IE)
81  SHRMCD = E/(2.0*(1.0+GNU))
        WRITE (NOUT,5) E,SHRMCD,GNU
        WRITE (NOUT,6)
        DO 149 I1=1,11
149  WRITE (NOUT,7) I1,TSIGN(I1),TEPSN(I1)
        K = 0
        PM = 0.0
152  IEXIT = 4
        CALL OVRTIM(K000FX)
        IF(KERRSW.EQ.2) GO TO 993
        RETURN
990  WRITE (NOUT,33) NAME,NROWS,NCOLS
        GO TO 998
993  WRITE (NOUT,34)
        GO TO 998
995  WRITE (NOUT,16) LROW
        GO TO 998
996  CONTINUE
        WRITE (NOUT,14) NAME
        WRITE (NOUT,4) (NR(I22),ND(I22),EL(I22),I22=1,3)
        GO TO 998
997  CONTINUE
        WRITE (NOUT,13) NAME
998  IF(KLU4.GE.1) REWIND NAUX

```

```

CALL EXIT
999 WRITE (NOUT,11) NA
GO TO 998
1 FORMAT(2I1,2I3,F6.3,11A6)
2 FORMAT(6X,A4,E10.6,E10.1,E10.1)
C * GISHC CARD FORMATS FOR TAPE 9
3 FORMAT(11X, A6,1X,2I3)
4 FORMAT(3(1X,2I3,E16.8) )
5 FORMAT(26H1 MODULUS OF ELASTICITY = ,F11.0,4H PSI,6X,16HSHEAR MODU
ILUS = ,F11.0,4H PSI,3X,5HNU = ,F6.3)
6 FORMAT(41H0 TABLE OF VALUES FOR STRESS-STRAIN CURVE //
15X,29HPOINT STRESS LEVEL STRAIN/16X,3HPSI,9X,7HIN./1N.//)
7 FORMAT(6X,I3,F15.8)
9 FORMAT(//79H THIS ISOTROPIC RUN USES ELASTIC UNLOADING (HOOKES L
1AW) WITH STRAIN HARDENING)
11 FORMAT(10H ERROR NA=I4)
12 FORMAT(26H ERROR- INCREMENT CARD NO.,I3,5H N.G.)
13 FORMAT(14H ERROR-MATRIX ,A6)
14 FORMAT(17H ERROR-NEG. INDEX A6)
16 FORMAT(33H ERROR-NO INTERVAL-INCREMENT CARD,I3)
20 FORMAT(//5X,7HMATRIX ,A6,1X,I3,8H ROWS X ,I3,19H COLUMNS FROM TAPE
1 ,I2)
21 FORMAT(1H1,29X,11A6)
22 FORMAT(5X,16HLOAD INCREMENTS ,F9.2,11H POUNDS TO ,F10.2,7H POUNDS)
23 FORMAT(1H+,61X,15HPRINT OUTPUT EVERY ,F9.2,7H POUNDS)
27 FORMAT(5X,35HSTORE MEMORY ON TAPE 11, THEN EXIT)
31 FORMAT(10X,I2)
33 FORMAT(14H INPUT MATRIX ,A6,2I4,46H CN TAPE IS NEITHER SQUARE NOR
1A COLUMN VECTOR)
34 FORMAT(74H INPUT ERROR - INDEX IN INPUT MATRIX DOES NOT MATCH CL
1UE IN CONTROL CARD)
37 FORMAT(//5X,24HRAMBERG-OSGOOD CONSTANTS)
38 FORMAT(5X,22HINPUT REFERENCE STRESS,E16.7/5X,14HINPUT EXPONENT,
1F10.5/5X,15HYOUNG'S MODULUS,F11.0)
40 FORMAT(//5X,I3,16H ROWS IN MATRIX ,A6,15H DOES NOT MATCH,I4,
119H ROWS IN INPUT CLUE/5X,21HRUN WILL NOT CONTINUE//)
C * COMAP CARD FORMATS FOR TAPE 9
43 FORMAT(11X,A6,4X,I3,2X,I3)
44 FORMAT(3(2I4,E15.7,1X))
C * SPECIAL FORMATS FOR AFFDL FORMAT INPUT ON AUXILIARY TAPE 9 * *
45 FORMAT(1X,2I4,57X,A6)
46 FORMAT(1X,3(2I4,E13.5,1X))
END

```

```

SUBROUTINE TWOL( IEXIT)
C GRUMMAN AEROSPACE CORP. DECK NO. 45128 LINK 2
C MATRIX ANALYSIS OF INELASTIC PLATE LINK 2
C THIS LINK READS IN A SAVED BINARY TAPE - FIRST PART
  DIMENSION T Alf12(11),T Alf23(11),T Alf31(11),T Alf44(11)
  DIMENSION AL1212(34),AL1223(34),AL1231(34),ALFA44(34)
  DIMENSION PMTM(10,3),KPMTH(10),SIM(102),SIJ(102,102),TSIGN(11)
  DIMENSION TEP SN(11),SIGUK(102),EPSUK(102),SGBARN(34),SGBARP(34)
  DIMENSION SGBARN(34),EPBARN(34),EPBARP(34),DELEPN(34)
  DIMENSION DIM(102),DIJ(102,102),DEFL(102)
  COMMON /COMA/ KLU3,KLU4,KLU5,KLU6,NA,NC,NDE,INTAPE,NIN,NOUT,NAUX
2 , K , KERRSW , KSET , RUNEST , E , EXP , S7
3 , GNU , SHRMOD , PM
4 , PMTM , KPMTH , SIM , DIM , DEFL , TSIGN , TEP SN
5 , T Alf12 , T Alf23 , T Alf31 , T Alf44 , AL1212 , AL1223 , AL1231
6 , ALFA44 , SIGUK , EPSUK , SGBARN , SGBARP , SGBARN , EPBARN
7 , EPBARP , DELEPN , SIJ , DIJ
  DIMENSION BL(6)
  DIMENSION PSTEP(3)
  DATA BL/36,LOAD VOID TAPE FINS ESTM /
  INTAPE = NAUX - 1
  READ (INTAPE) KLU3,KLU4,KLU5, NA,NC,NDE, NIN,NOUT
2 , K , KERRSW , KSET , RUNEST , E , EXP , S7
3 , GNU , SHRMOD , PM
C KLU6 = 1 WILL PRINT EVERY CYCLE, 0 WILL PRINT ONLY SELECTED CYCLES

```

```

  WRITE (NOUT,9)
  NCLU = 1
  KERRSW = 2
  DO 51 I42 = 1,10
51 KPMTH(I42) = 0
  PSTEP(1) = 0.0
  PSTEP(2) = 0.0
  PSTEP(3) = 0.0
  LROW = 0
52 READ (NIN,2) TEMP1,TEMP2,TEMP3,TEMP4
  LROW = LROW + 1
  DO 53 J22=1,6
  IF(TEMP1.EQ.BL(J22)) GO TO 54
53 CONTINUE
C BAD CONTROL CARD
  WRITE (NOUT,12) LROW
  WRITE (NOUT,2) TEMP1,TEMP2,TEMP3,TEMP4
  CALL EXIT
54 GO TO (55,66,55,60,60,65),J22
55 IF(LROW-10)56,56,53
C KPMTH( N ) = 1 FOR A LOAD STEP
C KPMTH( N ) = 2 NOT USED
C KPMTH( N ) = 3 TO DUMP MEMORY INTO A SAVE TAPE
C PMTM (N,1) = UPPER LIMIT OF STEP
C PMTM (N,2) = INTERVAL (INCREMENT) FOR CALCULATION
C PMTM (N,3) = INTERVAL (INCREMENT) FOR PRINT OUTPUT
56 KPMTH(LROW) = J22
  NCLU = 2

```

```

KSET = 1
PMTM(LROW,1)= TEMP2
TEMP5      = TEMP2-PSTEP(J22)
PMTM(LROW,2)= SIGN(TEMP3,TEMP5)
PSTEP(J22) = TEMP2
IF(PMTM(LROW,2))57,63,57
63 GO TO (995,995,57),J22
57 CONTINUE
PMTM(LROW,3)= TEMP4
GO TO 52
65 RUNEST = TEMP2 - 1.0
66 LROW   = LROW - 1
GO TO 52
60 CONTINUE
GO TO (61,62),NCLU
62 CONTINUE
READ (INTAPE) JUNK
READ (INTAPE) JUNK
GO TO 64
61 CONTINUE
READ (INTAPE) PMTM
READ (INTAPE) KPMTM
64 CONTINUE
WRITE (NOUT,1)
ON 361 J23 = 1,10
IF(KPMTM(J23))361,361,362
362 IF(KPMTM(J23)-4)363,361,361
363 J24 = KPMTM(J23)
GO TO (364,364,368),J24
364 WRITE (NOUT,22) PMTM(J23,2),PMTM(J23,1)
IF(PMTM(J23,3))361,361,365
365 WRITE (NOUT,23) PMTM(J23,3)
GO TO 361
368 WRITE (NOUT,27)
361 CONTINUE
READ (INTAPE) SIM
READ (INTAPE) SLL
READ (INTAPE) DIM
READ (INTAPE) DIJ
READ (INTAPE) DEFL
READ (INTAPE) TSIGN
READ (INTAPE) TEP SN
READ (INTAPE) TALF12
READ (INTAPE) TALF23
READ (INTAPE) TALF31
READ (INTAPE) TALF44
READ (INTAPE) AL1212
READ (INTAPE) AL1223
READ (INTAPE) AL1231
READ (INTAPE) ALFA64
READ (INTAPE) SIGUK
READ (INTAPE) EPSUK
C GRUMMAN AEROSPACE CORP. DECK NO. 45128 LINK 3
C MATRIX ANALYSIS OF INELASTIC PLATE LINK 3
C THIS IS THE SECOND HALF OF OLD LINK 2
C THIS LINK READS IN A SAVED BINARY TAPE - SECOND PART

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      READ (INTAPE) SGBARN
      READ (INTAPE) SGBARP
      READ (INTAPE) SGBARM
      READ (INTAPE) EPBARN
      READ (INTAPE) EPBARP
      READ (INTAPE) DELEPH
      REWIND INTAPE
      WRITE (NOUT,5) E,SHRMOD,GNU
      IF(KLU3.NE.1) GO TO 140
      WRITE (NOUT,37)
      WRITE (NOUT,38) S7,EXP,E
140  CONTINUE
      WRITE (NOUT,6)
      DO 149 I=1,11
149  WRITE (NOUT,7) I1,TSIGN(I1),TEPSN(I1)
      IEXIT = 4
      CALL OVRTIM(K000FX)
      RETURN
995  WRITE (NOUT,16) LROW
      CALL EXIT
      STOP
1  FORMAT(1H0,29X,37HCONTINUATION RUN - (INELASTIC ANALYSIS)
2  FORMAT(6X,A4,E10.6,E10.1,E10.1)
5  FORMAT(26H0 MODULUS OF ELASTICITY = ,F11.0,4H PSI,6X,16HSHEAR MODU
      ILUS = ,F11.0,4H PSI,6X,5HNU = ,F6.3)
6  FORMAT(41H0 TABLE OF VALUES FOR STRESS-STRAIN CURVE //
      15X,29HP0INT STRESS LEVEL STRAIN/16X,3HPSI,9X,7HIN./IN.//)
7  FORMAT(6X,13,F13.2,F15.8)
9  FORMAT(//79H THIS ISOTROPIC RUN USES ELASTIC UNLOADING (HOOKES L
      LAW) WITH STRAIN HARDENING)
12  FORMAT(26H ERROR- INCREMENT CARD NO.,13,5H N.G.)
16  FORMAT(33H ERROR-NO INTERVAL-INCREMENT CARD,13)
22  FORMAT(5X,16HLOAD INCREMENTS ,F9.2,11H POUNDS TO ,F10.2,7H POUNDS)
23  FORMAT(1H+,61X,19HPRINT OUTPUT EVERY ,F9.2,7H POUNDS)
27  FORMAT(5X,35HSTORE MEMORY ON TAPE A-6. THEN EXIT)
37  FORMAT(//5X,24HAMBERG-OSGOOD CONSTANTS)
38  FORMAT(5X,22HINPUT REFERENCE STRESS,E16.7/5X,14HINPUT EXPONENT,
      IF10.5/5X,15HYOUNG'S MODULUS,F11.0)
      END

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SUBROUTINE FIVEI( EXIT )
C GRUMMAN AEROSPACE CORP. DECK NO. 45128 LINK 5
C MATRIX ANALYSIS OF INELASTIC PLATE LINK 5
C THIS LINK WRITES A SAVE TAPE FOR RESTART
DIMENSION T Alf12(11),T Alf23(11),T Alf31(11),T Alf44(11)
DIMENSION AL1212(34),AL1223(34),AL1231(34),ALFA44(34)
DIMENSION PMTN(10,3),KPMTM(10),SIM(102),SIJ(102,102),TSIGN(11)
DIMENSION TEP SN(11),SIGUK(102),EPSUK(102),SGBARN(34),SGBARP(34)
DIMENSION SGBARN(34),EPBARN(34),EPBARP(34),DELEPN(34)
DIMENSION DIM(102),DIJ(102,102),DEFL(102)
COMMON /COMA/ KLU3,KLU4,KLU5,KLU6,NA,NC,NDE,INTAPE,NIN,NOUT,NAUX
2 , K , KERRSW , KSET , RUNEST , E , EXP , S7
3 , GNU , SHRMOD , PM
4 , PMTN , KPMTM , SIM , DIM , DEFL , TSIGN , TEP SN
5 , T Alf12 , T Alf23 , T Alf31 , T Alf44 , AL1212 , AL1223 , AL1231
6 , ALFA44 , SIGUK , EPSUK , SGBARN , SGBARP , SGBARN , EPBARN
7 , EPBARP , DELEPN , SIJ , DIJ
NSTAPE = 11
WRITE (NSTAPE) KLU3,KLU4,KLU5, NA,NC,NDE, NIN,NOUT
2 , K , KERRSW , KSET , RUNEST , E , EXP , S7
3 , GNU , SHRMOD , PM
WRITE (NSTAPE) PMTN
WRITE (NSTAPE) KPMTM
WRITE (NSTAPE) SIM
WRITE (NSTAPE) SIJ
WRITE (NSTAPE) DIM
WRITE (NSTAPE) DIJ
WRITE (NSTAPE) DEFL
WRITE (NSTAPE) TSIGN
WRITE (NSTAPE) TEP SN
WRITE (NSTAPE) T Alf12
WRITE (NSTAPE) T Alf23
WRITE (NSTAPE) T Alf31
WRITE (NSTAPE) T Alf44
WRITE (NSTAPE) AL1212
WRITE (NSTAPE) AL1223
WRITE (NSTAPE) AL1231
WRITE (NSTAPE) ALFA44
WRITE (NSTAPE) SIGUK
WRITE (NSTAPE) EPSUK
WRITE (NSTAPE) SGBARN
WRITE (NSTAPE) SGBARP
WRITE (NSTAPE) SGBARN
WRITE (NSTAPE) EPBARN
WRITE (NSTAPE) EPBARP
WRITE (NSTAPE) DELEPN
WRITE (NOUT,25) NSTAPE
PRINT 25,NSTAPE
END FILE NSTAPE
REWIND NSTAPE
WRITE (NOUT,25) NSTAPE
CALL OVRTIM(K000FX)
CALL EXIT
RETURN
25 FORMAT(19H SAVE LOGICAL TAPE ,I3, 10H FOR RERUN//)

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SUBROUTINE FOURL( IEXIT)
C   GRUMMAN AEROSPACE CORP.   DECK NO. 45128   LINK 4
C   MATRIX ANALYSIS OF INELASTIC PLATE   LINK 4
C   THIS LINK DOES THE CALCULATION AND WRITES PRINT OUTPUT
  DIMENSION T Alf12(11),T Alf23(11),T Alf31(11),T Alf44(11)
  DIMENSION AL1212(34),AL1223(34),AL1231(34),ALFA44(34)
  DIMENSION PMTN(10,3),KPMTN(10),SIM(102),SIJ(102,102),TSIGN(11)
  DIMENSION TEP SN(11),SIGUK(102),EPSUK(102),SGBARN(34),SGBARP(34)
  DIMENSION SGBARN(34),EPBARN(34),EPBARP(34),DELEPN(34)
  DIMENSION DIN(102),DIJ(102,102),DEFL(102)
  DIMENSION TEFSTN(34),TOTEPS(102),DELEPK(102)
  COMMON /COMA/ KLU3,KLU4,KLU5,KLU6,NA,NC,NDE,INTAPE,NIN,NOUT,NAUX
  2 . K , KERRSW , KSET , RUNEST , E , EXP , S7
  3 . GNU , SHRMOD , PM
  4 . PMTN , KPMTN , SIM , DIN , DEFL , SIJ , TEP SN
  5 . T Alf12 , T Alf23 , T Alf31 , T Alf44 , AL1212 , AL1223 , AL1231
  6 . ALFA44 , SIGUK , EPSUK , SGBARN , SGBARP , EPBARN
  7 . EPBARP , DELEPN , SIJ , DIJ
  GO TO (418,419),KERRSW
C   INITIALIZE WORK AREAS
  418 DO 102 I1 = 1,NA
    EPSUK(I1) = 0.0
  102 SIGUK(I1) = 0.0
    DO 103 I1 = 1,NC
      DELEPN(I1) = 0.0
      SGBARN(I1) = 0.0
      SGBARP(I1) = 0.0
      EPBARN(I1) = 0.0
      EPBARP(I1) = 0.0
      AL1212(I1) = 2.0*T Alf12(2)
      AL1223(I1) = T Alf12(2) + T Alf23(2)
      AL1231(I1) = T Alf12(2) + T Alf31(2)
      ALFA44(I1) = T Alf44(2)
    103 EPBARN(I1) = 0.0
      DO 105 I1 = 1,NDE
        105 DEFL(I1) = 0.0
      KSET = 1
  419 CONTINUE
    KREF = K
    IF(KLU3.EQ.0) GO TO 151
    AA = 3.*((1./S7)**(EXP-1.))/7.
  151 K = K + 1
    KLU2 = 1
C   IF KLU2 = 0, THE CYCLE OF OPERATIONS WILL BE PRINTED
C   KLU6 = 1 WILL PRINT EVERY CYCLE, 0 WILL PRINT ONLY SELECTED CYCLES
    IF(KLU6)248,248,249
  249 KLU2 = 0
  249 CONTINUE
    IF(K-1)270,270,271
  270 KLU2 = 0
  271 CONTINUE
    GO TO (416,417),KERRSW
  417 KLU2 = 0
    KERRSW = 1
  416 CONTINUE

```



```

C      KSET IS THE ROW OF KPMTM OR PMTM BEING USED (CURRENT LOAD LEVEL)
300 IF(KSET-1)301,319,319
301 IF(KPMTM(KSET))302,302,303
302 KSET = KSET + 1
    GO TO 300
303 IF(KPMTM(KSET)-4)304,302,302
C      VARIABLE (J24) INDICATES A LOAD CYCLE (1) OR WRITE MEMORY ON
C      A SAVE TAPE (3). (2) IS NOT USED,
304 J24 = KPMTM(KSET)
    IF(PMTM(KSET,2))342,342,341
341 GO TO (305,305,423),J24
342 GO TO (345,345,423),J24
305 IF(PMTM(KSET,1)-(PM+PMTM(KSET,2))) 316,309,308
345 IF(PMTM(KSET,1)-(PM+PMTM(KSET,2))) 308,309,316
309 KLU2 = 0
308 DELPM = PMTM(KSET,2)
    GO TO 154
316 KLU2 = 0
    GO TO 302
319 IF(KLU2)205,205,320
320 KLU2 = 0
    KSET = KSET - 1
    IF(KSET)205,205,321
321 IF(KPMTM(KSET)) 320,320,308
154 CONTINUE
335 CONTINUE
502 PM = PM + DELPM
324 CALL OUTPUT(PM,PMTM(KSET,3),KLU1)
326 IF(KLU2)327,328,327
327 KLU2 = KLU1
328 CONTINUE
    IF(KLU2)250,250,251
250 CONTINUE
C      LABEL = APPLIED LOAD
    WRITE (NOUT,22) PM
251 CONTINUE
C      SAVE PRECEDING CYCLE VALUES OF EPBARN
    DO 152 I1 = 1,NC
152 EPBARN(I1) = EPBARN(I1)
C      CALCULATE NODE STRESSES - MATRIX SIGUK - FRAME SIZE 105 X 1
C      COMPUTATION OF STRESSES AND DEFLECTIONS IN THE INELASTIC RANGE
C      REDUCE SIJ*EPSUK MULTIPLICATION BY SELECTING THE COLUMNS OF SIJ
C      CORRESPONDING TO THE NON-ZERO ELEMENTS OF EPSUK
    DO 861 I5=1,NA
861 SIGUK(I5)= SIJ(I5)*PM
    DO 631 I5=1,NDE
631 DEFL(I5) = DIM(I5)*PM
    DO 641 I4=1,NA
    IF (EPSUK(I4).EQ.0.0) GO TO 641
    DO 643 I2=1,NA
643 SIGUK(I2)= SIGUK(I2) + SIJ(I2,I4)*EPSUK(I4)
    DO 644 I3=1,NDE
644 DEFL(I3)=DEFL(I3) + DIJ(I3,I4)*EPSUK(I4)
641 CONTINUE
    IF(KLU2)254,254,255
254 CONTINUE

```

```

C      LABEL DEFLECTIONS
      WRITE (NOUT,23)
      KNDE = NDE/2
      DO 665 I1 = 1,KNDE
      KNDE2 = 2*I1
      KNDE1 = KNDE2-1
      WRITE (NOUT,24) (I1,DEFL(KNDE1),DEFL(KNDE2))
665  CONTINUE
255  CONTINUE
C      CALCULATE MAGNITUDE AND SIGN OF EFFECTIVE STRESS AT EACH NODE
C      CALCULATE EFFECTIVE STRESSES - MATRIX SGBARN - SAME SIZE 35 X 1
      DO 166 I7 = 1,NC
      SGBARP(I7) = SGBARN(I7)
      M3 = 3*I7
      M32 = M3-2
      M31 = M3-1
      SGBARN(I7)=SQRT(AL123(I7)*SIGUK(M32)**2-AL1212(I7)*SIGUK(M32)*SI
      GUK(M31)+AL1223(I7)*SIGUK(M31)**2+3.0*ALFA44(I7)*SIGUK(M3)**2)
166  CONTINUE
C      LABEL = EFF. STRESSES
C      CALCULATE EFFECTIVE INELASTIC STRAIN FOR EACH NODE - INTERPOLATE
C      IN TABLE (TSIGN VS. TEPN)
      DO 181 I8 = 1,NC
      C      SGBARP IS EFFECTIVE STRESS OF PREVIOUS CYCLE
      IF(SGBARN(I8)-SGBARP(I8))411,401,401
      C      EFFECTIVE STRESS IS ABOVE PREVIOUS LEVEL
      C      SGBARN IS EFFECTIVE STRESS OF LAST CYCLE TO SHOW AN INCREASE
401  IF(SGBARN(I8)-SGBARN(I8))411,402,402
      C      EFFECTIVE STRESS IS ABOVE KNEE OF PREVIOUS DROP-OFF. IF ANY
402  CONTINUE
403  SGBARN(I8) = SGBARN(I8)
      ESUPRK =(SGBARN(I8)/E) + EPSARP(I8)
      IF(KL03.EQ.1) GO TO 670
      DO 171 I9 = 1,11
      IF(ESUPRK-TEPSN(I9))173,172,171
171  CONTINUE
      GO TO 998
172  BARSGN = TSIGN(I9)
      IF(SGBARN(I8)-TSIGN(2))178,177,177
177  AL1212(I8)= 2.0*TALF12(I9)
      AL1223(I8)= TALF12(I9)+ TALF23(I9)
      AL1231(I8)= TALF12(I9)+ TALF31(I9)
      ALFA44(I8)= TALF44(I9)
178  CONTINUE
      GO TO 174
173  KKK2 = I9
      KKK1 = I9 - 1
      STNRAT = (ESUPRK-TEPSN(KKK1))/(TEPSN(KKK2)-TEPSN(KKK1))
      BARSGN = TSIGN(KKK1)+(TSIGN(KKK2)-TSIGN(KKK1))*STNRAT
      IF(SGBARN(I8)-TSIGN(2))176,175,175
175  CONTINUE
      ALFA12 = TALF12(KKK1)+(TALF12(KKK2)-TALF12(KKK1))*STNRAT
      ALFA23 = TALF23(KKK1)+(TALF23(KKK2)-TALF23(KKK1))*STNRAT
      ALFA31 = TALF31(KKK1)+(TALF31(KKK2)-TALF31(KKK1))*STNRAT
      ALFA44(I8)=TALF44(KKK1)+(TALF44(KKK2)-TALF44(KKK1))*STNRAT
      AL1212(I8)= 2.0*ALFA12

```

```

      AL1223(18)= ALFA12 + ALFA23
      AL1231(18)= ALFA12 + ALFA31
176  CONTINUE
      GO TO 174
670  CONTINUE
      BSGN1 = SGBARN(18)
      DO 674 I1=1,10
      F1BSN = AA*BSGN1**EXP+BSGN1-E*ESUPRK
      F2BSN = AA*EXP*BSGN1**((EXP-1.)*1.
      BARSGN = BSGN1 - F1BSN/F2BSN
      TEST = BARSGN/BSGN1
      BSGN1 = BARSGN
      IF(TEST.LT..99999) GO TO 674
      IF(TEST.GT.1.00001) GO TO 674
      GO TO 676
674  CONTINUE
676  CONTINUE
174  EPBARN(18) = 0.0
      IF(SGBARN(18).GT.TSIGN(2)) EPBARN(18)=ESUPRK-BARSGN/E
C      CALCULATE TOTAL EFFECTIVE STRAIN - MATRIX TEFSTN - FRAME SIZE 35X1
      TEFSTN(18) = ESUPRK
C      CALCULATE EFFECTIVE STRAIN CHANGES - MATRIX DELEPN - FRAME 35 X 1
C      CALCULATE INCREMENTAL EFFECTIVE INELASTIC STRAIN
      DELEPN(18) = EPBARN(18) - EPBARN(18)
      GO TO 181
C      DROP-OFF OF EFFECTIVE STRESS
C      OR STILL BELOW THE KNEE OF PREVIOUS DROP-OFF
411  EPBARN(18) = EPBARN(18)
      TEFSTN(18) = EPBARN(18)+(SGBARN(18)/E)
      DELEPN(18) = 0.0
181  CONTINUE
C
C
      IF(KL03.EQ.0) GO TO 689
C      USE OF THE RAMBERG-OSGOOD EQUATION IS ONLY VALID FOR
C      ISOTROPIC ANALYSIS
      DO 679 I8 = 1,NC
      AL1212(18) = 1.0
      AL1223(18) = 1.0
      AL1231(18) = 1.0
      ALFA44(18) = 1.0
679  CONTINUE
680  CONTINUE
C
C      LABEL = EFF. PLASTIC STRAIN
C      LABEL = TOTAL EFF. STRAIN

C      CALCULATE NODE STRAIN CHANGE - MATRIX DELEPK - FRAME SIZE 105 X 1
      DO 191 I11 = 1,NC
      TEMPA = DELEPN(I11)/ SGBARN(I11)
      M3 = 3*I11
      M32 = M3-2
      M31 = M3-1
      DELEPK(M32)=TEMPA*(AL1231(I11)*SIGUK(M32)-.5*AL1212(I11)*SIGUK(M31
1))

```

```

      DELEPK(M31)=TEMPA*(AL1223(I11)*SIGUK(M31)-.5*AL1212(I11)*SIGUK(M32
1))
      DELEPK(M3) =TEMPA*(3.0*ALFA04(I11)*SIGUK(M3))
191 CONTINUE
C   CALCULATE NODE PLASTIC STRAIN - MATRIX EPSUK - FRAME SIZE 105 X 1
C   CALCULATE NODE POINT STRAINS
      DO 192 I23=1,NA
      EPSUK(I23) = EPSUK(I23) + DELEPK(I23)
192 CONTINUE
C   LABSL = EFF. STRAIN CHANGES
C   LABEL = NODE STRAIN CHANGE
C   LABEL = NODE INELAS. STRAIN
C   CALCULATE TOTAL NODE STRAINS - MATRIX TOTEPS - FRAME SIZE 105 X 1
      DO 201 I14=1,NC
      M32 = 3*I14-2
      M31 = 3*I14-1
      M3 = 3*I14
      TOTEPS(M32)=EPSUK (M32)+SIGUK(M32)/E -GNU*SIGUK(M31)/E
      TOTEPS(M31)=EPSUK (M31)+SIGUK(M31)/E -GNU*SIGUK(M32)/E
201 TOTEPS(M3) =EPSUK (M3) +SIGUK(M3) /SHRMOO
      IF(KLU2)262,262,263
262 CONTINUE
C   LABSL = TOT. NODE STRAINS
      WRITE (NOUT,41)
      WRITE (NOUT,42)
      DO 258 I8 = 1,NC
      J3 = 3*I8
      J2 = J3-1
      J1 = J3-2
      WRITE (NOUT,46) I8,SIGUK(J1),SIGUK(J2),SIGUK(J3),TOTEPS(J1),
1 TOTEPS(J2),TOTEPS(J3)
258 CONTINUE
      WRITE (NOUT,43)
      WRITE (NOUT,44)
      DO 259 I8 = 1,NC
      J3 = 3*I8
      J2 = J3-1
      J1 = J3-2
      WRITE (NOUT,46) I8,EPSUK(J1),EPSUK(J2),EPSUK(J3),DELEPK(J1),
1 DELEPK(J2),DELEPK(J3)
259 CONTINUE
      WRITE (NOUT,45)
      DO 260 I8 = 1,NC
      WRITE (NOUT,46) I8,SGBARN(I8),TEESTN(I8),DELEPN(I8),EPBARN(I8)
260 CONTINUE
263 CONTINUE
      IF(KLU2)268,268,269
268 CONTINUE
      WRITE (NOUT,31) K,PM
269 CONTINUE
      IF(K.LE.(KREF+20)) GO TO 422
      KREF = K
      CALL OVRTIM(K000FX)
C   K000FX = 1 TO FORCE AN EXIT
      GO TO(421,422),K000FX
422 CONTINUE

```

```

      GO TO 151
C     STATEMENT 421 IS REACHED IF THE RUNNING TIME IS ONE MINUTE
C     BELOW THE TIME ESTIMATE
421 IF(KLU2)423,423,208
423 CONTINUE
205 CONTINUE
      CALL DVRTIN(K000FX)
      GO TO(207,209),K000FX
209 IF(J24-3)210,207,210
207 CONTINUE
      IEXIT = 5
      RETURN
210 CONTINUE
      IEXIT = 6
      RETURN
208 KERRSW = 2
C     KERRSW SET TO 2 TO MAKE KLU. = 0 AND PRINT A CYCLE
      GO TO 151
998 CONTINUE
      WRITE (NOUT,12) ESUPRX,SGBARN(18)
      WRITE (NOUT,13) K,18,PM
      GO TO 205
12 FORMAT(46H VALUE NOT FOUND IN TABLE FOR EPSILON BAR N = ,E15.8,
119H ( SIGMA BAR N = ,E15.8,2K ) )
13 FORMAT(16H CYCLE NUMBER = ,15.20H   ELEMENT INDEX = ,14.17H   LO
1AD LEVEL = ,F9.2)
21 FORMAT(// 1X,4A4,A2.5(1PE16.7)/(19X,5E16.7))
22 FORMAT(//21H APPLIED LOAD           ,F12.2)
23 FORMAT(//9X,30HDEFLECTIONS AT GEOMEYRIC NODES//14X,4HNODE,6X
1,7HDELTA X,9X,7HDELTA Y)
24 FORMAT(15X,14.2E16.7)
31 FORMAT(2H0 ,16.27H CYCLES COMPLETED -- LOAD = ,F9.2)
41 FORMAT(//22X,13HNODE STRESSES,33X,18HTOTAL NODE STRAINS)
42 FORMAT(98H NODE          SIGMA X          SIGMA Y          TAU XY
1 EPSILON X          EPSILON Y          GAMMA XY)
43 FORMAT(//18X,22HNODE INELASTIC STRAIN ,29X,18HNODE STRAIN CHANGE)
44 FORMAT(98H NODE          EPSILON X          EPSILON Y          GAMMA XY
1 EPSILON X          EPSILON Y          GAMMA XY)
45 FORMAT(//27X,39HTOTAL          EFFECTIVE          EFFECTIVE/9X,56HEFFEC
1TIVE          EFFECTIVE          STRAIN          PLASTIC/64H NODE          S
2TRESSES          STRAIN          CHANGES          STRAIN)
46 FORMAT(1X,13,1X,6(1PE16.7))
      END

```

```

SUBROUTINE OVRTIM(KOOCOFX)
C SUBROUTINE TO TEST THE SYSTEM CLOCK AND FORCE GENERATION OF A
C RESTARTABLE SAVE TAPE WHEN THE TIME ESTIMATE IS REACHED
COMMON /COMA/ KLU3,KLU4,KLU5,KLU6,NA,NC,NDE,INTAPE,NIN,NOUT,NAUX
2 , K , KERRSW , KSET , RUNEST , E , EXP , S7
3 , GNU , SHRMOD , PM
C KOOCOFX = 1 TO TERMINATE THE RUN
C = 2 TO LET THE RUN CONTINUE
LIM = RUNEST*600.
N = 0
C TIMING ROUTINE DISABLED PENDING INFORMATION ON WPAFB CLOCK ROUTINE
KOOCOFX = 2
IT = 0
IF(IT.EQ.0) GO TO 102
C END OF PATCH
C IT = ICHRON(N)
KOOCOFX = 2
IF(IT.GY.90000) GO TO 102
IF(IT.LT.LIM) GO TO 100
KOOCOFX = 1
100 TI = IT
TIME = TI/600.
WRITE (NOUT,1) TIME,K
102 RETURN
1 FORMAT(20H ELAPSED TIME READS ,F10.4,10H MINUTES ON CYCLE ,15)
END

```

NOTE: Subroutine OVRTIM is a portion of
the root segment of the program.

```

SUBROUTINE OUTPUT(VALUE1,STEP1,KLU1)
C THIS SUBROUTINE SETS KLU1 = 0 IF THE CURRENT CYCLE IS TO BE PRINTED
VALUE = ABS(VALUE1)
STEP = ABS(STEP1)
102 IF(VALUE-STEP)131,100,100
100 NTEST1 = (VALUE/STEP)*1.00001
NTEST2 = (VALUE/STEP)*.999
IF(NTEST1-NTEST2)131,130,131
130 KLU1 = 0
GO TO 135
131 KLU1 = 1
135 RETURN
END

```

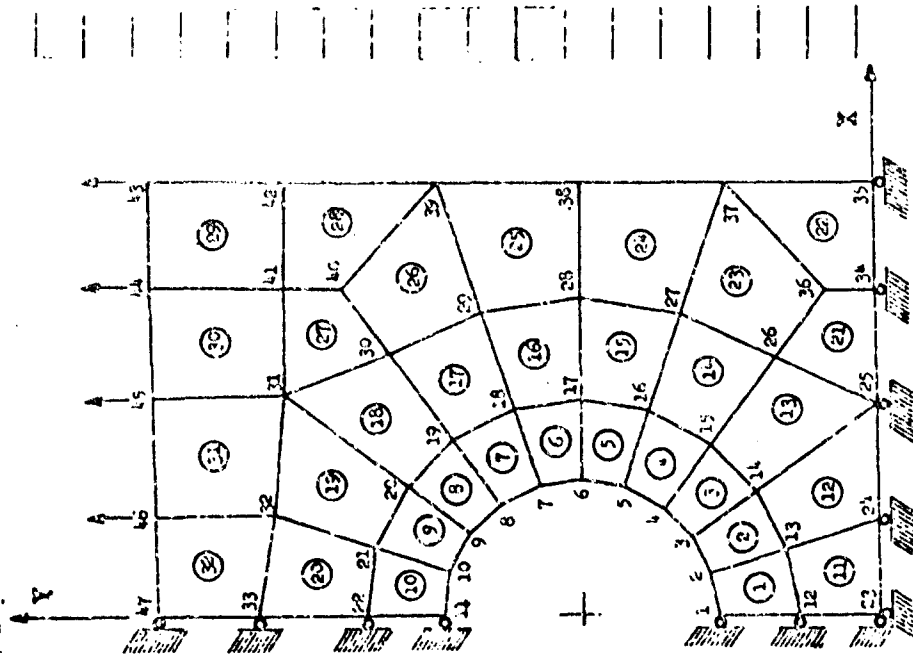
APPENDIX D

SAMPLE OUTPUT OF INELASTIC STRUCTURAL ANALYSIS PROGRAM

TEST PROBLEM FOR IBM 7394 VERSION - 34 MADE CHECK RUN FEB. 3, 1970

THIS ISOTROPIC RUN USES ELASTIC UNLOADING (MOCKES LAW) WITH STRAIN HARDENING
 LOAD INCREMENTS 500.00 POUNDS TO 5000.00 POUNDS PRINT OUTPUT EVERY 2500.00 POUNDS
 LOAD INCREMENTS 100.00 POUNDS TO 1000.00 POUNDS PRINT OUTPUT EVERY 1000.00 POUNDS
 LOAD INCREMENTS 100.00 POUNDS TO 10000.00 POUNDS PRINT OUTPUT EVERY 4000.00 POUNDS
 LOAD INCREMENTS 100.00 POUNDS TO 17000.00 POUNDS PRINT OUTPUT EVERY 2000.00 POUNDS
 LOAD INCREMENTS 50.00 POUNDS TO 18000.00 POUNDS PRINT OUTPUT EVERY 200.00 POUNDS

MATRIX DIM.	94. ROWS	7	1	COLUMNS	ENDX	TYPE	S
2	1	0.1561422E-07	-0	-0	-0	-0	-0
3	1	0.1321454E-07	-0	-0	-0	-0	-0
4	1	0.5494214E-08	-0	-0	-0	-0	-0
5	1	0.6246124E-07	-0	-0	-0	-0	-0
6	1	0.5141152E-07	-0	-0	-0	-0	-0
7	1	0.1102462E-06	-0	-0	-0	-0	-0
8	1	0.1212514E-06	-0	-0	-0	-0	-0
9	1	0.1246515E-06	-0	-0	-0	-0	-0
10	1	0.2495373E-06	-0	-0	-0	-0	-0
11	1	0.1759863E-06	-0	-0	-0	-0	-0
12	1	0.3525677E-06	-0	-0	-0	-0	-0
13	1	0.1805104E-06	-0	-0	-0	-0	-0
14	1	0.5480468E-06	-0	-0	-0	-0	-0
15	1	0.1531211E-06	-0	-0	-0	-0	-0
16	1	0.6990057E-06	-0	-0	-0	-0	-0
17	1	0.1111554E-06	-0	-0	-0	-0	-0
18	1	0.5281373E-06	-0	-0	-0	-0	-0
19	1	0.5732159E-07	-0	-0	-0	-0	-0
20	1	0.5122655E-06	-0	-0	-0	-0	-0
22	1	0.5502375E-06	-0	-0	-0	-0	-0
24	1	0.2424354E-07	-0	-0	-0	-0	-0
25	1	0.373284E-08	-0	-0	-0	-0	-0
26	1	0.1023543E-08	-0	-0	-0	-0	-0
27	1	0.2150532E-07	-0	-0	-0	-0	-0
28	1	0.6859054E-07	-0	-0	-0	-0	-0
29	1	0.5267732E-07	-0	-0	-0	-0	-0
30	1	0.16561074E-06	-0	-0	-0	-0	-0
31	1	0.16393784E-05	-0	-0	-0	-0	-0
32	1	0.27405577E-05	-0	-0	-0	-0	-0
33	1	0.2074725E-06	-0	-0	-0	-0	-0
34	1	0.35202104E-06	-0	-0	-0	-0	-0
35	1	0.15502477E-06	-0	-0	-0	-0	-0
36	1	0.50954276E-05	-0	-0	-0	-0	-0
37	1	0.1257580E-06	-0	-0	-0	-0	-0
38	1	0.6454512E-06	-0	-0	-0	-0	-0
39	1	0.60114750E-07	-0	-0	-0	-0	-0



NOT REPRODUCIBLE

40	1	0.79310115E-05	-0	-0	-0.	-0	-0	-0.
41	1	-0.19503205E-07	-0	-0	-0.	-0	-0	-0.
42	1	0.51644271E-06	-0	-0	-0.	-0	-0	-0.
44	1	0.56593702E-06	-0	-0	-0.	-0	-0	-0.
47	1	-0.17408250E-08	-0	-0	-0.	-0	-0	-0.
49	1	-0.43044237E-07	-0	-0	-0.	-0	-0	-0.
51	1	-0.10825294E-06	-0	-0	-0.	-0	-0	-0.
52	1	0.10203946E-06	-0	-0	-0.	-0	-0	-0.
53	1	-0.12808413E-06	-0	-0	-0.	-0	-0	-0.
54	1	0.23061265E-06	-0	-0	-0.	-0	-0	-0.
55	1	-0.23061265E-06	-0	-0	-0.	-0	-0	-0.
56	1	0.23061265E-06	-0	-0	-0.	-0	-0	-0.
57	1	-0.23061265E-06	-0	-0	-0.	-0	-0	-0.
58	1	0.49220110E-08	-0	-0	-0.	-0	-0	-0.
59	1	-0.12885652E-06	-0	-0	-0.	-0	-0	-0.
60	1	0.44111628E-06	-0	-0	-0.	-0	-0	-0.
61	1	-0.47273757E-07	-0	-0	-0.	-0	-0	-0.
62	1	0.60748929E-06	-0	-0	-0.	-0	-0	-0.
63	1	-0.12300820E-07	-0	-0	-0.	-0	-0	-0.
64	1	0.53601289E-06	-0	-0	-0.	-0	-0	-0.
66	1	0.56588587E-06	-0	-0	-0.	-0	-0	-0.
67	1	-0.12540402E-06	-0	-0	-0.	-0	-0	-0.
69	1	-0.16244166E-06	-0	-0	-0.	-0	-0	-0.
71	1	-0.12824389E-06	-0	-0	-0.	-0	-0	-0.
72	1	0.30845227E-07	-0	-0	-0.	-0	-0	-0.
73	1	-0.20082534E-06	-0	-0	-0.	-0	-0	-0.
74	1	0.20516308E-06	-0	-0	-0.	-0	-0	-0.
75	1	-0.26091772E-06	-0	-0	-0.	-0	-0	-0.
76	1	0.36732427E-06	-0	-0	-0.	-0	-0	-0.
77	1	-0.22063284E-06	-0	-0	-0.	-0	-0	-0.
78	1	0.40853229E-06	-0	-0	-0.	-0	-0	-0.
79	1	-0.12329541E-06	-0	-0	-0.	-0	-0	-0.
80	1	0.4222392E-06	-0	-0	-0.	-0	-0	-0.
81	1	-0.67678536E-07	-0	-0	-0.	-0	-0	-0.
82	1	0.67980732E-06	-0	-0	-0.	-0	-0	-0.
83	1	-0.12250506E-06	-0	-0	-0.	-0	-0	-0.
84	1	0.61747611E-04	-0	-0	-0.	-0	-0	-0.
85	1	0.12268174E-07	-0	-0	-0.	-0	-0	-0.
86	1	0.78612787E-06	-0	-0	-0.	-0	-0	-0.
87	1	0.35562034E-07	-0	-0	-0.	-0	-0	-0.
88	1	0.65571610E-06	-0	-0	-0.	-0	-0	-0.
89	1	0.82927116E-07	-0	-0	-0.	-0	-0	-0.
90	1	0.97824431E-06	-0	-0	-0.	-0	-0	-0.
91	1	0.36283374E-07	-0	-0	-0.	-0	-0	-0.
92	1	0.10638923E-05	-0	-0	-0.	-0	-0	-0.
94	1	0.10918055E-05	-0	-0	-0.	-0	-0	-0.

MATRIX DIJ 94 ROWS X 96 COLUMNS FROM TAPE 9

MATRIX SIM 96 ROWS X 1 COLUMNS FROM TAPE 9								
1	1	0.22051536E-00	-0	-0	-0.	-0	-0	-0.
2	1	0.61378789E-00	-0	-0	-0.	-0	-0	-0.
3	1	0.11614404E-00	-0	-0	-0.	-0	-0	-0.
4	1	-0.16528405E-01	-0	-0	-0.	-0	-0	-0.
5	1	0.62109423E-00	-0	-0	-0.	-0	-0	-0.
6	1	0.42355019E-00	-0	-0	-0.	-0	-0	-0.
7	1	-0.52512552E-00	-0	-0	-0.	-0	-0	-0.
8	1	0.31672780E-01	-0	-0	-0.	-0	-0	-0.
9	1	0.75245333E-00	-0	-0	-0.	-0	-0	-0.
10	1	0.30618185E-00	-0	-0	-0.	-0	-0	-0.
11	1	0.68312987E-01	-0	-0	-0.	-0	-0	-0.
12	1	0.53093573E-00	-0	-0	-0.	-0	-0	-0.
13	1	0.90510484E-00	-0	-0	-0.	-0	-0	-0.
14	1	0.59245485E-01	-0	-0	-0.	-0	-0	-0.
15	1	0.14156404E-00	-0	-0	-0.	-0	-0	-0.
16	1	0.12055702E-01	-0	-0	-0.	-0	-0	-0.
17	1	0.10601959E-02	-0	-0	-0.	-0	-0	-0.

18	1	-0.12255783E-01	-0	-0	-0.	-0	-0	-0.
19	1	0.12488060E-01	-0	-0	-0.	-0	-0	-0.
20	1	0.54726815E-01	-0	-0	-0.	-0	-0	-0.
21	1	-0.21487273E-01	-0	-0	-0.	-0	-0	-0.
22	1	0.40350556E-00	-0	-0	-0.	-0	-0	-0.
23	1	0.45795274E-01	-0	-0	-0.	-0	-0	-0.
24	1	-0.20217408E-01	-0	-0	-0.	-0	-0	-0.
25	1	-0.12447850E-01	-0	-0	-0.	-0	-0	-0.
26	1	0.12447850E-01	-0	-0	-0.	-0	-0	-0.
27	1	-0.11904111E-01	-0	-0	-0.	-0	-0	-0.
28	1	-0.70052013E-01	-0	-0	-0.	-0	-0	-0.
29	1	0.29256300E-00	-0	-0	-0.	-0	-0	-0.
30	1	-0.26649203E-00	-0	-0	-0.	-0	-0	-0.
31	1	-0.55601031E-00	-0	-0	-0.	-0	-0	-0.
32	1	0.76835633E-01	-0	-0	-0.	-0	-0	-0.
33	1	-0.29490650E-00	-0	-0	-0.	-0	-0	-0.
34	1	-0.53254112E-00	-0	-0	-0.	-0	-0	-0.
35	1	0.24567261E-01	-0	-0	-0.	-0	-0	-0.
36	1	-0.22046158E-00	-0	-0	-0.	-0	-0	-0.
37	1	-0.11633930E-01	-0	-0	-0.	-0	-0	-0.
38	1	0.51672256E-01	-0	-0	-0.	-0	-0	-0.
39	1	0.73959639E-02	-0	-0	-0.	-0	-0	-0.
40	1	-0.45056726E-00	-0	-0	-0.	-0	-0	-0.
41	1	0.65816440E-01	-0	-0	-0.	-0	-0	-0.
42	1	-0.74351584E-00	-0	-0	-0.	-0	-0	-0.
43	1	0.57886416E-00	-0	-0	-0.	-0	-0	-0.
44	1	0.66630563E-01	-0	-0	-0.	-0	-0	-0.
45	1	-0.56367997E-00	-0	-0	-0.	-0	-0	-0.
46	1	0.67365377E-00	-0	-0	-0.	-0	-0	-0.
47	1	0.64682445E-01	-0	-0	-0.	-0	-0	-0.
48	1	-0.34052330E-00	-0	-0	-0.	-0	-0	-0.
49	1	0.59090111E-01	-0	-0	-0.	-0	-0	-0.
50	1	0.65220727E-01	-0	-0	-0.	-0	-0	-0.
51	1	-0.19094899E-00	-0	-0	-0.	-0	-0	-0.
52	1	-0.79729091E-00	-0	-0	-0.	-0	-0	-0.
53	1	0.57992791E-01	-0	-0	-0.	-0	-0	-0.
54	1	-0.58869335E-00	-0	-0	-0.	-0	-0	-0.
55	1	-0.64703017E-00	-0	-0	-0.	-0	-0	-0.
56	1	0.41281175E-01	-0	-0	-0.	-0	-0	-0.
57	1	-0.16386002E-01	-0	-0	-0.	-0	-0	-0.
58	1	-0.36979900E-00	-0	-0	-0.	-0	-0	-0.
59	1	0.18133201E-01	-0	-0	-0.	-0	-0	-0.
60	1	-0.51672026E-00	-0	-0	-0.	-0	-0	-0.
61	1	-0.50945470E-00	-0	-0	-0.	-0	-0	-0.
62	1	0.62208836E-01	-0	-0	-0.	-0	-0	-0.
63	1	-0.33301412E-00	-0	-0	-0.	-0	-0	-0.
64	1	-0.29685431E-00	-0	-0	-0.	-0	-0	-0.
65	1	0.69792469E-01	-0	-0	-0.	-0	-0	-0.
66	1	-0.37222034E-00	-0	-0	-0.	-0	-0	-0.
67	1	-0.22472068E-00	-0	-0	-0.	-0	-0	-0.
68	1	0.24089822E-01	-0	-0	-0.	-0	-0	-0.
69	1	-0.59730778E-00	-0	-0	-0.	-0	-0	-0.
70	1	0.20813503E-00	-0	-0	-0.	-0	-0	-0.
71	1	0.52628106E-01	-0	-0	-0.	-0	-0	-0.
72	1	-0.11141205E-01	-0	-0	-0.	-0	-0	-0.
73	1	0.21072656E-00	-0	-0	-0.	-0	-0	-0.
74	1	0.45575524E-01	-0	-0	-0.	-0	-0	-0.
75	1	0.40620423E-02	-0	-0	-0.	-0	-0	-0.
76	1	-0.15477098E-00	-0	-0	-0.	-0	-0	-0.
77	1	0.51395853E-01	-0	-0	-0.	-0	-0	-0.
78	1	0.56883767E-00	-0	-0	-0.	-0	-0	-0.
79	1	-0.16256534E-00	-0	-0	-0.	-0	-0	-0.
80	1	0.55062751E-01	-0	-0	-0.	-0	-0	-0.
81	1	-0.42365104E-00	-0	-0	-0.	-0	-0	-0.
82	1	-0.43814982E-01	-0	-0	-0.	-0	-0	-0.
83	1	0.45753306E-01	-0	-0	-0.	-0	-0	-0.
84	1	0.27397618E-01	-0	-0	-0.	-0	-0	-0.
85	1	0.10101712E-01	-0	-0	-0.	-0	-0	-0.
86	1	0.45253629E-01	-0	-0	-0.	-0	-0	-0.
87	1	-0.67068736E-01	-0	-0	-0.	-0	-0	-0.

88	1	0.15930474E+00	-0.000000	-0.000000	-0.000000
89	1	0.42977658E+01	-0.000000	-0.000000	-0.000000
90	1	-0.52375240E+00	-0.000000	-0.000000	-0.000000
91	1	0.00040805E+00	-0.000000	-0.000000	-0.000000
92	1	0.00000000E+00	-0.000000	-0.000000	-0.000000
93	1	-0.00000000E+00	-0.000000	-0.000000	-0.000000
94	1	0.17045342E+01	-0.000000	-0.000000	-0.000000
95	1	0.00053390E+01	-0.000000	-0.000000	-0.000000
96	1	-0.00021923E+00	-0.000000	-0.000000	-0.000000

MATRIX STJ 96 ROWS X 96 COLUMNS FROM TAPE 5
 MODULUS OF ELASTICITY = 9727600. PSI SHEAR MODULUS = 3741325. PSI MU = 0.300

TABLE OF VALUES FOR STRESS-STRAIN CURVE

POINT	STRESS LEVEL PSI	STRAIN IN./IN.
1	0	0
2	40000.00	0.00411200
3	45000.00	0.00475000
4	50000.00	0.00560000
5	55000.00	0.00700000
6	59000.00	0.00875000
7	60000.00	0.01031200
8	80000.00	0.06569999
9	100000.00	0.42800000
10	120000.00	2.22199997
11	140000.00	8.86399594

EXIT = 4

APPLIED LOAD 300.00

DEFLECTIONS AT GEOMETRIC NODES

NODE	DELTA X	DELTA Y
1	0.	0.9207413E-03
2	-0.6807273E-05	0.2747107E-03
3	-0.3134502E-04	0.2570500E-04
4	-0.5512311E-04	0.4016457E-04
5	-0.7674264E-04	0.1247687E-03
6	-0.899316E-04	0.1944930E-03
7	-0.5025524E-04	0.2740234E-03
8	-0.7756605E-04	0.3455030E-03

7	6.244050E 02	4.2333A09E 03	-1.0743637E 03	-6.6A60304E-05	4.162A039E-0A	-2.071561E-04
8	2.0175276E 02	2.4857637E 03	-1.0108704E 03	-5.6044278E-05	2.4972633E-04	-2.7012618E-04
9	5.2180224E 02	5.6582022E 02	-5.9821269E 02	-9.1716744E-05	1.1846522E-04	-1.5585072E-04
10	-1.1046417E 03	1.4628190E 02	-1.8324601E 02	-1.3862887E-04	5.5273078E-05	-4.8978127E-05
11	-2.760515E 02	1.8419931E 01	-1.475325E 02	-2.9763881E-05	1.2523282E-05	-3.941141E-05
12	-4.6627356E 02	1.2283630E 03	1.1423384E 02	-8.5815561E-05	1.4065585E-04	3.0531701E-05
13	-5.1165699E 02	2.5836148E 03	1.6979969E 00	-1.3947751E-04	2.8333590E-04	5.8240330E-07
14	-2.4543363E 02	2.2508220E 03	3.7175991E 02	-1.2671987E-04	3.586661E-04	-9.9364260E-05
15	2.954320E 02	3.3215252E 03	-4.8183398E 02	-7.2990907E-05	3.3355551E-04	-1.2878654E-04
16	4.3682684E 02	3.2241222E 03	-1.7026165E 02	-5.4834674E-05	3.1899687E-04	-4.5507657E-05
17	2.5545055E 01	3.2610364E 03	-9.5024493E 01	-9.7533393E-05	3.3A22427E-04	-2.539216E-05
18	-3.7364545E 02	2.8566395E 03	-4.9432667E 02	-1.2783398E-04	3.0960701E-04	-1.3212399E-04
19	-3.2251508E 02	2.0640588E 03	-8.1925009E 02	-9.6913185E-05	2.2216105E-04	-2.1896976E-04
20	-1.8489700E 03	5.0666008E 02	-4.5836013E 02	-4.6968936E-05	9.6707148E-05	-1.2251083E-04
21	-4.547234E 02	3.1144418E 03	-1.6650700E 02	-1.4279524E-04	1.318332E-04	-4.4504129E-05
22	-1.5342710E 02	3.4856244E 03	-1.8611017E 02	-1.2339266E-04	3.6346607E-04	-4.9743661E-05
23	-1.612343E 02	3.204911E 03	-2.7885389E 02	-1.1540192E-04	3.3439512E-04	-7.9478813E-05
24	1.5406781E 02	2.6314053E 03	-5.5706024E 02	-6.5314575E-05	2.6575774E-04	-1.4289147E-04
25	1.0336398E 02	2.2789765E 03	2.0314213E 00	-5.9452448E-05	2.3103000E-04	5.4295975E-07
26	-2.2385237E 01	2.5657932E 03	2.8361883E 02	-8.7721890E-05	2.6671622E-04	7.5752351E-05
27	-4.1282617E 01	2.7531376E 03	-2.1182552E 02	-9.3262873E-05	2.8533007E-04	-5.6616879E-05
28	-2.1507491E 01	2.2876653E 03	1.3698809E 01	-7.2003884E-05	2.3584826E-04	3.6614278E-06
29	5.0502854E 00	2.2626315E 03	-3.3834347E 01	-6.9262055E-05	2.3248622E-04	-5.0432744E-06
30	7.5652369E 01	2.2588829E 03	-2.5199673E 02	-6.2709454E-05	2.3865332E-04	-6.9599948E-05
31	-4.512340E 02	2.1022911E 03	-4.9156138E 02	-1.8632888E-05	2.0201682E-04	-1.313848E-04
32	8.517671E 02	1.8026695E 03	-2.5410962E 02	3.1864620E-05	1.5907720E-04	-6.7518603E-05

NODE INELASTIC STRAIN			NODE STRAIN CHANGE		
NODE	EPSILON_X	EPSILON_Y	GAMMA_XY	EPSILON_X	GAMMA_XY
1	0.	0.	0.	0.	0.
2	0.	0.	0.	0.	0.
3	0.	0.	0.	0.	0.
4	0.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.
6	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.
8	0.	0.	0.	0.	0.
9	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.
11	0.	0.	0.	0.	0.
12	0.	0.	0.	0.	0.
13	0.	0.	0.	0.	0.
14	0.	0.	0.	0.	0.
15	0.	0.	0.	0.	0.
16	0.	0.	0.	0.	0.
17	0.	0.	0.	0.	0.
18	0.	0.	0.	0.	0.
19	0.	0.	0.	0.	0.
20	0.	0.	0.	0.	0.
21	0.	0.	0.	0.	0.

NODE	EFFECTIVE STRESSES	TOTAL EFFECTIVE STRAIN	EFFECTIVE STRAIN CHANGES	EFFECTIVE PLASTIC STRAIN	
22	0.	0.	0.	-0.	-0.
23	0.	0.	0.	-0.	-0.
24	0.	0.	0.	-0.	0.
25	0.	0.	0.	-0.	0.
26	0.	0.	0.	-0.	-0.
27	0.	0.	0.	-0.	-0.
28	0.	0.	0.	-0.	-0.
29	0.	0.	0.	-0.	-0.
30	0.	0.	0.	-0.	-0.
31	0.	0.	0.	-0.	-0.
32	0.	0.	0.	-0.	-0.

NODE	EFFECTIVE STRESSES	TOTAL EFFECTIVE STRAIN	EFFECTIVE STRAIN CHANGES	EFFECTIVE PLASTIC STRAIN
1	2.864810E-02	2.5611431E-05	0.	0.
2	1.0664806E-03	1.1169053E-04	0.	0.
3	1.8488251E-03	1.505974E-04	0.	0.
4	3.4376115E-03	2.5328743E-04	0.	0.
5	4.7527771E-03	4.6869562E-04	0.	0.
6	5.1217747E-03	5.2651986E-04	0.	0.
7	4.3765342E-03	4.4950894E-04	0.	0.
8	2.565661E-03	3.0500494E-04	0.	0.
9	1.7302519E-03	1.7717038E-04	0.	0.
10	1.4155120E-03	1.4522828E-04	0.	0.
11	3.9328269E-02	4.0425569E-05	0.	0.
12	1.5251107E-03	1.5719301E-04	0.	0.
13	2.012801E-03	2.000001E-04	0.	0.
14	1.4602156E-03	2.576922E-04	0.	0.
15	3.102502E-03	2.3523179E-04	0.	0.
16	3.0526175E-03	2.1351273E-04	0.	0.
17	3.2505342E-03	2.3415582E-04	0.	0.
18	1.2103220E-03	2.3074720E-04	0.	0.
19	2.6144805E-03	2.728134E-04	0.	0.
20	1.2161310E-03	1.221491E-04	0.	0.
21	1.172612E-03	2.4718381E-04	0.	0.
22	1.5623105E-03	2.6536845E-04	0.	0.
23	2.1233093E-03	2.4161712E-04	0.	0.
24	2.727827E-03	2.8103356E-04	0.	0.
25	2.2281667E-03	2.2905616E-04	0.	0.
26	2.6576500E-03	2.7221127E-04	0.	0.
27	2.0186461E-03	2.6575760E-04	0.	0.
28	2.2528159E-03	2.3631932E-04	0.	0.
29	2.2609192E-03	2.2242319E-04	0.	0.
30	2.1051822E-03	2.3657139E-04	0.	0.
31	2.1028715E-03	2.1617578E-04	0.	0.
32	1.0029716E-03	1.662446E-04	0.	0.

1 CYCLES COMPLETED - LOAD = 500.00

APPLIED LOAD 5500.00

DEFLECTIONS AT GEOMETRIC NODES

NODE	DELTA X	DELTA Y
1	-0.	0.0751782E-04
2	-0.6819702E-04	0.2709466E-04
3	-0.3137A26E-04	0.2562A35E-03
4	-0.5616135E-03	0.6601230E-03
5	-0.7679278E-03	0.1244639E-02
6	-0.8999C12E-03	0.1965615E-02
7	-0.9041115E-03	0.2752182E-02
8	-0.7778574E-03	0.3505041E-02
9	-0.5573C60E-03	0.4152337E-02
10	-0.2875716E-03	0.4601115E-02
11	-0.	0.4764025E-02
12	-0.	0.1205512E-03
13	-0.1655659E-04	0.3883897E-04
14	-0.1577292E-03	0.3421780E-03
15	-0.4637262E-03	0.0206606E-03
16	-0.4207240E-03	0.1393606E-02
17	-0.1046774E-02	0.1961724E-02
18	-0.5781868E-03	0.2538090E-02
19	-0.6776040E-03	0.3236209E-02
20	-0.3314501E-03	0.3974965E-02
21	-0.5833552E-04	0.4591980E-02
22	-0.	0.4839611E-02
23	-0.	-0.
24	-0.2871761E-04	-0.
25	-0.211653E-03	-0.
26	-0.5409221E-03	0.5162463E-03
27	-0.5417250E-03	0.1195004E-02
28	-0.1201778E-02	0.1840009E-02
29	-0.1064576E-02	0.2464597E-02
30	-0.6449034E-03	0.3211716E-02
31	-0.2361642E-03	0.4046435E-02
32	-0.6147018E-04	0.4689589E-02
33	-0.	0.4939317E-02
34	-0.4265664E-03	-0.
35	-0.8121352E-03	-0.
36	-0.6411183E-03	0.2545001E-03
37	-0.1005827E-02	0.1029028E-02
38	-0.1308330E-02	0.1839442E-02
39	-0.1105638E-02	0.2447632E-02
40	-0.6166593E-03	0.3216094E-02
41	-0.4882694E-03	0.3404184E-02
42	-0.2124988E-03	0.3091080E-02
43	0.6350513E-04	0.3934705E-02
44	0.1803583E-03	0.4283975E-02
45	0.2716215E-03	0.4898887E-02
46	0.1624233E-03	0.5330449E-02
47	-0.	0.5469438E-02

TOTAL NODE STRAINS

NODE STRESSES

NODE	SIGMA X	SIGMA Y	TAU XY	EPSILON X	EPSILON Y	GAMMA XY
1	1.058823E-03	3.076979E-03	5.693003E-02	1.825219E-05	2.818090E-04	1.575086E-04
2	-9.327762E-03	3.081882E-03	2.102655E-03	-9.511418E-04	5.736472E-04	5.620010E-04
3	-2.074068E-03	1.575254E-04	3.738423E-03	-7.585271E-04	1.704934E-03	9.952085E-04
4	1.437073E-03	2.404880E-04	4.659770E-03	-8.977120E-04	3.454525E-03	1.245466E-03
5	4.001150E-03	4.94207E-04	7.191175E-02	-1.138047E-03	5.114432E-03	2.061085E-04
6	6.001150E-03	5.228100E-04	-6.000000E-03	-1.077081E-03	5.952110E-03	-1.747937E-03
7	0.114884E-03	4.421130E-04	4.107403E-04	-6.104418E-04	4.121812E-03	-3.494545E-03
8	2.001093E-03	2.485798E-04	-1.009000E-04	-5.600270E-04	2.493668E-03	-2.656642E-03
9	-6.250536E-03	9.601670E-03	-8.970816E-03	-9.399479E-04	1.182945E-02	-1.859590E-03
10	-1.308524E-04	1.455787E-03	-1.808436E-03	-1.390063E-03	5.532034E-04	-4.687056E-04
11	-7.728145E-03	3.702642E-02	-1.477871E-03	-2.980413E-04	1.240800E-04	-3.950055E-04
12	-4.652316E-03	1.225058E-04	1.134074E-03	-8.571839E-04	1.403177E-03	3.033569E-04
13	-5.816180E-03	2.575611E-04	3.630244E-01	-1.393459E-03	3.833212E-03	9.754761E-04
14	-2.475941E-03	3.285376E-04	-3.702267E-03	-1.269385E-03	3.457970E-03	-9.895447E-04
15	2.851381E-03	2.342110E-04	-4.824959E-03	-7.334765E-04	3.346534E-03	-1.289378E-03
16	4.414680E-03	2.253126E-04	-1.723859E-03	-5.494939E-04	3.208265E-03	-4.607542E-04
17	2.662101E-02	2.261695E-04	-9.783282E-02	-9.742933E-04	3.346778E-03	-2.614877E-04
18	-3.745461E-03	2.897016E-04	-4.461154E-03	-1.278476E-03	3.036512E-03	-1.326020E-03
19	-3.242292E-03	2.062179E-04	-8.197043E-03	-9.693882E-04	3.219945E-03	-2.190511E-03
20	-1.861220E-03	9.055185E-03	-4.503420E-03	-4.703966E-03	9.882757E-04	-1.225059E-03
21	-4.545345E-03	2.115341E-04	-1.664381E-03	-1.428016E-03	3.302783E-03	-9.448536E-04
22	-1.541170E-03	2.456480E-04	-1.988220E-03	-1.236750E-03	3.641921E-03	-4.954485E-04
23	-1.620076E-03	3.205686E-04	-2.787741E-03	-1.156413E-03	3.349526E-03	-7.431182E-04
24	1.547007E-03	2.635002E-04	-5.601182E-03	-6.530345E-04	2.661077E-03	-1.457082E-03
25	1.060043E-03	2.275385E-04	3.028370E-01	-5.939910E-04	2.310522E-03	9.028925E-06
26	-0.354754E-02	2.574416E-04	2.839345E-03	-8.798363E-04	2.872722E-03	7.589028E-04
27	-8.152208E-02	2.754315E-04	-2.130719E-03	-9.332379E-04	2.896565E-03	-5.716384E-04
28	-2.228082E-02	2.205902E-04	1.359402E-02	-7.292210E-04	2.340925E-03	3.660147E-05
29	5.173285E-01	2.263576E-04	-3.437281E-02	-6.327707E-04	2.325367E-03	-5.187192E-05
30	8.000715E-02	2.245044E-04	-2.632593E-03	-6.261622E-04	2.338581E-03	-7.036430E-04
31	4.524562E-03	2.167852E-04	-4.923718E-03	-1.829198E-04	2.327035E-03	-1.316282E-03
32	8.523001E-03	1.802149E-04	-2.543598E-03	3.214107E-04	1.569458E-03	-6.702547E-04

NODE INELASTIC STRAIN

MODE STRAIN CHANGE

NODE	EPSILON X	GAMMA XY	EPSILON Y	MODE STRAIN CHANGE
1	0.	0.	0.	0.
2	0.	0.	0.	0.
3	0.	0.	0.	0.
4	0.	0.	0.	0.
5	-7.062347E-05	8.690300E-06	-5.615582E-05	6.175594E-05
6	-1.370570E-04	3.419559E-04	-1.264204E-04	-8.217353E-05
7	-2.525534E-05	6.620901E-05	-2.529538E-05	-5.478302E-05
8	0.	0.	0.	0.
9	0.	0.	0.	0.
10	0.	0.	0.	0.
11	0.	0.	0.	0.
12	0.	0.	0.	0.

25	2.0000000E-04	2.0000000E-03	0.	0.
26	2.0000000E-04	2.0000000E-03	0.	0.
27	2.0000000E-04	2.0000000E-03	0.	0.
28	2.0000000E-04	2.0000000E-03	0.	0.
29	2.0000000E-04	2.0000000E-03	0.	0.
30	2.0000000E-04	2.0000000E-03	0.	0.
31	2.0000000E-04	2.0000000E-03	0.	0.
32	2.0000000E-04	2.0000000E-03	0.	0.

10 CYCLES COMPLETED LOAD = 5000.00

APPLIED LOAD 10000.00

DEFLECTIONS AT GEOMETRIC NODES

NODE	DELTA X	DELTA Y
1	-0.	0.034286E-02
2	-0.135420E-03	0.984087E-03
3	-0.213171E-02	0.437643E-02
4	-0.610504E-02	0.141337E-01
5	-0.105651E-01	0.292661E-01
6	-0.128803E-01	0.436401E-01
7	-0.115719E-01	0.580365E-01
8	-0.774442E-02	0.733097E-01
9	-0.417680E-02	0.840816E-01
10	-0.159201E-02	0.698184E-01
11	-0.	0.910561E-01
12	-0.	0.221589E-02
13	0.651469E-04	0.902852E-03
14	-0.130563E-02	0.498343E-02
15	-0.712764E-02	0.157087E-01
16	-0.141120E-01	0.297336E-01
17	-0.176052E-01	0.436226E-01
18	-0.150557E-01	0.567353E-01
19	-0.655757E-02	0.709125E-01
20	-0.303040E-02	0.827452E-01
21	-0.442923E-03	0.818495E-01
22	-0.	0.916362E-01
23	-0.	-0.
24	-0.108109E-03	-0.
25	-0.236573E-02	-0.
26	-0.107647E-01	0.110974E-01
27	-0.205821E-01	0.274180E-01
28	-0.256724E-01	0.426079E-01
29	-0.212842E-01	0.570174E-01
30	-0.119184E-01	0.732802E-01
31	-0.455564E-02	0.854443E-01
32	-0.138973E-02	0.917662E-01
33	-0.	0.928303E-01
34	-0.124856E-01	-0.
35	-0.157759E-01	-0.
36	-0.136170E-01	0.570242E-02
37	-0.256000E-01	0.223716E-01
38	-0.355760E-01	0.411102E-01
39	-0.242689E-01	0.576244E-01
40	-0.119741E-01	0.740062E-01
41	-0.426726E-02	0.775891E-01
42	-0.122447E-01	0.711926E-01
43	-0.424674E-02	0.830654E-01
44	-0.204221E-02	0.876450E-01
45	0.125359E-03	0.944933E-01
46	0.195278E-03	0.974740E-01
47	-0.	0.978510E-01

NODE STRESSES

NODE	SIGMA X	SIGMA Y	JAU, KY	EPSILON X	EPSILON Y	TOTAL NODE STRAINS	GAMMA XY
1	1.450000E-04	1.422860E-04	6.9800750E-03	-0.052021E-03	1.0152784E-03	1.0677779E-03	1.0677779E-03
2	-2.1555422E-04	1.7730039E-04	2.4416731E-04	-9.780122E-03	1.1537116E-02	1.1717501E-02	1.1717501E-02
3	-0.9731195E-03	7.5316840E-04	1.7862140E-04	-2.7071201E-02	4.879442E-02	3.5084094E-02	3.5084094E-02
4	8.300266E-03	1.0151915E-03	9.244221E-03	-3.7230509E-02	6.567075E-01	3.0285604E-02	3.0285604E-02
5	1.1732501E-04	1.0943502E-05	1.3569608E-03	-3.8403002E-02	1.082579E-01	6.2776454E-03	6.2776454E-03
6	1.651626E-04	1.0815108E-05	-3.1267073E-03	-3.6997817E-02	1.0545021E-01	-1.7416362E-02	-1.7416362E-02
7	9.8037131E-03	1.0047675E-05	-1.1437873E-04	-3.4263436E-02	8.8403459E-02	-3.5957675E-02	-3.5957675E-02
8	-2.3499079E-02	7.8228650E-04	-2.2233093E-04	-2.33524571E-02	4.9760257E-02	-4.6738130E-02	-4.6738130E-02
9	-1.467774E-04	4.305743E-04	-2.0347247E-04	-1.3238312E-02	1.7541306E-02	-2.8934352E-02	-2.8934352E-02
10	-4.358409E-04	1.5768084E-04	-1.5566204E-04	-7.2386625E-03	4.4034375E-03	-5.3401092E-03	-5.3401092E-03
11	3.2510612E-03	2.4168291E-04	-5.6671171E-02	-7.1946071E-04	3.4122227E-02	-1.5147112E-04	-1.5147112E-04
12	-4.5761956E-03	4.5303785E-04	6.4651110E-03	-9.2505785E-03	2.0259312E-02	5.0893118E-03	5.0893118E-03
13	-1.3716266E-04	7.7586634E-04	3.3084001E-03	-3.2107606E-02	5.2957940E-02	7.7514830E-03	7.7514830E-03
14	9.331314E-02	5.5552619E-04	-1.2945508E-03	-3.5999758E-02	7.5738037E-02	-5.8176969E-02	-5.8176969E-02
15	1.0335209E-04	5.9400467E-04	-3.4412607E-03	-2.9001316E-02	7.4769365E-02	-1.1194506E-02	-1.1194506E-02
16	9.2033794E-03	5.8133469E-04	1.2712133E-03	-2.7916404E-02	7.0268548E-02	1.0850223E-03	1.0850223E-03
17	8.1679448E-02	5.4109449E-04	-1.1964001E-03	-3.4000587E-02	7.2394625E-02	-2.4729502E-03	-2.4729502E-03
18	-4.7287619E-03	6.265283E-04	-9.7430637E-03	-2.5492462E-02	5.4133062E-02	-2.0813346E-02	-2.0813346E-02
19	1.4617430E-03	7.1564340E-04	-1.6911842E-04	-1.5028862E-02	3.0925773E-02	-2.5832348E-02	-2.5832348E-02
20	-2.9153300E-02	5.1071254E-04	-1.4407918E-04	-5.6642335E-03	1.3022771E-02	-1.0255439E-02	-1.0255439E-02
21	-2.555422E-03	9.1400709E-04	-6.4795702E-03	-3.5157801E-02	6.9225117E-02	-1.3628222E-02	-1.3628222E-02
22	-7.0240982E-03	9.0611372E-04	-6.5701942E-03	-4.0993475E-02	7.7256805E-02	-1.4551795E-02	-1.4551795E-02
23	-2.2544351E-03	9.2813559E-04	-7.7228595E-03	-3.5444289E-02	7.3816285E-02	-1.7173950E-02	-1.7173950E-02
24	3.1503409E-03	8.810823E-04	-1.5701046E-04	-2.6126802E-02	5.9396268E-02	-3.2627374E-02	-3.2627374E-02
25	2.7031907E-03	8.4081665E-04	6.9725612E-02	-2.4382550E-02	5.4646152E-02	1.5825503E-03	1.5825503E-03
26	1.8927714E-03	8.7083228E-04	1.0579419E-04	-2.2591619E-02	5.7273250E-02	2.0555146E-02	2.0555146E-02
27	1.2501454E-04	8.927824E-04	1.7349433E-02	-1.8045175E-02	4.8523077E-02	-1.5244626E-03	-1.5244626E-03
28	1.0478217E-04	8.8239101E-04	5.5663302E-03	-1.8576231E-02	4.8685813E-02	7.7893624E-03	7.7893624E-03
29	-7.1551847E-03	6.7957775E-04	-9.1024467E-03	-1.4407811E-02	2.7761305E-02	-8.7835752E-03	-8.7835752E-03
30	-7.0153007E-03	6.5349370E-04	-3.1359237E-04	-1.2270336E-02	2.3510051E-02	-1.2210904E-02	-1.2210904E-02
31	9.1857012E-02	6.7282411E-04	-1.0911027E-04	-7.17995E-03	1.8346844E-02	-8.988927E-03	-8.988927E-03
32	1.6512595E-04	7.3951444E-04	-2.4291205E-03	-2.6839480E-03	1.4484971E-02	-1.5524446E-03	-1.5524446E-03

NODE INELASTIC STRAIN

NODE	EPSILON X	EPSILON Y	GAMMA XY	NODE STRAIN CHANGE	EPSILON X	EPSILON Y	GAMMA XY
1	0.	0.	0.	0.	0.	0.	0.
2	-6.3502808E-03	6.9803257E-03	1.0652857E-02	-3.1863937E-04	3.0023508E-04	5.7171831E-04	5.7171831E-04
3	-2.3825587E-02	4.0751379E-02	3.1310488E-02	-5.9637085E-04	1.0266055E-02	4.8531653E-04	4.8531653E-04
4	-1.4545261E-02	7.9450725E-02	2.7780072E-02	-7.7622191E-04	1.7246422E-03	5.1360104E-04	5.1360104E-04
5	-3.6414087E-02	5.7568380E-02	5.9149418E-03	-7.4138210E-04	1.9906848E-03	8.0750255E-05	8.0750255E-05
6	-3.5401958E-02	5.4254141E-02	-1.6780119E-02	-7.0570113E-04	1.9039667E-03	-1.7526613E-04	-1.7526613E-04
7	-3.0178406E-02	7.0376726E-02	-3.3794525E-02	-7.0545257E-04	1.6674236E-03	-5.9264487E-04	-5.9264487E-04
8	-2.1005484E-02	4.1709442E-02	4.0765458E-02	-5.0778360E-04	1.0106388E-03	-8.6396517E-04	-8.6396517E-04
9	-1.0401542E-02	1.2662317E-02	-2.24263E-02	-3.2544200E-04	4.5710348E-04	-7.7485217E-04	-7.7485217E-04
10	-2.234415E-03	1.5669265E-03	-1.4468216E-03	-1.7634451E-04	1.2840222E-04	-1.4465588E-04	-1.4465588E-04
11	0.	0.	0.	0.	0.	0.	0.
12	-7.074917E-03	1.3406641E-02	3.8612701E-03	-2.8123414E-04	5.1123224E-04	1.4674441E-04	1.4674441E-04
13	-2.8292445E-02	4.4517070E-02	4.8670531E-03	-6.594530E-04	1.0616616E-03	1.2421501E-04	1.2421501E-04
14	-3.162434E-02	6.5943380E-02	-5.4710892E-03	-7.4115829E-04	1.5044628E-03	-6.1447949E-05	-6.1447949E-05
15	-2.7070315E-02	6.4870327E-02	-1.6254775E-02	-6.1301523E-04	1.4655133E-03	-1.6105856E-04	-1.6105856E-04
16	-2.074070E-02	6.1024554E-02	7.4924350E-04	-5.9732032E-04	1.4014545E-03	3.7145675E-05	3.7145675E-05
17	-3.111110E-02	7.2345340E-02	-2.1511736E-03	-6.4116125E-04	1.3445742E-03	-5.2882422E-05	-5.2882422E-05

IR	MODE	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	STRAIN	STRESS	S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120 CYCLES COMPLETED - LOAD = 16000.00

APPENDIX E CONTACT PROBLEM - LOADED HOLE BOUNDARY COMPUTER PROGRAM

(A) Program Description

This is a brief description of the program for doing a nonlinear contact problem of a symmetric single hole plate structure which is loaded along its axis of symmetry. The program can accommodate varying amounts of clearance or interference and up to twenty points in contact on the semicircle as well as up to ten different load levels.

The program was originally written and debugged on the IBM 1130 machine and then converted to the IBM 7094. It is fully operable on the Grumman IBM 7094 system and can easily be expanded to handle more contact points around the periphery of the fastener and plate hole.

A constraint on the program as it stands at present is that the contact points must be numbered consecutively from the contact point on the center line at the bottom of the hole to the point on the center line at the top of the hole counterclockwise.

The program consists of a main program and two subroutines, one for reading and stacking input, and the other for performing the calculations. A matrix inversion routine "LINEAR" is included to avoid dependence on a library routine.

(B) Sequencing and Details of the Data Cards

The symbols used in the program for various items of input data are listed on page 205 and are shown on the sample key-punching sheet page 209.

The data cards are used in the following sequence, immediately after the \$DATA card required by IBSYS:

1. A load-fit card (FORMAT 23) containing a clue K, a load QIN(1), a fit FITIN(1), and a 60 character title to be printed for identification. The clue K is ignored in this card. Up to 9 additional load-fit cards in the same

format may be provided. They are used in a slightly different manner. This title is ended by one completely blank card.

2. The four data matrices are next. They must be in proper sequence, as shown below. Each has a header card (FORMAT 1), one or more data cards (FORMAT 2) and a blank card to end it. If one of the matrices is null, it must be represented by a blank title card and a blank card to represent the end card. Required sequence is as follows:

- a) Matrix A - influence coefficient for unit radial loads, maximum size 20 x 20
- b) Matrix DL - radial displacements around hole for transfer loads, maximum size 20 x 10. Note that sequence of loads (columns) must correspond to sequence of load-fit cards already read in.
- c) Matrix THETA - angles to defined points around hole, maximum size 20 x 1
- d) Matrix DELB - radial displacements around hole due to bypass loads, maximum size 20 x 1. This matrix may be null as previously indicated. All others must be provided to give meaningful results.

(C) Symbols and Format of the Data Cards

(1) Load-fit cards - FORMAT 23

Cols.	Field	Symbol	
1	I1	K	A clue, ignored in the first card. In the other cards, any digit except 0 indicates a new fit is provided (a blank it means 0 clearances) If K = 0, the previous fit is re-used.
2-10	E9.4	Q or QIn	This is the load to be applied with the corresponding column of matrix DL. A value must be provided. If none is provided, this is treated as a blank card (end of load table).

Cols.	Field	Symbol	
11-20	E10.4	AKFIT or FITIN	This is the fit of the pin in the hole. A positive value indicates an interference fit. A negative value indicates lack of fit (sloppy fit). Zero indicates exact or sliding fit. Fit is always accepted from the first load-fit card. It is ignored on the other cards unless Column 1 is punched to show that a new value of fit is provided. This new value is then used until a superceding value is provided.
21-80	30A2	TITLE	A title or caption that is printed to identify the run. In addition, the last 8 columns are used to identify the punched output from this run. The title is used from the first card only.
(2) Data Matrices Header Card			FORMAT 1
Cols.	Field	Symbol	
1-4	I4	NROWS	The number of rows in this matrix. The number of rows in matrix A sets the size of the problem.
5-8	I4	NCOLS	The number of columns in this matrix. The number of columns in matrix DL must agree with the number of load-fit cards read in.
9-80			Ignored
(3) Data Matrices - Data Cards			FORMAT 2
Cols.	Field	Symbol	
1-4	I4	MR(1)	Row index for the first element in this card. If this field is blank or zero, the card is considered blank (end of matrix)

Cols.	Field	Symbol	
5-8	I4	MC(1)	Column index for the first element in this card.
9-23	E15.7	EL(1)	The first matrix element in this card.
24	IX		Ignored
25-28	I4	MR(2)	Row index for the second element in this card. If this field is blank or zero, the element is ignored and the program looks at MR(3)
29-32	I4	MC(2)	Column index for the second element in this card.
33-47	E15.7	EL(2)	The second matrix element in this card.
48	IX		Ignored
49-52	I4	MR(3)	Row index for the third element in this card. If this field is blank or zero, the element is ignored and the program reads the next card.
53-56	I4	MC(3)	Column index for the third element in this card.
57-71	E15.7	EL(3)	The third matrix element in this card.
72	IX		Ignored
73-80	4A2		Ignored on input. For punch output, this takes 8 columns of identification from the title card.

The last card of a matrix should be completely blank (tested in Cols. 1-4 as an integer field). Note that the program does no checking on the validity of elements, their indices or their sequencing, other than checking the row index for a zero or blank as described above (negative row index is considered zero). Specifically, no checking is done for duplicate indices, column index of zero, elements out of sequence, element indices outside of the array or elements in row or column sort. Valid input is the responsibility of the user. The ability to read matrices in row or column sort, and one to three elements per card is convenient, but requires careful data preparation.

(D) Punched Output

Upon finding the points in contact for each applied load, the program calculates and prints out P_0 (the center-line contact force), and then the X and Y components of the force at each point. It then punches out cards containing these values in FORMAT 2 for input as applied loads to further analysis programs. These cards carry identification in Columns 73-80 from the input title, Columns 73-80.

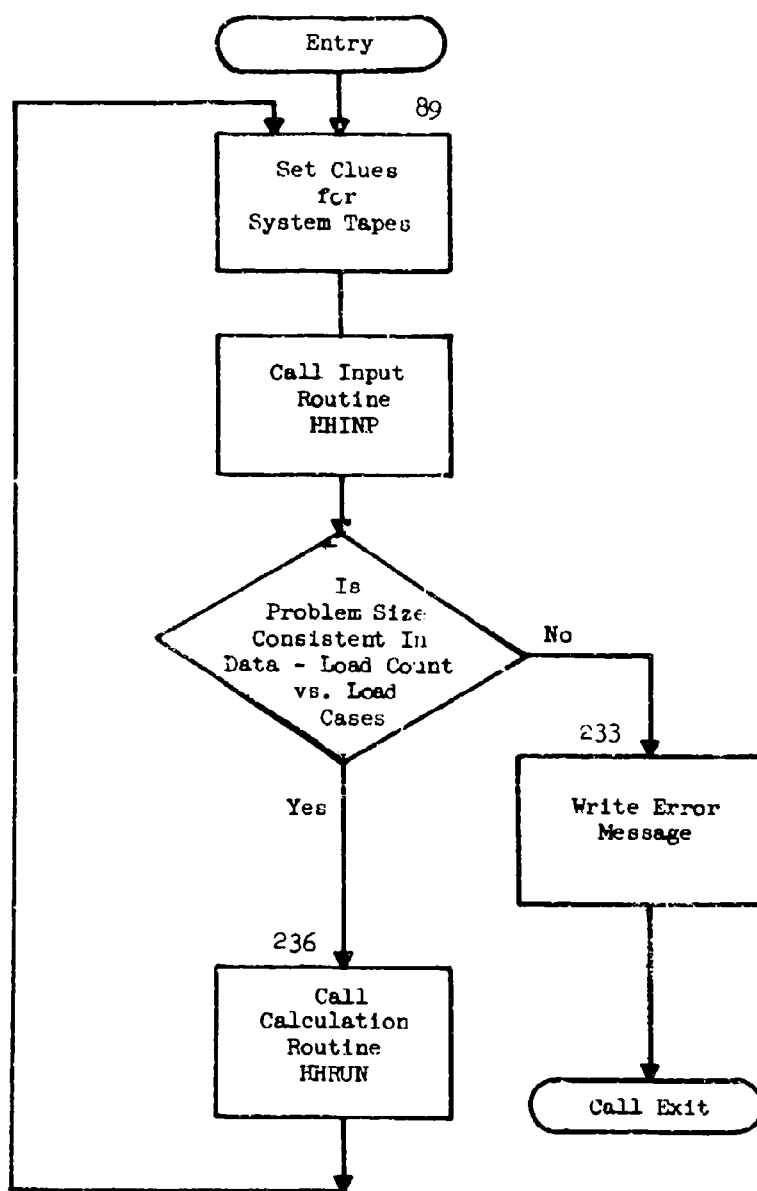
The punchout is described as follows. The row number corresponds to the load case (columns of matrix DL). Column 1 is the center-line contact force. Columns 2 and 3 are the X and Y components at Point 1, columns 4 and 5 are the X and Y components at point 2, etc. Thus the punched-out load matrix is N rows by M columns, where N is the input number of load cases and M is 2 times the number of points around the semicircle, plus 1.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
LEAD-IN CARDS - FORMAT 23 - ONE REQUIRED FOR EACH LOAD CONDITION IN MATRIX OF																																																																															
TITLE (USED FROM FIRST CARD ONLY)																																																																															
PUNCHES OUTPUT																																																																															
MATRIX HEADER CARDS - FORMAT 1 - DEFINE THE START AND SIZE OF A MATRIX																																																																															
MATRIX DATA CARDS - FORMAT 2																																																																															
EACH MATRIX IS TERMINATED BY ONE COMPLETELY BLANK CARD																																																																															

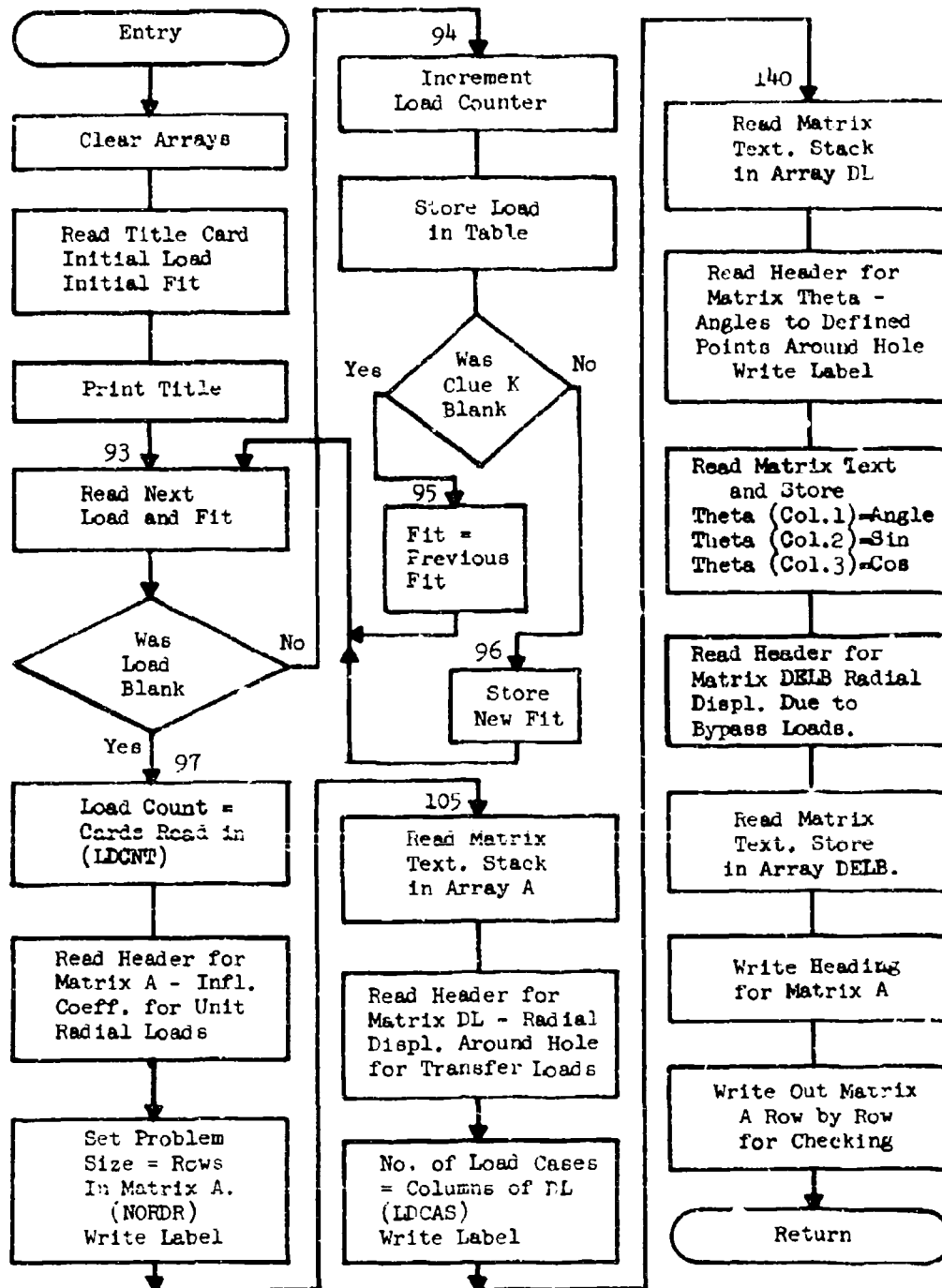
FORMS FOR CONTACT PROBLEM COMPUTER PROGRAM

APPENDIX F CONTACT PROBLEM - LOADED HOLE BOUNDARY PROGRAM FLOW CHARTS

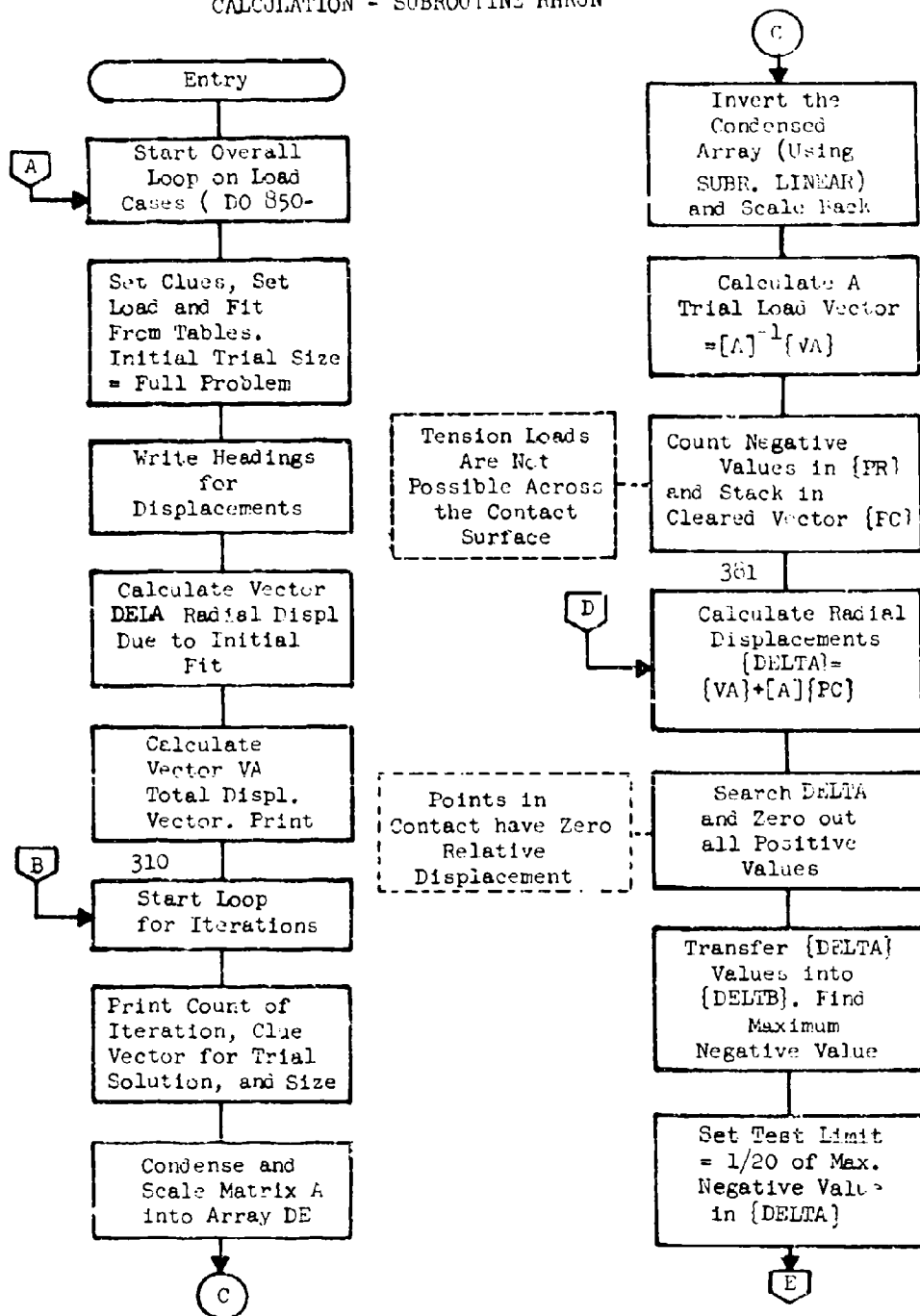
MAIN PROGRAM

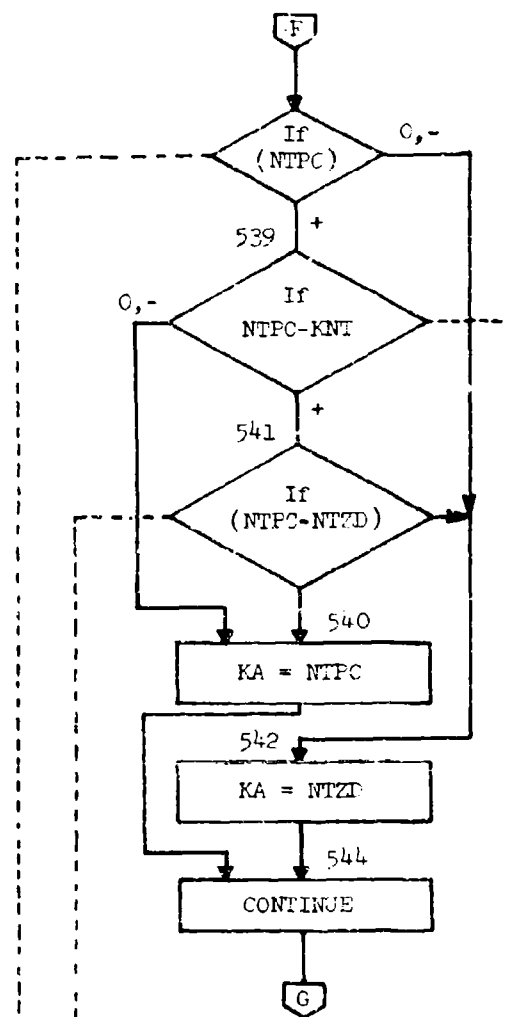
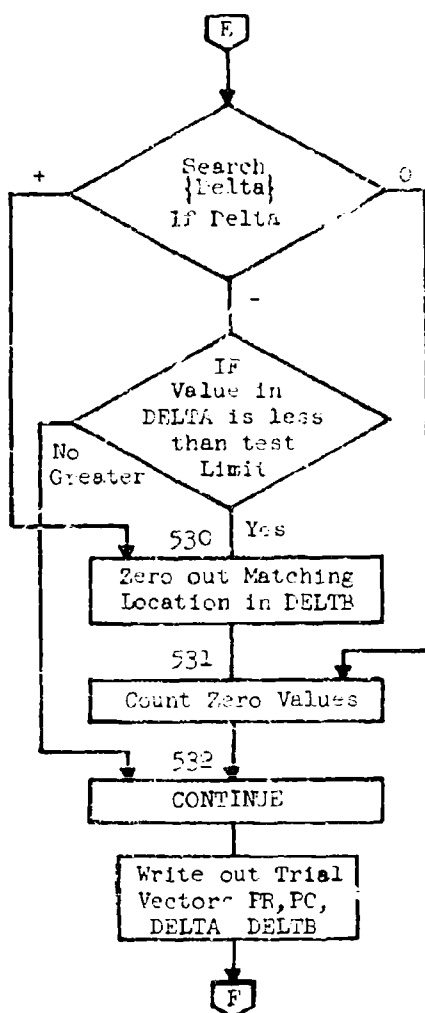


INPUT - SUBROUTINE HHINP



CALCULATION - SUBROUTINE HHRUN

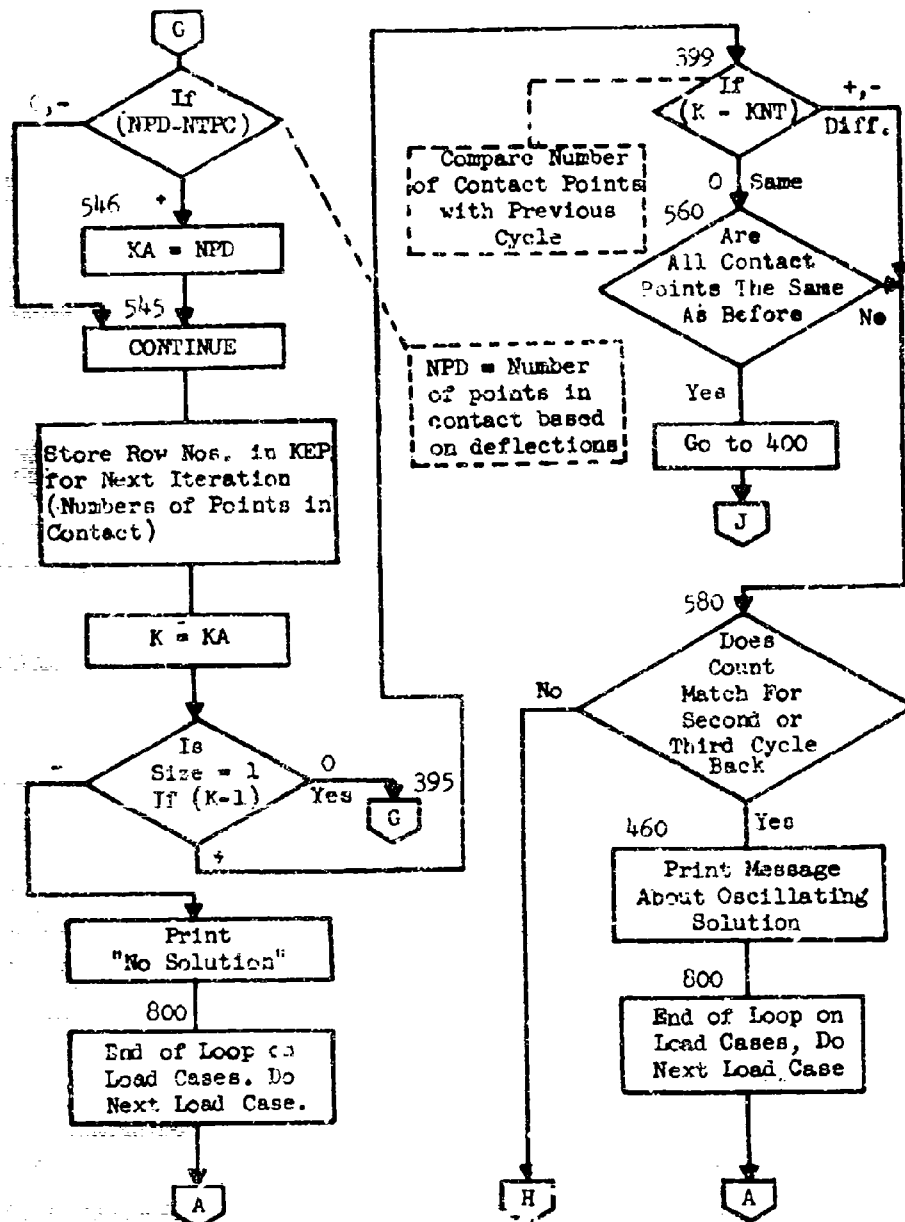


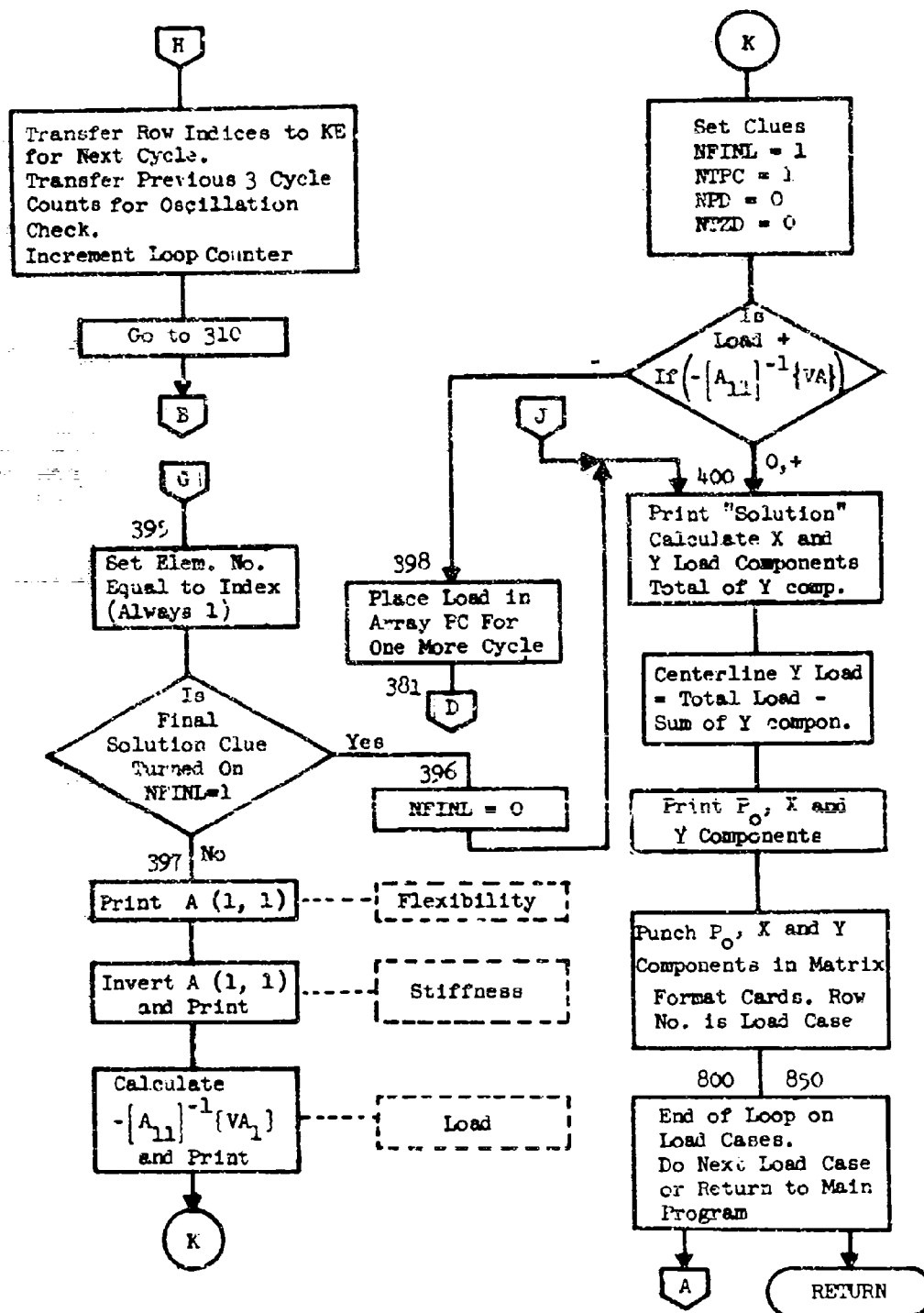


NTPC = Number of
Compression (Neg-
ative) Loads In
PC

NTZD = Number of Points In
Contact, Based on Deflec-
tion and Including those
Less Than 1/20 of Maximum
Positive Deflection

KNT = Number Of
Contact Points
In Previous
Cycle





APPENDIX G FORTRAN LISTING OF ELASTIC CONTACT PROBLEM PROGRAM

```

COMMON NIN,NOUTP,NOUT,LDCNT,LDCAS,NORDR,Q,FIT
COMMON      A(20,20),DL(20,10),DELA(20),DELB(20),DE(20,20),
1  KR(20),VA(20),KE(20),KEP(20),
2  MR(3),MC(3),EL(3),LVA(20),LVH(20)
COMMON      PC(20),DELTA(20),THETA(20,3),TITLE(30)
COMMON      QIN(10),FITIN(10)
89 NIN = 5
NOUTP = 7
NOUT = 6
CALL HHINP
IF (LDCNT-LDCAS) 233,236,233
233 WRITE (NOUT,33)
CALL EXIT
236 CONTINUE
CALL HHRUN
GO TO 89
35 FORMAT(23H INPUT ERROR-LOAD CASES)
END

```

NOT REPRODUCIBLE

```

SUBROUTINE HHINP
COMMON NIN,NOUTP,NOUT,LDCNT,LDCAS,NORDR,Q,FIT
COMMON      A(20,20),DL(20,10),DELA(20),DELB(20),DE(20,20),
1  KR(20),VA(20),KE(20),KEP(20),
2  MR(3),MC(3),EL(3),LVA(20),LVH(20)
COMMON      PC(20),DEL A(20),THETA(20,3),TITLE(30)
COMMON      QIN(10),FITIN(10)
DO 91 J=1,20
DELA(1) = 0.0
DELB(1) = 0.0
DE(1) = 0.0
VA(1) = 0.0
KE(1) = 0.0
KEP(1) = 0.0
PC(1) = 0.0
DELTA(1) = 0.0
DO 92 K=1,3
THETA(1,K) = 0.0
DL 92 JJ = 1,10
92 DL(1,JJ) = 0.0
DO 91 J=1,20
A(1,J) = 0.0
DL(1,J) = 0.0
91 CONTINUE
C WORK AREAS ARE NOW CLEAR
READ (NIN,23) K,QIN(1),FITIN(1),(TITLE(I),I=1,30)

```



```

WRITE (NOUT,32) (TITLE(I),I=1,30)
1 = 1
93 READ (NIN,23) K,J,AKFIT
   IF(Q) 94,97,94
94 I = I + 1
   QIN(I) = Q
   IF(K) 95,95,96
95 FITIN(I) = FITIN(I-1)
   GO TO 93
96 FITIN(I) = AKFIT
   GO TO 93
97 CONTINUE
   LDCNT = I
C   READ IN MATRIX (A)
   READ (NIN,1) NROWS,NCOLS
C   PROBLEM SIZE IS SET BY THE NUMBER OF ROWS IN INPUT MATRIX (A)
   NORDR = NROWS
   WRITE (NOUT,6) NROWS,NCOLS
   WRITE (NOUT,7)
105 READ (NIN,2) (MR(I),MC(I),EL(I),I=1,3)
   IF(MR(1))130,130,110
110 DO 120 I=1,3
   IF(MR(I))120,120,115
115 NROW = MR(I)
   NCOL = MC(I)
   A(NROW,NCOL) = EL(I)
120 CONTINUE
   WRITE (NOUT,13) (MR(I),MC(I),EL(I),I=1,3)

...

GO TO 105
130 CONTINUE
   WRITE (NOUT,5)
C   READ IN MATRIX (DL)
   READ (NIN,1) NROWS,NCOLS
   WRITE (NOUT,6) NROWS,NCOLS
   WRITE (NOUT,8)
   LDCAS = NCOLS
140 READ (NIN,2) (MR(I),MC(I),EL(I),I=1,3)
   IF(MR(1))160,160,150
150 DO 170 I=1,3
   IF(MR(I))170,170,160
160 NROW = MR(I)
   NCOL = MC(I)
   DL(NROW,NCOL) = EL(I)
170 CONTINUE
   WRITE (NOUT,13) (MR(I),MC(I),EL(I),I=1,3)
   GO TO 140
180 CONTINUE
   WRITE (NOUT,5)
C   READ IN MATRIX (THETA)
   READ (NIN,1) NROWS,NCOLS
   WRITE (NOUT,6) NROWS,NCOLS

```

NOT REPRODUCIBLE

```

      WRITE (NOUT,9)
190  READ (NIN,2) (MR(I),MC(I),EL(I),I=1,3)
      IF(MR(1))230,230,200
200  DO 220 I=1,3
      IF(MR(I))220,220,210
210  NROW = MR(I)
      THETA(NROW,1) = EL(I)
      ANGLE = .0174533*EL(I)
      THETA(NROW,2) = SIN(ANGLE)
      THETA(NROW,3) = COS(ANGLE)
220  CONTINUE
      WRITE (NOUT,13) (MR(I),MC(I),EL(I),I=1,3)
      GO TO 190
230  CONTINUE
      WRITE (NOUT,5)
C    READ IN MATRIX (DELB)
      READ (NIN,1) NROWS,NCOLS
      WRITE (NOUT,6) NROWS,NCOLS
      WRITE (NOUT,10)
240  READ (NIN,2) (MR(I),MC(I),EL(I),I=1,3)
      IF(MR(1)) 280,280,250
250  DO 270 I = 1,3
      IF(MR(I)) 270,270,260
260  NROW = MR(I)
      DELB(NROW) = EL(I)
270  CONTINUE
      WRITE (NOUT,13) (MR(I),MC(I),EL(I),I=1,3)
      GO TO 240
280  CONTINUE
      WRITE (NOUT,5)
      WRITE (NOUT,5)
C    FOUR INPUT MATRICES ARE NOW IN CORE
      WRITE (NOUT,7)

      DO 290 I = 1,NORDR
      WRITE (NOUT,16) (A(I,J),J=1,NORDR)
290  CONTINUE
      WRITE (NOUT,5)
      RETURN
1  FORMAT(2I4)
2  FORMAT(3(2I4,E15.7,1X),4A2)
3  FORMAT(1H /1H )
6  FORMAT(13H INPUT MATRIX,6X,I4,2H X,15)
7  FORMAT(1H+,13X,1HA)
8  FORMAT(1H+,13X,2HDL)
9  FORMAT(1H+,13X,5HTHETA)
10 FORMAT(1H+,13X,4HDELB)
13 FORMAT(1X,3(2I4,1X,E16.7))
16 FORMAT(/1X,7E15.7/2X,7E15.7/3X,7E15.7)
23 FORMAT(11,E9.4,E10.4 30A2)
32 FORMAT(1H1,4X,30A2)
      END

```

```

SUBROUTINE MHRUN
  DIMENSION DELTH(20),BLVB(20)
  DIMENSION LOC(20),SCALE(20)
  COMMON NIN,NOUT,NOUT,LOCNT,LOCAS,NORDR,Q,FIT
  COMMON A(20,20),DL(20,10),DELA(20),DELB(20),DE(20,20),
1 PR(20),VA(20),KE(20),KEP(20),
2 MR(3),MC(3),EL(3),LVA(20),LVB(20)
  COMMON PC(20),DEL A(20),THETA(20,3),TITLE(30)
  COMMON QIN(10),FITIN(10)
C   THIS IS THE START OF THE OVERALL LOOP ON LOAD CASES
  DO 850 LOOP = 1,LOCAS
    KNT = NORDR
C   KNTB IS THE CLUE OF SIZE FROM THE SECOND ITERATION BACK
C   KNTC IS THE CLUE OF IZE FROM THE THIRD ITERATION BACK
C   BOTH USED TO CHECK FOR OSCILLATIONS IN THE SOLUTION
    KNTB = 25
    KNTC = 30
    NFNL = 0
    FIT = FITIN(LOOP)
    Q = QIN (LOOP)
    WRITE (NOUT,24) (TITLE(I),I=1,30),Q

    IF(FIT)101,103,102
101 WRITE (NOUT,25) FIT
    GO TO 104
102 WRITE (NOUT,26) FIT
    GO TO 104
103 WRITE (NOUT,27)
104 CONTINUE
C   CALCULATE MATRIX (DELA)
    DO 240 I = 1,NORDR
      DELA(I) = FIT - FIT*THETA(I,3)
240 CONTINUE
C   FORM (DL)X(L) + (DELTA A) + DELB
    DO 300 I=1,NORDR
      VA(I) = DELA(I) + DELB(I) + DL(I,LOOP)
      WRITE (NOUT,28) LOOP
      DO 305 I=1,NORDR
        WRITE (NOUT,10) I,DELA(I),DELB(I),DL(I,LOOP),VA(I)
305 KE(I) = I
      WRITE (NOUT,5)
C   THIS IS THE START OF THE MAIN LOOP
      KLOOP = 1
310 CONTINUE
C   KNT IS THE NUMBER OF NEGATIVE VALUES IN PRECEDING (PR)
C   ARRAY (KE) INDICATES THE ROWS AND COLUMNS OF (A) TO BE COMPACTED
      WRITE (NOUT,15) KLOOP,(KE(I),I=1,20),KNT
C   CONDENSE (A) ROW BY ROW

```

```

      K = 1
      DO 320 I=1,KNT
      N = KE(I)
      DO 315 J=1,NORDR
      DE(K,J) = A(N,J)*1.0E+07
315 CONTINUE

```

```

      K = K + 1
320 CONTINUE
C      NOW CONDENSE (A) COLUMN BY COLUMN
      K = 1
      DO 340 I=1,KNT
      N = KE(I)
      DO 335 J=1,KNT
      DE(J,K)=DE(J,N)
335 CONTINUE
      K = K + 1
340 CONTINUE
C      NOW INVERT THE CONDENSED ARRAY IN (DE)
      CALL LINEAR (DE,DE,K T,0,0,LOC,SCALE,20)
C      INVERSE IS IN ARRAY (DE)
      DO 345 I = 1,KNT
      IF(LOC(I))345,350,345
345 CONTINUE
      GO TO 360
350 WRITE (NDUT,11) I
      GO TO 800
360 CONTINUE
      DO 362 I = 1,KNT
      DO 362 J = 1,KNT
362 DE(I,J) = DE(I,J)*1.0E+07
      DO 368 I = 1,NORDR
363 PR(I) = 0.0
      DO 370 I = 1,KNT
      KSP(I) = 0
      DO 370 J = 1,KNT
370 PR(I) = PR(I) - DE(I,J)*VA(J)
C      NTNC IS THE COUNT OF NEGATIVE VALUES IN P
      NTNC = 0
C      NTZD IS THE COUNT OF POSITIVE OR ZERO VALUES IN DELTA,
C      WITH VALUES SMALLER THAN ONE-TENTH THE LARGEST NEGATIVE
C      VALUE ALSO SET TO ZERO
      NTZD = 0
      NPD = 0
      DELMX = 0.0
C      TEST (PR) FOR NEGATIVE VALUES
C      START 'STEP 1'
      WRITE (NDUT,14) LOOP

```

```

      DO 505 I = 1,NORDR
505 PC(I) = 0.0
      K = 0
      DO 330 I = 1,KNT
      IF (PR(I)) 375,380,380
375 NPC = KE(I)
      PC(NPC) = PR(I)
      NTPC = NTPC + 1
380 CONTINUE
381 CONTINUE
      KA = 0
C      START 'STEP 2'
      DO 514 I = 1,NORDR
      DELTA(I) = VA(I)
      DO 515 J = 1,NURDR

          DELTA(I) = DELTA(I) + A(I,J) * PC(J)
515 CONTINUE
514 CONTINUE
      DO 525 I = 1,NORDR
      IF (DELTA(I)) 518,517,517
517 DELTA(I)=0.0
      NPD = NPD + 1
518 CONTINUE
525 CONTINUE
C      START 'STEP 3'
      DO 527 I = 1,NORDR
      DELTB(I) = DELTA(I)
      IF (DELMX-DELTA(I)) 527,527,526
526 DELMX = DELTA(I)
527 CONTINUE
      DELMX = 0.05*DELMX
      DO 533 I = 1,NORDR
      IF (DELTA(I)) 529,531,530
529 IF (DELMX-DELTA(I)) 530,532,532
530 DELTB(I) = 0.0
531 NTZD = NTZD + 1
532 CONTINUE
      WRITE (NOUT,18) I,KE(I),PR(I),PC(I),DELTA(I),DELTB(I)
533 CONTINUE
C      DETERMINE SIZE FOR NEXT ITERATION
      IF (NTPC) 542,542,539
539 CONTINUE
      IF (NTPC-KNT) 540,540,541
541 CONTINUE
      IF (NTPC-NTZD) 542,542,540
540 KA = NTPC
      GO TO 544
542 KI = NTZD

```

```

544 CONTINUE
    IF(NPD-NTPC) 545,545,546
546 KA = NPD
545 CONTINUE
    K = 0
    DO 550 I=1,KA
    KEP(I)=I
550 CONTINUE
    WRITE (NOUT,5)
552 K = KA
554 CONTINUE
C    IF ORDER IS REDUCED TO ONE, THIS IS THE END
    IF(K-1)480,395,399
395 KEL = KEP(K)
    IF(NFINL) 397,397,396
396 NFINL = 0
    GO TO 460
397 CONTINUE
    RA = A(KEL,KEL)
    WRITE (NOUT,17)
    WRITE (NOUT,16) RA
    WRITE (NOUT,5)
    RAV = 1./RA

    WRITE (NOUT,17)
    WRITE (NOUT,19)
    WRITE (NOUT,16) RAV
    WRITE (NOUT,5)
    RPR = -RAV * VA(KEL)
    WRITE (NOUT,3) KEL,RPR
    NFINL = 1
    NTPC = 1
    NPD=0
    NTZD = 0
    IF(RPR) 398,400,400
398 PC(KEL) = RPR
    GO TO 381
399 CONTINUE
C    COMPARE WITH PRECEDING CYCLE
    IF(K-KNT) 580,560,580
560 CONTINUE
    DO 565 I = 1,NORDR
    IF(KE(I)-KEP(I)) 580,565,580
565 CONTINUE
    GO TO 460
580 CONTINUE
    IF(K-KNT3) 585,460,585
585 IF(K-KNTC) 590,460,590
590 CONTINUE

```

```

DO 385 I = 1,NORDR
385 KE(I) = KEP(I)
KNTC = KNTB
KNTB = KNT
KNT = K
C GO BACK FOR ANOTHER ITERATION
KLOOP = KLOOP + 1
GO TO 310
400 CONTINUE
WRITE (NOUT,4) LOOP
C STORE P(I)*SIN(THETA(I)) IN DE(I,1) (= PIX)
C STORE P(I)*COS(THETA(I)) IN DE(I,2) (= PIY)
PSUM = 0.0
DO 410 I = 1,NORDR
DE(I,1) = -PC(I)*THETA(I,2)
DE(I,2) = PC(I)*THETA(I,3)
PSUM = PSUM + DE(I,2)
410 CONTINUE
PZERO = -Q - PSUM
WRITE (NOUT,29) PZERO
WRITE (NOUT,30)
DO 420 I = 1,NORDR
WRITE (NOUT,31) I,DE(I,1),DE(I,2)
420 CONTINUE
WRITE (NOUT,5)
C WRITE PUNCH OUTPUT FOR COMAP
MR(1) = LOOP
MR(2) = LOOP
MR(3) = LOOP
NELCT = 1
MC(1) = 1

MC(2) = 2
MC(3) = 3
EL(1) = PZERO
EL(2) = DE(1,1)
EL(3) = DE(1,2)
WRITE (NOUTP,2) (MR(J),MC(J),EL(J),J=1,3),(TITLE(I),I=27,30)

DO 440 IA = 2,NORDR
DO 440 IB = 1,2
EL(NELCT) = DE(IA,IB)
MC(NELCT) = 2*IA + IB - 1
NELCT = NELCT + 1
IF (NELCT-3) 440,440,430
430 NFLCT = 1
WRITE (NOUTP,2) (MR(J),MC(J),EL(J),J=1,3),(TITLE(I),I=27,30)

```

```

      DO 439 I = 1,3
      MC(I) = 0
      EL(I) = 0.0
439 CONTINUE
440 CONTINUE
C      PUNCH THE LAST CARD, IF ANY
      IF(NELCT-1) 455,455,450
450 WRITE (NOUT,2) (MR(J),MC(J),EL(J),J=1,3),(TITLE(I),I=27,30)

455 CONTINUE
      GO TO 800
460 WRITE (NOUT,21) LOOP
      GO TO 800
480 WRITE (NOUT,20)
800 CONTINUE
850 CONTINUE
      RETURN
      2 FORMAT(3(2I4,E15.7,1X),4A2)
      3 FORMAT(1X,13,2X,E15.7)
      4 FORMAT(4HCSOL,I4/1H0
      5 FORMAT(1H /1H )
      10 FORMAT(1X,13,E16.7,3E18.7)
      11 FORMAT(23H)MATRIX SINGULAR? AT ROW,13)
      12 FORMAT(17H VECTOR DL + DELA)
      14 FORMAT(10H0LOAD CASE,13,1X,8HTRIAL PR,4X,14HVECTOR P1 (PC),
      111X,7HDELTA 1,11X,7HDELTA 2)
      15 FORMAT(34H0PCS,OR ZERO DELTA,OR NEG,P( CYCLE,13,2H ),2113)
      16 FORMAT(/1X,7E15.7/2X,7E15.7/3X,7E15.7)
      17 FORMAT(17H0MATRIX A REDUCED)
      18 FORMAT(1X,2I4,4E18.7)
      19 FORMAT(1H+,17X,12HAND INVERTED)
      20 FORMAT(7H NO NEG)
      21 FORMAT(10H0LOAD CASE,14,11H OSCILLATES?)
      24 FORMAT(1H1,4X,30A2///7H LOAD =,E16.7,7H POUNDS///)
      25 FORMAT(22H INITIAL LACK OF FIT =,E16.7,7H INCHES//)
      26 FORMAT(19H INTERFERENCE FIT =,E16.7,7H INCHES)
      27 FORMAT(12H SLIDING FIT)
      28 FORMAT(73H0ROW CALCULATED DELA      INPUT DELB      LOAD VECTOR
      1      TOTAL VVECTOR/45X,6HDL (N.,12,22H)      DELA + DELB + DL)
      29 FORMAT(9H P ZERO =,E16.7//)

      30 FORMAT(5H NODE,6X,3HPI1X,14X,3HPI1Y)
      31 FORMAT(14,2E17.7)
      END

```



```

SUBROUTINE LINEAR (A,B,MO,NO,D,LOC,SCALE,MAT)
DIMENSION A(MAT,1),B(MAT,1),LOC(1),SCALE(1)
DIMENSION SUM(1)
DOUBLE PRECISION SUM
M = MO
N = NO
INC = 0
IF (N) 10,30,30
10 IF (N + 199) 20,230,230
20 INC = 1
30 INC = INC + 1
DO 60 I = 1,M
X = 0.0
DO 50 J = 1,M
IF (X - ABS(A(J,I))) 40,50,50
40 X = ABS(A(J,I))
50 CONTINUE
X = POWER(X)
SCALE(I) = X
DO 60 J = 1,M
60 A(J,I) = A(J,I) / X
DO 210 I = 1,M
I1 = I - 1
NEXTI = I + 1
SUM(1) = -A(1,I)
CALL DOT (I1,A(1,I),MAT,A(1,I),1,SUM)
X = ABS(SUM)
Y = -SNGL(SUM)
K = I
IF (I - M) 70,110,110
70 DO 110 J = NEXTI,M
SUM(1) = -A(J,I)
CALL DOT (I1,A(J,I),MAT,A(1,I),1,SUM)
IF (X - ABS(SUM)) 80,90,90
80 A(K,I) = Y
K = J
X = ABS(SUM)
Y = -SNGL(SUM)
GO TO 100
90 A(J,I) = -SNGL(SUM)
100 CONTINUE
110 LOC(I) = K
IF (256.0 * X - ABS(A(K,I))) 120,120,150
120 IF (15777216.0 * X - ABS(A(K,I))) 130,130,140
130 LOC(I) = 0
GO TO 400
140 LOC(I) = -LOC(I)
150 A(K,I) = Y

```

```

      IF (I - K) 160,180,130
160 DO 170 J = 1,M
      X = A(I,J)
      A(I,J) = A(K,J)
170 A(K,J) = X
      D = -D
180 IF (I - M) 190,210,210

190 DO 200 J = NEXTI,M
      A(J,I) = A(J,I) / Y
      SUM(I) = -A(I,J)
      CALL DOT (I,A(I,I),MAT,A(I,J),I,SUM)
200 A(I,J) = -SNGL(SUM)
210 D = D * Y * SCALE(I)
      DO 220 I = 1,M
      DO 220 J = 1,I
220 A(J,I) = A(J,I) * SCALE(I)
      GO TO (230,460),INC
230 LIM = IABS(N)
      IF (LIM) 240,280,240
240 INC = 0
      DO 270 I = 1,M
      K = IABS(LUC(I))
      IF (I - K) 250,270,130
250 DO 260 J = 1,LIM
      X = B(I,J)
      B(I,J) = B(K,J)
260 B(K,J) = X
270 CONTINUE
280 DO 300 I = 1,M
      SCALE(I) = -A(I,I)
      A(I,I) = 1.0
      DO 290 J = 1,M
290 A(I,J) = A(I,J) / SCALE(I)
      DO 300 J = 1,I
300 A(I,J) = -A(I,J)
      DO 330 I = 2,M
      INK = I
      IF (LIM) 130,330,310
310 DO 320 J = 1,LIM
      INK = INK + INC
      SUM = B(I,J)
      LIMB = I - INK
      CALL DOT (LIMB,A(I,INK),MAT,B(INK,J),I,SUM)
320 B(I,J) = SNGL(SUM)
330 LIM = LIM + INC
      I = M
      DO 410 M)I = 1,M
      IF (LIM) 340,360,340
340 DO 350 K = 1,LIM
      SUM(I) = 0.00

```

```

      CALL DOT (MI,A(I,I),MAT,B(I,K),I,SUM)
350 B(I,K) = SNGL(SUM)
360 IF (I) 410,370,410
370 LIM = LIM - INC
      MI = M - I
      DO 400 K = I,M
      SUM(I) = 0.00
      IF (K - I) 380,380,390
380 SUM = A(I,I)
390 CALL DOT (MI,A(I,I+1),MAT,B(I+1,K),I,SUM)
400 SCALE(K) = SNGL(SUM)
      DO 405 K = I,M
405 B(I,K) = SCALE(K)

```

```

410 I = I - 1
      IF (N) 460,420,460
420 I = M
      DO 450 MII = I,M
      K = IABS(LJC(I))
      IF (I - K) 430,450,130
430 DO 440 J = I,M
      X = A(J,I)
      A(J,I) = A(J,K)
440 A(J,K) = X
450 I = I - 1
460 RETURN
      END

```

```

      SUBROUTINE DOT(L,X,IX,Y,IY,SUM)
      DOUBLE PRECISION SUM
      REAL X(IX,1),Y(1)
      IF (L) 100,120,100
100 DO 110 J = 1,L
110 SUM = SUM + X(1,J)*Y(J)
120 RETURN
      END

```

APPENDIX II EXAMPLE OF OUTPUT FROM NONLINEAR CONTACT PROGRAM

PROBLEM 4.3 CASE 2 .27 NEGATIVE FIT										P2.3 C 2		
INPUT MATRIX A										10 X 10		
1	1	0.269466E-05	1	2	0.206199E-05	1	3	0.165324E-05				
1	4	0.991567E-06	1	5	0.200355E-06	1	6	-0.627744E-06				
1	7	-0.139083E-05	1	8	-0.200050E-05	1	9	-0.239157E-05				
1	10	-0.252600E-05	0	0	-0.	-0	0	-0.				
2	1	0.206199E-05	2	2	0.382427E-05	2	3	0.287373E-05				
2	4	0.193607E-05	2	5	0.683050E-06	2	6	-0.689350E-06				
2	7	-0.198832E-05	2	8	-0.303531E-05	2	9	-0.370707E-05				
2	10	-0.343756E-05	0	0	-0.	-0	0	-0.				
3	1	0.165323E-05	3	2	0.287372E-05	3	3	0.416416E-05				
3	4	0.265755E-05	3	5	0.123529E-05	3	6	-0.400343E-06				
3	7	-0.196093E-05	3	8	-0.326850E-05	3	9	-0.409542E-05				
3	10	-0.437874E-05	0	0	-0.	-0	0	-0.				
4	1	0.991566E-06	4	2	0.193607E-05	4	3	0.265755E-05				
4	4	0.348645E-05	4	5	0.170325E-05	4	6	0.192911E-06				
4	7	-0.130715E-05	4	8	-0.253668E-05	4	9	-0.332874E-05				
4	10	-0.359577E-05	0	0	-0.	-0	0	-0.				
5	1	0.200354E-06	5	2	0.683047E-06	5	3	0.123529E-05				
5	4	0.170325E-05	5	5	0.251857E-05	5	6	0.957123E-06				
5	7	-0.982231E-07	5	8	-0.974189E-06	5	9	-0.153390E-05				
5	10	-0.172553E-05	0	0	-0.	-0	0	-0.				
6	1	-0.627743E-06	6	2	-0.689354E-06	6	3	-0.400343E-06				
6	4	0.192912E-06	6	5	0.957124E-06	6	6	0.230030E-05				
6	7	0.144873E-05	6	8	0.117560E-05	6	9	0.100492E-05				
6	10	0.951071E-06	0	0	-0.	-0	0	-0.				
7	1	-0.139083E-05	7	2	-0.198832E-05	7	3	-0.198093E-05				
7	4	-0.130715E-05	7	5	-0.982235E-07	7	6	0.144873E-05				
7	7	0.363677E-05	7	8	0.356056E-05	7	9	0.382052E-05				
7	10	0.362837E-05	0	0	-0.	-0	0	-0.				
8	1	-0.200350E-05	8	2	-0.303530E-05	8	3	-0.326850E-05				
8	4	-0.253668E-05	8	5	-0.974190E-06	8	6	0.117560E-05				
8	7	0.356056E-05	8	8	0.635254E-05	8	9	0.656058E-05				
8	10	0.692165E-05	0	0	-0.	-0	0	-0.				
9	1	-0.239157E-05	9	2	-0.370707E-05	9	3	-0.469542E-05				
9	4	-0.332874E-05	9	5	-0.153390E-05	9	6	0.100492E-05				
9	7	0.386052E-05	9	8	0.656658E-05	9	9	0.930023E-05				
9	10	0.914373E-05	0	0	-0.	-0	0	-0.				
10	1	-0.252600E-05	10	2	-0.393757E-05	10	3	-0.437874E-05				
10	4	-0.359577E-05	10	5	-0.125531E-05	10	6	0.951072E-06				
10	7	0.396283E-05	10	8	0.682165E-05	10	9	0.914373E-05				
10	10	0.117668E-04	0	0	-0.	-0	0	-0.				

INPUT MATRIX OL 10 X 8

1	1	0.2488759E-05	1	2	0.0221896E-03	1	3	0.1244379E-02
1	4	0.2483758E-02	1	5	0.4977517E-02	1	6	0.9955034E-02
1	7	0.1110941E-01	1	8	0.1244379E-01	0	0	-0.
2	1	0.3812806E-05	2	2	0.9532014E-03	2	3	0.1906403E-02
2	4	0.3812806E-02	2	5	0.7625510E-02	2	6	0.1525122E-01
2	7	0.1715762E-01	2	8	0.1906403E-01	0	0	-0.
3	1	0.4156169E-05	3	2	0.1039042E-02	3	3	0.2078084E-02
3	4	0.4156169E-02	3	5	0.8312337E-02	3	6	0.1662467E-01
3	7	0.1870276E-01	3	8	0.2078084E-01	0	0	-0.
4	1	0.3304573E-05	4	2	0.8261432E-03	4	3	0.1352287E-02
4	4	0.3304573E-02	4	5	0.6609146E-02	4	6	0.1321829E-01
4	7	0.1487058E-01	4	8	0.1652287E-01	0	0	-0.
5	1	0.1419197E-05	5	2	0.3547992E-03	5	3	0.7035986E-03
5	4	0.1419197E-02	5	5	0.2830395E-02	5	6	0.5070787E-02
5	7	0.0386368E-02	5	8	0.7095965E-02	0	0	-0.
6	1	-0.1170033E-05	6	2	-0.2925002E-03	6	3	-0.3650143E-03
6	4	-0.1170033E-02	6	5	-0.2349061E-02	6	6	-0.4688131E-02
7	1	-0.3265140E-02	7	2	-0.5850162E-02	0	0	-0.
7	4	-0.3945521E-05	7	5	-0.9913003E-03	7	3	-0.1992761E-02
7	7	-0.3965519E-02	7	8	-0.7931642E-02	7	6	-0.15778E-01
8	1	0.1784848E-01	8	2	-0.1982100E-01	0	0	-0.
8	4	-0.6417051E-05	8	5	-0.1604473E-02	8	3	-0.3208945E-02
8	7	-0.6417051E-02	8	8	-0.1283578E-01	8	6	-0.2567156E-01
9	1	-0.2308051E-01	9	2	-0.3208945E-01	0	0	-0.
9	4	-0.8074130E-09	9	5	-0.3018532E-02	9	3	-0.4037064E-02
9	7	-0.8074127E-02	9	8	-0.1614826E-01	9	6	-0.3229652E-01
10	1	-0.3633358E-01	10	2	-0.4037065E-01	0	0	-0.
10	4	-0.8657122E-05	10	5	-0.2164280E-02	10	3	-0.4378560E-02
10	7	-0.8657120E-02	10	8	-0.1731424E-01	10	6	-0.3662648E-01
10	7	-0.3895704E-01	10	8	-0.4326561E-01	0	0	-0.

NOT REPRODUCIBLE

INPUT MATRIX THETA 10 X 1

1	1	0.1800000E 02	-0	-0	-0.
2	1	0.3600000E 02	-0	-0	-0.
3	1	0.5400000E 02	-0	-0	-0.
4	1	0.7200000E 02	-0	-0	-0.
5	1	0.9000000E 02	-0	-0	-0.
6	1	0.1080000E 03	-0	-0	-0.
7	1	0.1260000E 03	-0	-0	-0.
8	1	0.1440000E 03	-0	-0	-0.
9	1	0.1620000E 03	-0	-0	-0.
10	1	0.1800000E 03	-0	-0	-0.

INPUT MATRIX DELT -0.1 -0

A

0.269460E-05 0.206163E-05 0.1033240E-05 0.9913675E-06 0.2003556E-06 -0.527745E-06 -0.1300839E-05

-0.2000503E-05 -0.239157E-05 -0.2526008E-05

0.2061944E-05 0.382471E-05 0.2873731E-05 0.1936479E-05 0.1863095E-06 -0.6893550E-06 -0.1988329E-05

-0.1015310E-05 -0.1701077E-05 -0.3593756E-05

0.1658231E-05 0.287372E-05 0.4164133E-05 0.2637554E-05 0.1235284E-05 -0.4003437E-06 -0.1980932E-05

-0.1260505E-05 -0.4095426E-05 -0.4376774E-05

0.291566E-06 0.1936876E-05 0.2637553E-05 0.348631E-05 0.1701233E-05 0.1029116E-06 -0.1307150E-05

-0.2536002E-05 -0.3324797E-05 -0.3594776E-05

0.2703547E-05 0.6856479E-06 0.1235286E-05 0.1703250E-05 0.2516570E-05 0.9371231E-06 -0.9822310E-07

-0.9741944E-06 -0.1533908E-05 -0.1725531E-05

-0.0277434E-06 -0.6893942E-06 -0.4003434E-06 0.1929122E-06 0.9571240E-06 0.2300309E-05 0.1448734E-05

0.1175006E-05 0.1071692E-05 0.9510716E-06

-0.1390037E-05 -0.198032E-05 -0.198030E-05 -0.1307157E-05 -0.1982235E-05 0.1048733E-05 0.3636777E-05

0.2500503E-05 0.3460326E-05 0.3562437E-05

-0.2000502E-05 -0.1035306E-05 -0.3208502E-05 -0.2536680E-05 -0.19741907E-06 0.1175006E-05 0.3560562E-05

0.635535E-05 0.686488E-05 0.6823854E-05

-0.391374E-05 -0.3707870E-05 -0.4095432E-05 -0.3328743E-05 -0.1933906E-05 0.1004992E-05 0.1060524E-05

0.6566580E-05 0.6300023E-05 0.914373E-05

-0.2526008E-05 -0.3937437E-05 -0.4378709E-05 -0.359973E-05 -0.172531E-05 0.9510726E-06 0.3962836E-05

0.692151E-05 0.914373E-05 0.1176886E-05

PROBLEM 2.0 CASE 2 +21 NEGATIVE FIT P2.3 C 2

LOAD = 0.1000000E 01 POUNDS

INITIAL LACK OF FIT = -0.1308000E-02 INCHES

ROW	CALCULATED DELT	INPUT DELT	LOAD VECTOR DL (N) 1	TOTAL VECTOR DELT + DELB + OL
1	-0.7341530E-04	0.	0.2488759E-05	-0.7092654E-04
2	-0.2864740E-05	0.	0.3812600E-05	-0.2826200E-05
3	-0.6193226E-05	0.	0.4156149E-05	-0.6141664E-05
4	-0.1036475E-02	0.	0.3304575E-05	-0.103317E-02
5	-0.1500001E-02	0.	0.1419197E-05	-0.1498862E-02

PGS.O.R ZERO DELTA.ON NEG.PP CYCLE 1) 1 2 3 4 5 6 7 A 9 10 0 0 0 0 0 0 0 0 0 0

LOAD CASE	INITIAL PR	VECTORS PL (PC)	DELTA 1	DELTA 2
1	0.24374132	0.	-0.7092634E-04	0.
2	0.2524691E	0.	-0.2826620E-03	-0.6241664E-02
3	0.2624066E	0.	-0.6141664E-02	-0.6241664E-02
4	0.2765368E	0.	-0.1923171E-02	-0.1933171E-02
5	0.3547688E	0.	-0.1496582E-02	-0.1498582E-02
6	0.3390201E	0.	-0.1966097E-02	-0.1964697E-02
7	0.3338007E	0.	-0.2385644E-02	-0.2385644E-02
8	0.3306480E	0.	-0.2715944E-02	-0.2715944E-02
9	0.3266746E	0.	-0.2934659E-02	-0.2934659E-02
10	0.3161604E	0.	-0.3009657E-02	-0.3009657E-02

MATRIX A REDUCED

0.2694605-05

MATRIX A REDUCED AND INVERTED

0:3711343E 06

! 0.2632115E 02

105

2 ZERO 5 -0-182308E 41

NODE	PIX	PIV
1	-0.	0.
2	-0.	0.
3	-0.	0.
4	-0.	0.
5	-0.	-0.
6	-0.	-0.
7	-0.	-0.
8	-0.	-0.
9	-0.	-0.
10	0.	-0.

PROBLEM 2.3 CASE 2 .21 NEGATIVE FIT P2.3 C 2

LOAD = 0.250000E 03 POUNDS

INITIAL LACK OF FIT = -0.1500000E-02 INCHES

ROW	CALCULATED DELTA	INPUT DELTA	LOAD VECTOR DL (N. 2)	TOTAL VECTOR DELTA + DELB + DL
1	-0.7341530E-04	0.	0.5221896E-03	0.5487743E-03
2	-0.2304749E-03	0.	0.9532014E-03	0.6667267E-03
3	-0.6133226E-03	0.	0.1039042E-02	0.4207195E-03
4	-0.1036473E-02	0.	0.8261321E-03	-0.2103321E-03
5	-0.150001E-02	0.	0.3547992E-03	-0.1145202E-02
6	-0.136327E-02	0.	-0.2925082E-03	-0.2256035E-02
7	-0.2381679E-02	0.	-0.9913603E-03	-0.3373059E-02
8	-0.2713526E-02	0.	-0.1604473E-02	-0.4317999E-02
9	-0.2926505E-02	0.	-0.2010532E-02	-0.4945118E-02
10	-0.3010000E-02	0.	-0.2164280E-02	-0.5164280E-02

POS. OR ZERO DELTA, OR NEG. PI CYCLE 1 1 2 3 4 5 6 7 8 9 10 0 0 0 0 0 0 0 0 10

LOAD CASE	2	TRIAL PR	VECTOR	PI (PC)	DELTA 1	DELTA 2
1	1	0.2812304E 03	0.	0.	0.	0.
2	2	0.3361294E 03	0.	0.	0.	0.
3	3	0.3637718E 03	0.	0.	0.	0.
4	4	0.3594925E 03	0.	0.	-0.1145202E-02	0.
5	5	0.3467671E 03	0.	0.	-0.1145202E-02	-0.1145202E-02
6	6	0.3423903E 03	0.	0.	-0.2256035E-02	-0.2256035E-02
7	7	0.3684400E 03	0.	0.	-0.3373059E-02	-0.3373059E-02
8	8	0.3932125E 03	0.	0.	-0.4317999E-02	-0.4317999E-02
9	9	0.4110511E 03	0.	0.	-0.4945118E-02	-0.4945118E-02
10	10	0.2083124E 03	0.	0.	-0.5164280E-02	-0.5164280E-02

POS. OR ZERO DELTA, OR NEG. PI CYCLE 2 1 2 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 3

LOAD CASE	2	TRIAL	PR	VECTOR	PI (PC)	DELTA 1	DELTA 2
1	1	-0.1226254E 03	-0.1226254E 03	-0.1226254E 03	-0.7670546E-04	0.	
2	2	-0.1430878E 03	-0.1430878E 03	-0.1430878E 03	-0.1333326E-03	0.	
3	3	0.4639705E 02	0.	0.	-0.1932049E-03	0.	
4	4	0.	0.	0.	-0.6084952E-03	-0.6084952E-03	
5	5	0.	0.	0.	-0.1267506E-02	-0.1267506E-02	

6	0	0.	0.	-0.2080419E-02	-0.2080419E-02
7	0	0.	0.	-0.2918002E-02	-0.2918002E-02
8	0	0.	0.	-0.3638372E-02	-0.3638372E-02
9	0	0.	0.	-0.4121413E-02	-0.4121413E-02
10	0	0.	0.	-0.4291111E-02	-0.4291111E-02

POS,OR ZERO DELTA,OR NEG,PT CYCLE 3 1 2 0 0 0 0 0 0 0 0 0 0 0 0 0 2

LOAD CASE	2 TRIAL PR	VECTOR PT 1 PCY	DELTA 1	DELTA 2
1	1	-0.119838E 03	0.	0.
2	2	-0.1098629E 03	0.	0.
3	3	0.	-0.9265717E-04	0.
4	4	0.	-0.5416104E-03	-0.5416103E-03
5	5	0.	-0.1244203E-02	-0.1244203E-02
6	6	0.	-0.2105237E-02	-0.2105237E-02
7	7	0.	-0.2988294E-02	-0.2988294E-02
8	8	0.	-0.3745304E-02	-0.3745304E-02
9	9	0.	-0.4251854E-02	-0.4251854E-02
10	10	0.	-0.4429619E-02	-0.4429619E-02

SOL 2

P ZERO = -0.4238808E 02

NODE	PIX	PLX
1	0.3695345E 02	-0.1137310E 03
2	0.6557582E 02	-0.8888093E 02
3	0.	0.
4	0.	0.
5	0.	0.
6	0.	0.
7	0.	0.
8	0.	0.
9	0.	0.
10	0.	0.

PROBLEM 2.3 CASE 2 .21 NEGATIVE FIN P2.3 C 2

LOAD = 0.5000000E 03 POUNDS

INITIAL LACK OF FIT = -0.1500000E-02 INCHES

ROW	CALCULATED DELTA	INPUT DELTA	LOAD VECTOR OL (IN. 3)	TOTAL VECTOR DELTA + DELTA + OL
1	-0.7341530E-04	0.	0.1244379E-02	0.1170904E-02
2	-0.22865748E-03	0.	0.1902403E-02	0.1619928E-02
3	-0.6183226E-03	0.	0.2078084E-02	0.1459762E-02
4	-0.1030475E-02	0.	0.1852287E-02	0.6158113E-03
5	-0.1380001E-02	0.	0.7095986E-03	-0.7904023E-03
6	-0.1963527E-02	0.	-0.5850163E-03	-0.2548543E-02
7	-0.2331679E-02	0.	-0.1982761E-02	-0.4364439E-02
8	-0.2713526E-02	0.	-0.3208045E-02	-0.5922472E-02
9	-0.2926585E-02	0.	-0.4037064E-02	-0.6963649E-02
10	-0.3000000E-02	0.	-0.5328550E-02	-0.7328560E-02

POS. OR ZERO DELTA, OR NEG. PT CYCLE 1 1 2 3 4 5 6 7 8 9 10 0 0 0 0 0 0 0 0 0 0 0 0

LOAD CASE	3	TRIAL PR	VECTOR PT (R2)	DELTA 1	DELTA 2
1	1	0.2686693E 03	0.	0.	0.
2	2	0.3186698E 03	0.	0.	0.
3	3	0.346579E 03	0.	0.	0.
4	4	0.3423274E 03	0.	0.	0.
5	5	0.3387352E 03	0.	-0.7904023E-03	-0.7904023E-03
6	6	0.3577983E 03	0.	-0.2548543E-02	-0.2548543E-02
7	7	0.4632305E 03	0.	-0.4364439E-02	-0.4364439E-02
8	8	0.4560279E 03	0.	-0.5922472E-02	-0.5922472E-02
9	9	0.4960540E 03	0.	-0.6963649E-02	-0.6963649E-02
10	10	0.2550034E 03	0.	-0.7328560E-02	-0.7328560E-02

POS. OR ZERO DELTA, OR NEG. PT CYCLE 2 1 1 2 3 4 5 6 7 8 9 10 0 0 0 0 0 0 0 0 0 0 0 0

LOAD CASE	3	TRIAL PR	VECTOR PT (R2)	DELTA 1	DELTA 2
1	1	-0.1671922E 03	-0.1671922E 03	-0.1836764E-03	0.
2	2	-0.2593273E 03	-0.2593273E 03	-0.3586331E-03	-0.3586331E-03
3	3	-0.2234298E 03	-0.2234298E 03	-0.4922812E-03	-0.4922812E-03
4	4	-0.1852386E 03	0.	-0.6458248E-03	-0.6458248E-03
5	5	0.	0.	-0.1277035E-02	-0.1277035E-02
6	6	0.	0.	-0.2175372E-02	-0.2175372E-02
7	7	0.	0.	-0.3173676E-02	-0.3173676E-02
8	8	0.	0.	-0.4070585E-02	-0.4070585E-02
9	9	0.	0.	-0.4687412E-02	-0.4687412E-02
10	10	0.	0.	-0.4906768E-02	-0.4906768E-02

POS. OR ZERO DELTA, OR NEG. PT CYCLE 3 1 1 2 3 4 5 6 7 8 9 10 0 0 0 0 0 0 0 0 0 0 0 0

LOAD CASE 3 TRIAL PR VECTOR PT (PC) DELTA 1 DELTA 2

1	1	-0.160482E 03	-0.180648E 03	0.	0.
2	2	-0.2423254E 03	-0.2423254E 03	0.	0.
3	3	-0.1110122E 03	-0.1116020E 04	0.	0.
4	4	0.	0.	-0.3290826E-03	-0.3290826E-03
5	5	0.	0.	-0.1129978E-02	-0.1129978E-02
6	6	0.	0.	-0.2223315E-02	2223315E-02
7	7	0.	0.	-0.3410209E-02	3410209E-02
8	8	0.	0.	-0.4460781E-02	-0.4460781E-02
9	9	0.	0.	-0.5176240E-02	-0.5176240E-02
10	10	0.	0.	-0.5629391E-02	-0.5629391E-02

SOL 3

P ZERO = -0.6654997E 02

NODE PIX PIY

1	0.5582337E 02	-0.1718066E 03
2	0.1424353E 03	-0.1960453E 03
3	0.9028812E 02	-0.6559810E 02
4	-0.	0.
5	-0.	-0.
6	-0.	-0.
7	-0.	-0.
8	-0.	-0.
9	-0.	-0.
10	0.	-0.

ROULEN 2.3 CASE 2 .21 NEGATIVE FTY P2.3 C 2

LOAD = 0.1000000E 04 POUNDS

INITIAL LACK OF FIT = -0.1500000E-02 INCHES

ROW	CALCULATED DELA	INPUT DELB	LOAD VECTOR DL (N, 4)	TOTAL VECTOR DELA + DELB + DL
1	-0.7341530E-04	0.	0.2488758E-02	0.2415343E-02
2	-0.2064749E-03	0.	0.3812806E-02	0.3526331E-02
3	-0.6183226E-03	0.	0.4156169E-02	0.3537846E-02
4	-0.1836475E-02	0.	0.3304573E-02	0.2268098E-02
5	-0.150091E-02	0.	0.1419197E-02	-0.8080369E-04
6	-0.103327E-02	0.	-0.1170033E-02	-0.3133559E-02
7	-0.2381679E-02	0.	-0.3965519E-02	-0.6347198E-02

8	-0.2713524E-02	0.	-0.6417889E-02	-0.9131415E-02
9	-0.2420585E-02	0.	-0.8074127E-02	-0.1100071E-01
10	-0.3166700E-02	0.	-0.8657120E-02	-0.1165712E-01

POSITION ZERO DELTA TOR NEG. PI (PC) CYCLE 1 2 3 4 5 6 7 8 9 10 0 0 0 0 0 0 0 0 10

LOAD CASE	TRIAL PR	VECTOR PI (PC)	DELTA 1	DELTA 2
1	0.2435470E 03	0.	0.	0.
2	0.2638407E 03	0.	0.	0.
3	0.3046300E 03	0.	0.	0.
4	0.3079971E 03	0.	0.	0.
5	0.3225715E 03	0.	-0.8080369E-04	0.
6	0.3766149E 03	0.	-0.3133559E-02	-0.3733559E-02
7	0.4727985E 03	0.	-0.6347198E-02	-0.6347198E-02
8	0.5816574E 03	0.	-0.9131415E-02	-0.9131415E-02
9	0.6666611E 03	0.	-0.1100071E-01	-0.1100071E-01
10	0.7483853E 03	0.	-0.1165712E-01	-0.1165712E-01

POSITION ZERO DELTA TOR NEG. PI (PC) CYCLE 2 1 2 3 4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 4

LOAD CASE	TRIAL PR	VECTOR PI (PC)	DELTA 1	DELTA 2
1	-0.2960439E 03	-0.2960439E 03	-0.8868687E-05	0.
2	-0.4416215E 03	-0.4416215E 03	-0.1737658E-04	0.
3	-0.433004E 03	-0.433004E 03	-0.23 964E-04	0.
4	0.3994199E 01	0.	-0.37.0347E-04	0.
5	0.	0.	-0.96E-03	-0.9766490E-03
6	0.	0.	-0.37E-02	-0.2469937E-02
7	0.	0.	-0.4199618E-02	-0.4199618E-02
8	0.	0.	-0.5783460E-02	-0.5783460E-02
9	0.	0.	-0.682259E-02	-0.682259E-02
10	0.	0.	-0.7274393E-02	-0.7274393E-02

POSITION ZERO DELTA TOR NEG. PI (PC) CYCLE 3 1 2 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 3

LOAD CASE	TRIAL PR	VECTOR PI (PC)	DELTA 1	DELTA 2
1	-0.2960439E 03	-0.2960439E 03	0.	0.
2	-0.4416215E 03	-0.4416215E 03	-0.1455192E-10	0.
3	-0.433004E 03	-0.433004E 03	-0.165712E-11	0.
4	0.	0.	-0.1588877E-04	0.
5	0.	0.	-0.9693490E-03	-0.9693490E-03
6	0.	0.	-0.2472257E-02	-0.2472257E-02
7	0.	0.	-0.4211043E-02	-0.4211043E-02
8	0.	0.	-0.5302301E-02	-0.5302301E-02
9	0.	0.	-0.6805862E-02	-0.6805862E-02
10	0.	0.	-0.7274393E-02	-0.7274393E-02

SOL 4

M ZERO = -0.1098752E 03

NODE	PIX	PIV
1	0.9163342E 02	-0.2821724E 03
2	0.2590662E 03	-0.3566151E 03
3	0.3450364E 03	-0.2513373E 03
4	0.	0.
5	0.	0.
6	0.	0.
7	0.	0.
8	0.	0.
9	0.	0.
10	0.	0.

PROBLEM 2.3 CASE 2 .21 NEGATIVE FLY P2.3 C 2

LOAD = 0.2000000E 04 POUNDS

INITIAL LACK OF FIT = -0.1500000E-02 INCHES

ROW	CALCULATED DELTA	INPUT DELTA	LOAD VECTOR DL (IN. SI)	TOTAL VECTOR DELTA + DELT * DL
1	-0.7341520E-04	0.	0.4977517E-02	0.4977517E-02
2	-0.2067474E-03	0.	0.7625610E-02	0.7339135E-02
3	-0.0103246E-03	0.	0.8312337E-02	0.7694014E-02
4	-0.1036475E-02	0.	0.6609146E-02	0.5572670E-02
5	-0.1500001E-02	0.	0.2838395E-02	0.138394E-02
6	-0.1203327E-02	0.	-0.2340666E-02	-0.4303593E-02
7	-0.3081670E-02	0.	-0.7931042E-02	-0.1031272E-01
8	-0.2713576E-02	0.	-0.1281570E-01	-0.1554931E-01
9	-0.2926505E-02	0.	-0.1614826E-01	-0.1907444E-01
10	-0.2000000E-02	0.	-0.1731424E-01	-0.2031424E-01

POSITIVE ZERO DELTATION NEGATIVE CYCLE 1 1 2 3 4 5 6 7 8 9 10 0 0 0 0 0 0 0 0 0 0 10

LOAD CASE	S TOTAL PR	VECTOR PI (PC)	DELTA 1	DELTA 2
1	0.1933019E 03	0.	0.	0.
2	0.2141256E 03	0.	0.	0.
3	0.2257736E 03	0.	0.	0.
4	0.2335380E 03	0.	0.	0.
5	0.2405436E 03	0.	0.	0.

POSITIVE ZERO DELTA OR NEGATIVE

LOAD CASE	5	TOTAL PW	VECTOR	FI (PK)	DELTA 1	DELTA 2
1	1	-0.4994059E 03	-0.4994059E 03	-0.117126E-03	0.	0.
2	2	-0.7630051E 03	-0.7630051E 03	-0.3902854E-03	0.	0.
3	3	-0.8859670E 03	-0.8859670E 03	-0.7221094E-03	0.	0.
4	4	-0.6428667E 03	-0.6428667E 03	-0.5956577E-03	-0.7221094E-03	-0.7221094E-03
5	5	-0.584625E 03	-0.584625E 03	-0.1472265E-02	-0.1472265E-02	-0.1472265E-02
6	6	0.	0.	-0.323344E-02	-0.323344E-02	-0.323344E-02
7	7	0.	0.	-0.5505633E-02	-0.5505633E-02	-0.5505633E-02
8	8	0.	0.	-0.7707709E-02	-0.7707709E-02	-0.7707709E-02
9	9	0.	0.	-0.9233548E-02	-0.9233548E-02	-0.9233548E-02
10	10	0.	0.	-0.9854670E-02	-0.9854670E-02	-0.9854670E-02

[illegible]

LEAD CASE	5	TOTAL FR	VECTOR OF FR	DELTA 1	DELTA 2
1	1	-0.5537403E 03	-0.55527483E 03	-0.1818900E-10	0.
2	2	-0.8062881E 03	-0.808605081E 03	-0.2910303E-10	0.
3	3	-0.8510129E 03	-0.8533399E 03	-0.1010036E-09	0.
4	4	-0.5823263E 03	-0.58433643E 03	-0.5820760E-10	0.
5	0	0.	0.	-0.9611905E-03	-0.9611905
6	0	0.	0.	-0.3125361E-02	3125361
7	0	0.	0.	-0.5802311E-02	5802311
8	0	0.	0.	-0.837501E-02	-0.837501
9	0	0.	0.	-0.1012205E-01	-0.1012805
10	0	0.	0.	-0.1077260E-01	-0.1077260

5 JUL 5

P ZERO = -0.2140499E 03

MODE	PIX	PIV
1	-0.771177E 03	-0.526659E 03
2	-6.473777E 03	-0.652259E 03
3	-0.660398E 03	-0.500876E 03
4	-0.258244E 03	-0.1061915E 03

5 -0.
6 -0.
7 -0.
8 -0.
9 -0.
10 -0.

PROBLEM 2.3 CASE 2 .21 NEGATIVE F1/ P2.5 C 2

LOAD = 0.000000E 04 POUNDS

INITIAL LACK OF FIT = -0.150000E-02 INCHES

ROW	CALCULATED DELTA	INPUT DELB	LOAD VECTOR DL (N/6)	TOTAL VECTOR DELA + DELB + DL
1	-0.7341510E-02	0.	0.9955034E-02	0.9881618E-02
2	-0.2003474E-02	0.	0.1525122E-01	0.1496475E-01
3	-0.0185226E-03	0.	0.1662467E-01	0.1600635E-01
4	-0.1036475E-02	0.	0.1321929E-01	0.1218182E-01
5	-0.1500001E-02	0.	0.5076787E-02	0.4176786E-02
6	-0.1065275E-02	0.	-0.06400131E-02	-0.0643657E-02
7	-0.2381674E-02	0.	-0.1586208E-01	-0.1824376E-01
8	-0.2713526E-02	0.	-0.2567156E-01	-0.2838509E-01
9	-0.2027559E-02	0.	-0.3224652E-01	-0.3522310E-01
10	-0.3000000E-02	0.	-0.3462448E-01	-0.3762448E-01

POSITION ZERO DELTA, IN NEG. PI CYCLE 1 2 3 4 5 6 7 8 9 10 0 0 0 0 0 0 0 0 0 0

LOAD CASE	INITIAL PR	VECTOR PI (PC)	DELTA 1	DELTA 2
1	0.9281323E 02	0.	0.	0.
2	0.7464859E 02	0.	0.	0.
3	0.6406289E 02	0.	0.	0.
4	0.1020185E 03	0.	0.	0.
5	0.2262003E 03	0.	0.	0.
6	0.4483084E 03	0.	-0.6663657E-02	-0.6643657E-02
7	0.8002127E 03	0.	-0.1924376E-01	-0.1824376E-01
8	0.1333437E 04	0.	-0.2838509E-01	-0.2838509E-01
9	0.1606107E 04	0.	-0.3522310E-01	-0.3522310E-01
10	0.9096737E 03	0.	-0.3762448E-01	-0.3762448E-01

POSITION ZERO DELTA, IN NEG. PI CYCLE 2 1 2 3 4 5 6 0 0 0 0 0 0 0 0 0 0 0 0 0 0

LOAD CASE	6	TRIAL PH	VECTOR PI (PC)	DELTA 1	DELTA 2
1	1	-0.101483E 04	-0.101483E 04	-0.1170661E-03	0.
2	2	-0.1492203E 04	-0.1492203E 04	-0.3091001E-03	0.
3	3	-0.1724234E 04	-0.1724234E 04	-0.7217734E-03	0.
4	4	-0.1347923E 04	-0.1347923E 04	-0.9951941E-03	-0.9951941E-03
5	5	0.5842902E 03	0.	-0.1471570E-02	-0.1471570E-02
6	6	0.	0.	-0.4547688E-02	-0.4547688E-02
7	7	0.	0.	-0.8687769E-02	-0.8687769E-02
8	8	0.	0.	-0.1277070E-01	-0.1277070E-01
9	9	0.	0.	-0.1571599E-01	-0.1571599E-01
10	10	0.	0.	-0.1678712E-01	-0.1678712E-01

POS. OR NEG. DELTA. OR NEG. CYCLE 3 1 2 3 4 0 0 0 0 0 0 0 0 0 0 0 4

LOAD CASE	6	TRIAL PH	VECTOR PI (PC)	DELTA 1	DELTA 2
1	1	-0.1069155E 04	-0.1069155E 04	0.	0.
2	2	-0.1535306E 04	-0.1535306E 04	0.	0.
3	3	-0.1610425E 04	-0.1610425E 04	0.	0.
4	4	-0.1048819E 04	-0.1048819E 04	0.	0.
5	5	0.	0.	-0.9607425E-03	-0.9607425E-03
6	6	0.	0.	-0.4439656E-02	-0.4439656E-02
7	7	0.	0.	-0.8984310E-02	-0.8984310E-02
8	8	0.	0.	-0.1340020E-01	-0.1340020E-01
9	9	0.	0.	-0.1656010E-01	-0.1656010E-01
10	10	0.	0.	-0.1773363E-01	-0.1773363E-01

SOL 6

P ZERO = -0.4233114E 03

NODE	PIX	PIV
1	0.3303672E 03	-0.1016827E 04
2	0.9024774E 03	-0.1242153E 04
3	0.1367563E 04	-0.9936065E 03
4	0.9974867E 03	-0.3241025E 03
5	-0.	-0.
6	-0.	-0.
7	-0.	-0.
8	-0.	-0.
9	-0.	-0.
10	0.	-0.

MUS. OR ZERO DELTA OR NEG. PL CYCLE 3 1 2 3 4

LOAD CASE	7	TRIAL PR	VECTOR PT (PC)	DELTA 1	DELTA 2

	SECTION 11 (PC)	DELTA 1	DELTA 2
1	-0.1198006E 04	-0.2910383E-10	0.
2	-0.1717680E 04	0.	0.
3	-0.1629495E 04	-0.0910383E-10	0.
4	-0.1225115E 04	0.	0.
5	0.	-0.9606206E-03	0.
6	0.	-0.4768231E-02	4768231E-02
7	0.	-0.9779809E-02	9779809E-02
8	0.	-0.1466587E-01	-0.1466587E-01
9	0.	-0.1816911E-01	-0.1816911E-01
10	0.	-0.1943763E-01	-0.1943763E-01

705

P ZERU = -0.4735273E 03

NCDE PIX PLY

1	0.370209E 03	-0.1139371E 04
2	0.1009627E 04	-0.1389632E 04
3	0.1537128E 04	-0.1116788E 04
4	0.115154E 04	-0.3785808E 03
5	-0.	-0.
6	-0.	-0.
7	-0.	-0.
8	-0.	-0.
9	-0.	-0.
10	0.	-0.

PROBLEM 2.3 CASE 2 .2(NEGATIVE FIT

.QAD = 0.500000E 04 POUNDS

INITIAL LACK OF FIT = -0.150200E-02 INCHES

ROW	CALCULATED DELTA	INPUT DELTA	LOAD VECTOR DL (N, S)	TOTAL VECTOR DELTA + DEL0 + DL
1	-0.7341530E-04	0.	0.1244379E-01	0.1237038E-01
2	-0.2567748E-03	0.	0.19076403E-01	0.1877756E-01
3	-0.5181226E-03	0.	0.2078084E-01	0.2016252E-01
4	-0.1016475E-02	0.	0.182287E-01	0.1548639E-01
5	-0.1507101E-02	0.	0.7093985E-02	0.5595984E-02

8	0	0.	0.	-0.1593155E-01	-0.1593155E-01
9	0	0.	0.	-0.1977612E-01	-0.1977612E-01
10	0	0.	0.	-0.2117064E-01	-0.2117064E-01

SOL R

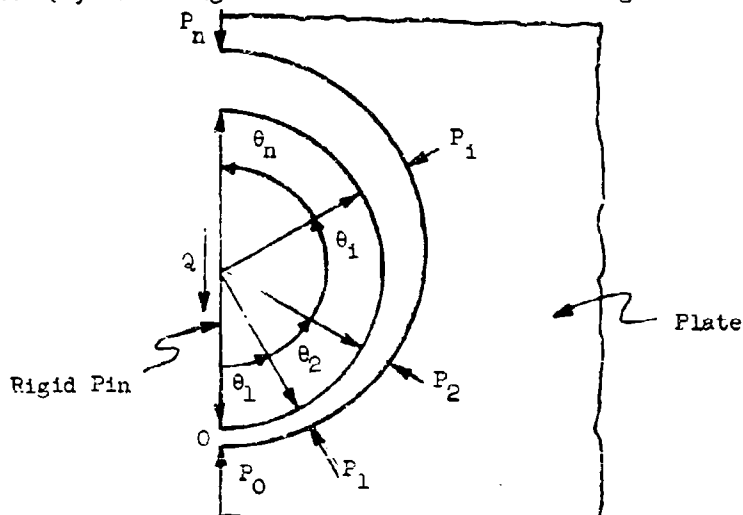
P ZERO = -0.5279421E 03

NODE	PIX	PIV
1	0.4100217E 03	-0.1261916E 04
2	0.1116778E 04	-0.1537112E 04
3	0.1706675E 04	-0.1239971E 04
4	0.1332819E 04	-0.4330583E 03
5	-0.	-0.
6	-0.	-0.
7	-0.	-0.
8	-0.	-0.
9	-0.	-0.
10	0.	-0.

APPENDIX I CONTACT PROBLEM INPUT MATRICES

In Chapter 3 the method used to solve the contact problem was briefly described. This Appendix gives the details for setting up the matrices needed as input for the analysis. The present study uses the Grumman matrix structural analysis program called COMAP-ASTRAL but any elastic analysis computer program that has similar planar finite elements as those described in Section 2.3.1 could be used.

The first matrix to be generated is the matrix of influence coefficients, $[A]$ which gives the radial displacements around the hole for unit redundants $\{P\}$ following the notation shown in the figure below.



The relation of $\{P\}$ to the radial displacements in the plate, $\{\delta_r\}$ is of the form:

$$\{P\} = [A] \{\delta_r\} \quad (I.1)$$

where,

$$\{P\} = \begin{Bmatrix} P_0 \\ P_1 \\ \vdots \\ P_n \end{Bmatrix}$$

We can rewrite Eq. I.1 by eliminating one of the redundants from the statically determinate equilibrium condition which is:

$$P_0 = [-\cos\theta_1 \quad -\cos\theta_2 \quad \dots \quad -\cos\theta_n \quad -1] \{P'\} \quad (I.2)$$

$$\text{where, } \{P'\} = \begin{Bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \\ Q \end{Bmatrix}$$

Thus the new redundants are related to the old by the expression below:

$$\begin{Bmatrix} P_0 \\ P_1 \\ \vdots \\ P_n \end{Bmatrix} = \begin{bmatrix} -\cos\theta_1 & -\cos\theta_2 & \dots & -1 \\ 1 & & & 0 \\ \vdots & & & \vdots \\ 0 & & & 0 \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \\ Q \end{Bmatrix} \quad (I.3)$$

= $[\alpha] \{P'\}$, where $[\alpha]$ is a transformation matrix.

Once the order of the redundants in the compatibility equation (Eq. 10) are changed the radial displacements must be reordered. Let $\{\Delta_r\}$ be the relative radial displacements between the fastener and the plate. The relation between $\{\Delta_r\}$ and $\{\delta_r\}$ becomes:

$$\begin{Bmatrix} \Delta_{r1} \\ \Delta_{r2} \\ \vdots \\ \Delta_{rn} \\ \Delta_{rQ} \end{Bmatrix} = \begin{bmatrix} -\cos\theta_1 & 1 & \dots & 0 \\ -\cos\theta_2 & 0 & 1 & \dots & 0 \\ \vdots & & & \vdots \\ 0 & & & 0 \end{bmatrix} \begin{Bmatrix} \delta_{r0} \\ \delta_{r1} \\ \vdots \\ \delta_{rn} \end{Bmatrix} \quad (I.4)$$

= $[\beta] \{\delta_r\}$, where $[\beta]$ is a transformation matrix.

Note that the relative radial displacement $\Delta_{r0} = 0$ so that Δ_{rQ} or any point on the rod fastener has zero relative displacement to point 0.

The expression for relative radial displacements due to the action of of the redundant $\{P'\}$ therefore becomes:

$$\{\Delta_r\} = [\beta] [A] [\alpha] \{P'\} \quad (1.5)$$

APPENDIX J NONLINEAR ANALYSIS OF A SINGLE FASTENER - THREE-DIMENSIONAL EFFECTS

Starting with a load deflection curve (Figure J-1) for a single rigid fastener, (Figure J-2) as determined by finite-element methods described earlier in Chapter 2 for which the detailed sheet stresses are known (e.g. Figure J-3) for each load level, a "secant spring modulus" \bar{k}_s is available. If one isolates the fastener and replaces the surrounding plate material sheets by a continuous series of non-linear transverse springs, the problem becomes quite tractable. It is emphasized here that the engineering assumption made is essentially the same as is made in "beam on elastic foundation" theory, i.e., the shear effects through the sheet thickness play only a secondary role in the load distribution and can be ignored for the present.

Continuing along these lines, Figure J-4 shows the idealization of a typical fastener in single shear. Denoting the fastener deflection adjacent to the i^{th} plate by y_i ($i=1$ and 2 for a fastener in single shear; $i=1, 2$ and 3 for a fastener in double shear, etc.) and Y_1 as the plate deflection at some reference section, it becomes possible to relate $Y_1 - y_1$ to the local secant spring stiffness:

$$q_i = \bar{k}_i (Y_1 - y_i) \quad (J-1)$$

where \bar{k}_i and y_i may vary through the plate thickness x_i .

If the reference section is sufficiently far (i.e., several hole diameters) from the fastener, it can be assumed to remain plane, leading to the following:

$$q_i = \bar{k}_i (a_i + b_i x_i - y_i) \quad (J-2)$$

where a_1 and b_1 are constants to be evaluated such that

$$P_1 = \int_0^{h_1} q_1 dx_1 \quad (J-3)$$

$$M_1 = \int_0^{h_1} q_1 \left(\frac{h_1}{2} - x_1 \right) dx_1 \quad (J-4)$$

where P_1 and M_1 are the net force and moment at the plate reference sections.

Since the problem may be nonlinear, a step-by-step linearized numerical solution technique is proposed wherein P_1 and M_1 are built up simultaneously in sufficiently small steps (ΔP_1 , ΔM_1), such that \bar{k}_1 and \bar{v}_1 may be approximated in the integrands of Equations (J-3) and (J-4) by values which correspond to the previous step. In this way a_1 and b_1 may be evaluated in terms of \bar{p}_1 and \bar{v}_1 from Equations (J-5) and (J-6):

$$\begin{bmatrix} a_{111} & a_{121} \\ a_{211} & a_{221} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} P_1 + \bar{p}_1 \\ M_1 + \bar{v}_1 \end{bmatrix} \quad (J-5)$$

where

$$\begin{aligned} a_{111} &= \int_0^{h_1} E_1 dx_1 \\ a_{121} &= \int_0^{h_1} x_1 \bar{k}_1 dx_1 \\ a_{211} &= \frac{h_1}{2} a_{111} - a_{121} \\ a_{221} &= \frac{h_1}{2} a_{121} - \int_0^{h_1} x_1^2 \bar{k}_1 dx_1 \end{aligned} \quad (J-6)$$

$$\begin{aligned} \beta_1 &= \int_0^{h_1} y_1 \bar{k}_1 dx_1 \\ \gamma_1 &= \frac{h_1}{2} \beta_1 - \int_0^{h_1} x_1 y_1 \bar{k}_1 dx_1 \end{aligned}$$

Thus q_1 , as a function of x_1 , is known after each increment in the loading.

To solve for the y_1 , a short thick-beam theory, such as Timoshenko's (Reference 12), appears suitable since it accounts for shear deflection. For simplicity's sake, it is assumed that the fastener-head effects may be approximated by a rotational spring and edge shear resistance to simulate head friction.

Referring to Section 4.3, the beam-segment equations, corresponding to each plate, are:

$$\frac{d}{dx_1} (EI_1 \frac{dy_1}{dx_1}) + \lambda GA_1 (\frac{dy_1}{dx_1} - \gamma_1) = 0 \quad (J-7)$$

and

$$\frac{d}{dx_1} (A_1 [\frac{dy_1}{dx_1} - \gamma_1]) = - \frac{q_1}{\lambda G} \quad (J-8)$$

where γ_1 is the cross-sectional beam rotation under the usual assumption that plane sections remain plane, A_1 is the cross-sectional area, E and G are the material stiffnesses, I_1 the bending moment of inertia, and λ is a factor which depends on the geometry of the beam cross-section.

The interface boundary conditions are

$$\begin{aligned} y_1 &= -y_{1+1} \quad , \quad \gamma_1 = \gamma_{1+1} \\ \frac{dy_1}{dx_1} &= -\frac{dy_{1+1}}{dx_{1+1}} \quad , \quad \frac{dy_1}{dx_1} - \gamma_1 = \frac{dy_{1+1}}{dx_{1+1}} - \gamma_{1+1} \end{aligned} \quad (J-9)$$

and the boundary conditions at the fastener heads can be written as

$$C_{11} EI \frac{d^2 y_1}{dx_1^2} + C_{21} y_1 = 0$$

$$\lambda A_1 G \left(\frac{dy_1}{dx_1} - y_1 \right) = F \quad (J-10)$$

where the C's are adjusted to account for head effects and F is the shear load between fastener and head. For example, if the head rotational stiffness is a constant, say S, $C_{21} = S$ and $C_{11} = 1$ regardless of the level of P_1 and M_1 . In general, these C's as well as F, must be approximated by engineering means or be empirically adjusted to permit correlation with test data.

Equations (J-7) through (J-10) may be solved numerically by finite difference techniques. However, because of the nature of the assumptions and degree of approximation to this point, an analytical solution is easily available if the coefficients of the differential equation are assumed constant, i.e., A_1 and I_1 are assumed constant. Thus, the y_1 are evaluated as functions of x_1 for each step in the P_1 , M_1 iteration process.

Once a solution for q_1 as a function of the plate thickness is obtained in terms of the final P_1, M_1 combination, the stresses through the sheets near the fastener holes may be determined by reference to the previous two-dimensional stress solution at each thickness level. Furthermore, an effective fastener spring rate, including fastener shear and rotation, may be defined in terms of relative deflections on a given fastener.

Linearization: As a first step to the overall problem, the equations may be greatly simplified if, as appears justified from Chapter 3,

the elastic foundation springs, corresponding to each plate, are assumed to have constant rates. Thus, the first three integrations required by Equations (J-6) may be performed explicitly once and for all, and a_1 and b_1 become uniquely determined function of integrals in y_1 . Substituting these into Equation (J-2) and thence into Equation (J-8) yields

$$\frac{d}{dx_1} (A_1 \left[\frac{dy_1}{dx_1} - \psi_1 \right]) + \frac{k_1}{\lambda G} y_1 = \frac{k_1 a_1}{\lambda G} + \frac{k_1 b_1}{\lambda G} x_1 \quad (J-11)$$

where Equation (J-11) will involve integrals of y_1 .

For constant area, A_1 , and bending inertia, I_1 , it becomes a simple matter to obtain closed-form analytic solutions to Equations (J-7) and (J-11) subject to linear boundary Equations (J-10). In this way, once the y_1 are known the stress variation through the thickness, q_1 , may be computed.

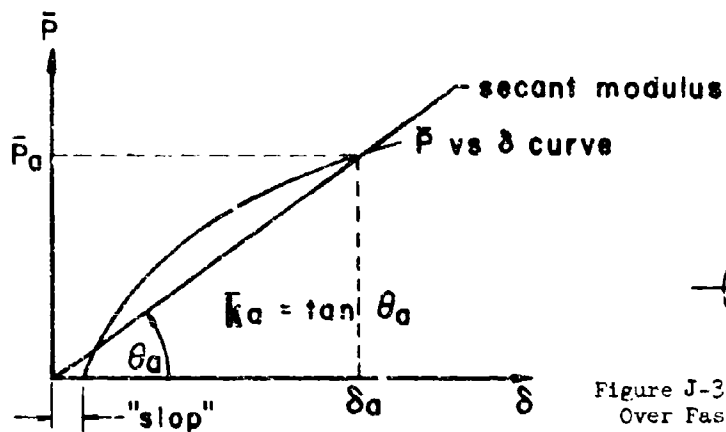


Figure J-1 - Load Deflection Curve for A Typical Fastener. Includes Local Planar Effects Only.

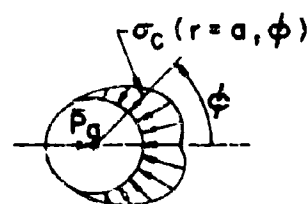


Figure J-3 - Typical Compressive Stresses Over Fastener-Plate Contact Region. Includes Planar Effects Only.

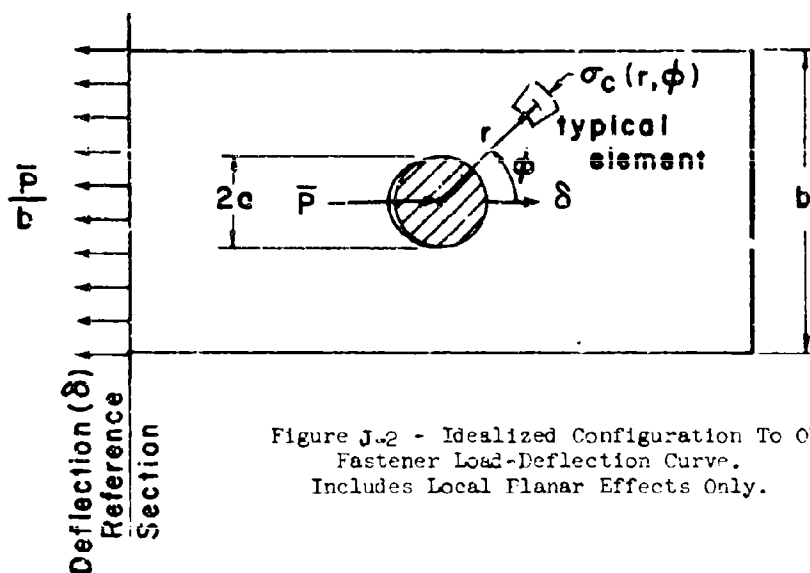
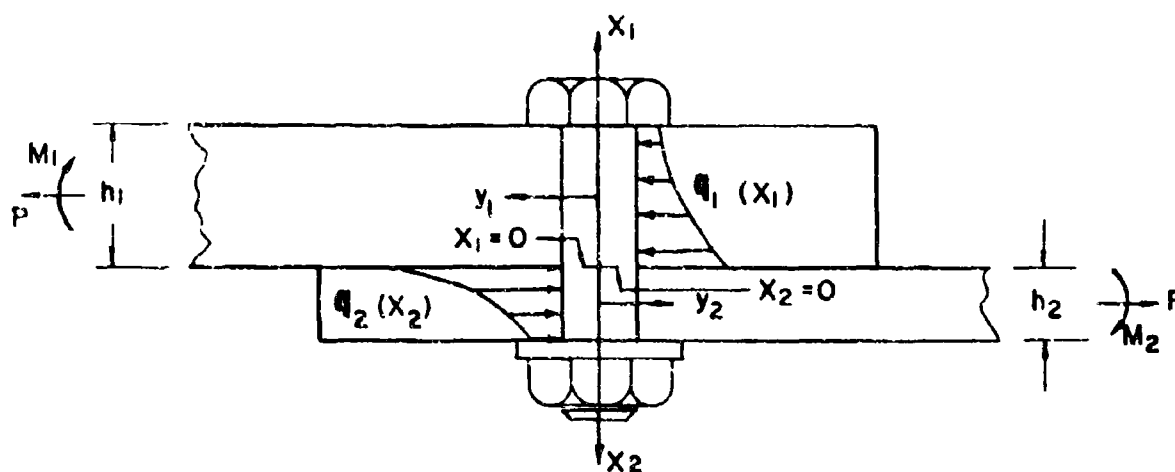
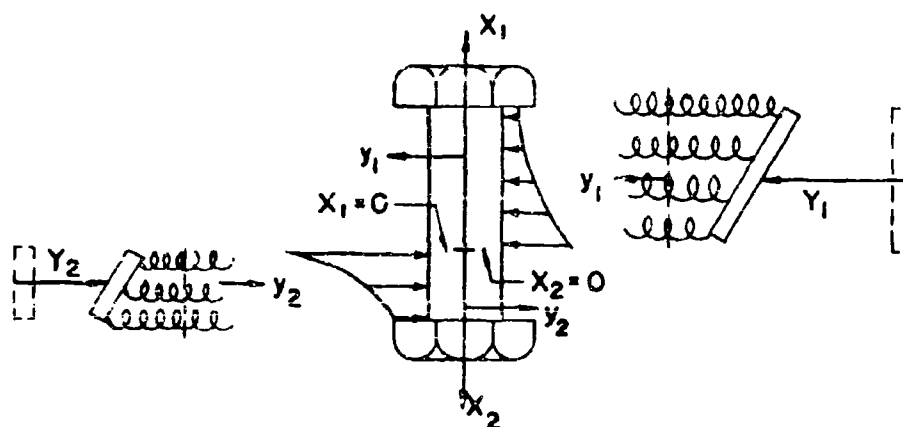


Figure J-2 - Idealized Configuration To Obtain Fastener Load-Deflection Curve. Includes Local Planar Effects Only.

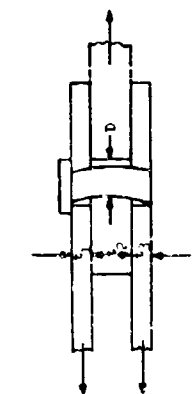


(a) Plate-Fastener Load Interaction Through Plate Thickness



(b) Idealized Model of Plate as a Nonlinear Elastic Foundation Interacting With a Short Beam

Figure J-4 - Idealized Model Including Local Effects Along the Fastener Length and Through the Plate's Thickness



$$\begin{array}{c} \textcircled{\alpha/\alpha'} \\ \vdots \\ \alpha/\alpha' \\ \vdots \\ \beta \end{array}$$

Basic Policy/Comments

Typical Dimensions

PART DATA				PARTS DATA				MEASURED INTERFERENCES				LOAD-DEFLECTION DATA						TEST INFORMATION		TESTING DATA											
Part No.	Part Name	Part Size	Part Material	Part No.	Part Name	Part Size	Part Material	Part No.	Part Name	Part Size	Part Material	Part No.	Part Name	Part Size	Part Material	Part No.	Part Name	Part Size	Part Material	Part No.	Part Name	Part Size	Part Material	Part No.	Part Name	Part Size	Part Material	Part No.	Part Name	Part Size	Part Material
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
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55	55	55	55	55	55	55	55	55	55	55	55	55	55																		

THE EFFECT OF "SPRING CONSTANTS" - REDUCTION OF EXPERIMENTAL LEAD-DISTRIBUTION CURVES

[illegible]

PAPERMASTER "SIPPING CONSTANTS" - REDUCTION OF EXPANDZITAL LOAD-DEFLECTION CURVES

[illegible]

[illegible][illegible]

PARTIAL "SAVINGS" - REDUCTION OF EXPERIMENTAL LOAD-DEFLECTION CURVES

[illegible]

FASTENER "SPRING CONSTANTS" - REDUCTION OF EXPERIMENTAL LOAD-DEFLECTION CURVES

PLATE DATA										FASTENER DATA				MEASURED INTERFERENCE				LOAD-DEFLECTION				TEST INFORMATION				IDENTIFICATION			
Material	E	t ₁	t ₂	t ₃	c	y	Material	F	Dis	Head Type	Tor-que	No. Fast	(-ve)	(+ve)	pos.	k _s	Δ ₀	F ₁	F ₂	A	B	Gage L _g	Type	Ref.	Date	Code	Test No.		
7050 Ti-6Al-4V	15	.062	.063	.063	.250	3/8	Titanium	15	.117	OSE	"	1	"	"	"	107000	1160	1140	1	12.59	"	"	AUTO	31	5/8/68	REL-1000-1	REL-1000-1	REL-1000-1	REL-1000-1
																87000	1120	1200	"	23.28	"	"	"	"	"	"	"	"	"
																74400	1008	1009	"	"	"	"	"	"	"	"	"	"	"
																89200	1032	1126	"	11.20	"	"	"	"	"	"	"	"	"
																79000	1080	1150	"	14.59	"	"	"	"	"	"	"	"	"
																50800	900	960	"	14.20	"	"	"	"	"	"	"	"	"
																51600	864	954	"	9.25	"	"	"	"	"	"	"	"	"
																45600	694	978	"	10.20	"	"	"	"	"	"	"	"	"
																126000	1680	1800	"	13.28	"	"	"	"	"	"	"	"	"
																141700	1800	1620	"	18.78	"	"	"	"	"	"	"	"	"
																130000	1640	1800	"	2.84	"	"	"	"	"	"	"	"	"
																121900	1670	1800	"	12.22	"	"	"	"	"	"	"	"	"
																144000	1640	1755	"	13.28	"	"	"	"	"	"	"	"	"
																144500	1650	1670	"	16.01	"	"	"	"	"	"	"	"	"
																91000	1350	1500	"	8.70	"	"	"	"	"	"	"	"	"
																75000	1575	1692	"	12.40	"	"	"	"	"	"	"	"	"
																84000	1420	1660	"	6.92	"	"	"	"	"	"	"	"	"
																88000	1245	1340	"	12.16	"	"	"	"	"	"	"	"	"
																87000	1230	1330	"	11.12	"	"	"	"	"	"	"	"	"
																97500	1215	1370	"	11.05	"	"	"	"	"	"	"	"	"
																135000	2490	2670	"	13.13	"	"	"	"	"	"	"	"	"
																168000	2550	2755	"	12.43	"	"	"	"	"	"	"	"	"
																130900	2360	2750	"	11.81	"	"	"	"	"	"	"	"	"
																171000	2385	2610	"	10.16	"	"	"	"	"	"	"	"	"
																138000	2205	2540	"	6.98	"	"	"	"	"	"	"	"	"
																96000	1970	2100	"	8.70	"	"	"	"	"	"	"	"	"
																102000	1890	2100	"	8.70	"	"	"	"	"	"	"	"	"
Ti-6Al-4V	15	"	"	"	"	"	Titanium	15	"	"	"	"	"	"	"	171000	1910	2130	"	6.70	"	"	"	31	"	"	"	"	

PAPADOPOULOS "SPRING CONSTANTS" - REDUCTION OF SUPERMERITAL LOAD-DEFLECTION CURVES

PLATE DATA				PASTERS: IN DATA				MEASURED IMPROVEMENT				LOAD-DEFLECTION RUNNERS				TEST		IDENTIFICATION					
Material	E, psi	t ₁ , in.	t ₂ , in.	c, in.	W, in.	Material	Z, in.	Dis Head (in) Type	Tor. No. (rev) (in) 10-10	Top. No. (rev) (in) 10-10	Zero (rev) (in) 10-10	E ₀ , lb/in	Δ ₀ , in.	Δ ₁ , lb.	Δ ₂ , lb.	A, in.	B, in.	Cage Type	Ref.	Date	Code	Test No.	
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10	31	6/28	10-10	1
T1-6A-4V	15	.061	.055	.055	.055	T1-6A-4V	.055	T1-6A-4V	0	0	0	0.000	.000	.000	.000	.000	.000	.000	10-10				

PASZKIEWICZ "STRUNG CONSTANTS" - REDUCTION OF EXPERIMENTAL LOAD-DEFORMATION CURVES

PLATE DATA							PASTER DATA				MEASURED INTERFERENCE					LOAD-REFLECTION PARAMETERS					TEST INFORMATION		IDENTIFICATION					
Material	E, psi	ν	t_1	t_2	t_3	e	$\frac{h}{Z}$	Material	E, psi	Head (D)	Type	Tr. No. Que	Fast. No.	(-val) (n)	(-val) (n)	(-val) (n)	λ_0	δ_D	P_1	P_2	A	B	Age	Type	Ref.	Date	Code	Test No.
	psi		in.	in.	in.	in.			psi	in.		in-lb		in.	in.	in.	lb/in	in.	lb.	lb.	in.		in.					
Ti-6Al-4V	15	.32	.125	.125	.125	.008		Titanium	15	.375	CH	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
																	300000	5860	6660	1005	7.16	4	AUTC	31	8/68		2	
																	300000	5860	6660		7.16							3
																	800000	19500	20500		8.32							7/8/68
																	800000	19500	21000		8.36							2
																	700000	20400	21500		6.01							3
																	700000	17500	18000		7.09							
																	800000	19500	17000		8.53							
																	870000	19500	18000		9.34							
																	100000	2050	235		7.01							
																	80000	1680	1875		8.34							
																	72700	1760	1950		8.04							
																	80000	1650	1910		6.26							
																	53500	881	880		4.90							
																	65000	850	902		13.92							
																	98000	786	908		6.77							
																	45600	822	888		11.86							
																	32400	500	742		4.61							
																	31500	654	762		10.11							
																	80000	1275	1360		2.99							
																	111000	1270	1350		15.00							
																	60000	1230	1330		12.9							
																	165000	2100	2190		21.89							
																	108000	1800	1950		11.45							
																	117000	1200	1600		3.13							
																	97000	1950	1790		5.18							
																	97500	1095	1185		11.50							
																	108000	1080	1200		9.53							
																	110000	1050	1200		8.70							
																	90000	1110	1117		17.41							
Ti-6Al-4V	15	.32	.125	.125	.125	.008		Titanium	15																			31

FASSTER "SPRING CONSTANTS" - RELATIONSHIP OF EXPERIMENTAL LOAD-DEFLECTION CURVES

PLATE DATA							FASTENER DATA				MEASURED INTERFERENCES				LOAD-DEFLECTION PARAMETERS						TEST INFORMATION		IDENTIFICATION				
Material	E x10 ⁶ psi	t ₁ in.	t ₂ in.	t ₃ in.	e in.	v %	Material	E x10 ⁶ psi	Dis (N) in.	Head (N) in.	For- que in-in	For- que in-in	For- que in-in	(in) zero	(in) top	(in) zero	(in) top	k _c	δ ₀	δ ₁	δ ₂	A	B	Cage Length in.	Type Ref.	Test Code	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
75S-T6		0.188	0.188	MOORE	3/4	1 1/8	BS-A7-12		373	CHE		1	0			175,000	5500	6350	.005	7.89		4	HEAD	30			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	159,700	5300	5700	"	12.78	"	"	"	"			
"		0.102	0.102	"	2	1	BS-A8-6		188	PROD		"	"	"	"	118,200	2200	2780	"	3.92	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	120,000	1600	2040	"	3.78	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	110,000	1650	2180	"	3.09	"	"	"	"			
"		0.188	0.188	"	"	"	BS-A9-10		311	"	"	"	"	"	"	208,000	4950	5100	"	5.03	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	200,000	4670	5650	"	4.81	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	200,000	4800	5740	"	5.12	"	"	"	"			
"		"	"	"	"	"	BS-A8-12		372	"	"	"	"	"	"	215,300	6500	7200	"	8.05	"	"	"	"			
75S-T6		"	"	"	"	"	"		"	"	"	"	"	"	"	210,000	5770	7400	"	10.30	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	250,000	6000	6900	"	6.56	"	"	"	"			
R301-T6		0.040	0.040	MOORE	3/8	9/16	BS-A8-5		375	PROD		1	0			37,000	750	830	.005	9.04		4	HEAD	30			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	53,000	700	780	"	8.47	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	44,450	730	845	"	6.86	"	"	"	"			
"		.091	.091	"	"	"	"		"	"	"	"	"	"	"	55,000	940	1060	"	7.61	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	49,000	1100	1200	"	10.53	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	54,000	1100	1120	"	8.02	"	"	"	"			
"		.063	.063	"	"	"	"		"	"	"	"	"	"	"	62,000	1280	1390	"	11.11	"	"	"	"			
R301-T5		"	"	"	"	"	"		"	"	"	"	"	"	"	48,000	1180	1320	"	9.46	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	60,000	1270	1380	"	11.01	"	"	"	"			
75S-T6		.102	.102	MOORE	2	1	BS-A8-6		188	PROD		1	0			80,000	2120	2140	.005	9.28		4	HEAD	30			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	90,000	1940	2190	"	8.26	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	88,000	1840	2100	"	6.93	"	"	"	"			
"		.188	.188	"	"	"	BS-A8-10		311	"	"	"	"	"	"	148,000	4520	5550	"	7.60	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	200,000	4540	5300	"	5.98	"	"	"	"			
"		"	"	"	"	"	"		"	"	"	"	"	"	"	190,000	4800	5600	"	5.97	"	"	"	"			
75S-T6		.125	.125	MOORE	2	1	BS-A8-8		25	PROD		1	0			152,000	3120	3560	.005	6.95		4	HEAD	30			

PAPER: "SPRING CONSTANT" - REDUCTION OF EXPERIMENTAL LOAD-DEFLECTION CURVE

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<p>This report presents analytical techniques for predicting both the linear and nonlinear stresses and deformations of mechanically fastened joints. The idealization used is a set of stacked parallel plates which transfer planar loads among themselves by means of transverse fasteners. The plates are treated by finite element methods of matrix structural analysis in which each element is assumed to be in plane stress for both elastic and plastic stress states. The fasteners, which are treated by short-beam theory, interact with the plates under the assumption that the plates may be represented by an equivalent elastic foundation.</p> <p>Application of the present analytical techniques was made to a variety of problems including: the combined elastic-plastic behavior of plates with unloaded holes, the load-deflection behavior of single-fastener joints, the residual stress distributions in plates with squeeze rivets, the effect of fastener bending and shear deformation on the bearing stress distribution between the fastener and the plate, and the prediction of the fatigue life of typical mechanically fastened joints. In all these cases, comparisons with test results generally gave very good correlation.</p> <p>Parametric studies were performed to determine the effects on stress and deflection distributions of variables such as: initial clearance or interference between fastener and hole, load level, geometry and material properties. The effect of these variables upon fatigue life of single and multi-fastener joints under realistic loadings was also assessed.</p> <p>For the range of parameters studied, the effects of hole clearance and fastener interference and geometric configuration appear to play the dominant roles in determining the stress distribution and hence, the fatigue life of mechanically fastened joints.</p>		

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